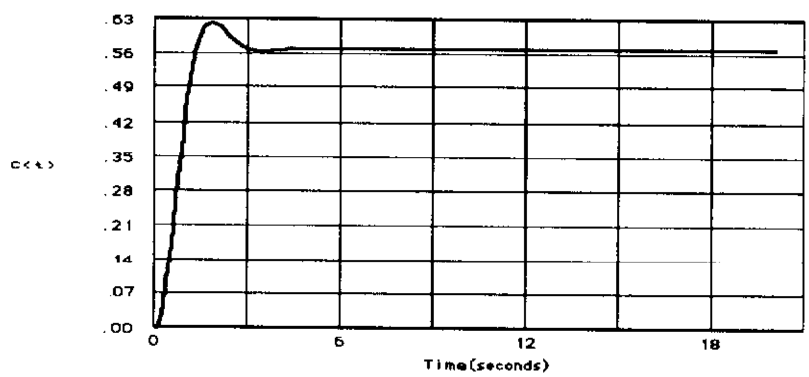
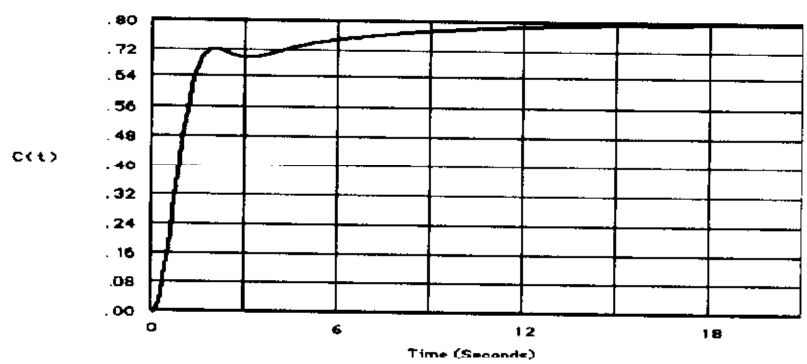
2. **a.** Searching along the 126.16° line (10% overshoot, $\zeta = 0.59$), find the operating point at -1.4+j1.92 with K = 20. Hence, $K_p = \frac{20}{1 \times 5 \times 3} = 1.333$.

b. A 3x improvement will yield $K_p = 4$. Use a lag compensator, $G_c(s) = \frac{s+0.3}{s+0.1}$.

c.



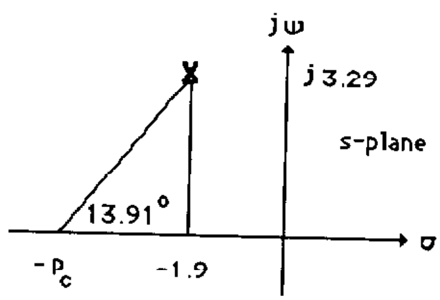
Uncompensated



Lag Compensated

13. Lead compensator design: Searching along the 120° line ($\zeta = 0.5$), find the operating point at $-1.531+j2.652$ with K = 354.49. Thus, $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{1.531} = 2.61$ seconds. For the settling time to decrease by 0.5 second, $T_s = 2.11$ seconds, or $Re = -\zeta\omega_n = -\frac{4}{2.11} = -1.9$. The imaginary part is $-1.9 \tan 60° = 3.29$. Hence, the compensated dominant poles are $-1.9 \pm j3.29$.

The compensator zero is at -5. Using the uncompensated system's poles along with the compensator zero, the summation of angles to the design point, $-1.9 \pm j3.29$ is -166.09°. Thus, the contribution of the compensator pole must be 166.09°-180° = -13.91°. Using the following geometry, $\frac{3.29}{p_c - 1.9} = \tan 13.91°$, or $p_c = 15.18$.



Adding the compensator pole and using $-1.9 \pm j3.29$ as the test point, K = 1416.63.

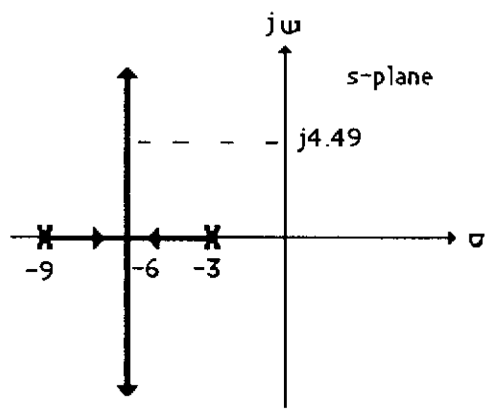
Computer simulations yield the following: Uncompensated: $T_s = 3$ seconds, %OS = 14.6%. Compensated: $T_s = 2.3$ seconds, %OS = 15.3%.

Lag compensator design: The lead compensated open-loop transfer function is $G_{LC}(s) = \frac{1416.63(s+5)}{(s+2)(s+4)(s+6)(s+8)(s+15.18)}$. The uncompensated K_p = 354.49/(2x4x6x8) = 0.923. Hence, the uncompensated steady-state error is $\frac{1}{1+K_p}$ = 0.52. Since we want 30 times improvement, the lag-lead compensated system must have a steady-state error of 0.52/30 = 0.017. The lead compensated K_p = 1416.63*5/(2*4*6*8*15.18) = 1.215. Hence, the lead-compensated error is $\frac{1}{1+K_p}$ = 0.451. Thus, the lag compensator must improve the lead-compensated error by 0.451/0.017 = 26.529 times. Thus 0.451/ ($\frac{1}{1+K_{pllc}}$) = 26.529, where K_{pllc} = 58.824 is the lead-lag compensated system's position constant. Thus, the improvement in K_p from the lead to the lag-lead compensated system is 58.824/1.215 = 48.415. Use a lag compensator, whose zero is 48.415 times farther than its pole, or $G_{lag} = \frac{(s+0.048415)}{(s+0.001)}$. Thus, the lead-lag compensated open-loop transfer function is $G_{LLC}(s) = \frac{1416.63(s+5)(s+0.048415)}{(s+2)(s+4)(s+6)(s+8)(s+15.18)(s+0.001)}$.

20. a. Since %OS = 1.5%, $\zeta = \frac{-\ln(\frac{\%OS}{100})}{\sqrt{\pi^2 + \ln^2(\frac{\%OS}{100})}} = 0.8$. Since $T_s = \frac{4}{\zeta\omega_n} = \frac{2}{3}$ second, $\omega_n = 7.49$ rad/s. Hence,

the location of the closed-loop poles must be $-6\pm j4.49$. The summation of angles from open-loop poles to $-6\pm j4.49$ is -226.3^o. Therefore, the design point is not on the root locus.

b. A compensator whose angular contribution is $226.3^o - 180^o = 46.3^o$ is required. Assume a compensator zero at -5 canceling the pole. Thus, the breakaway from the real axis will be at the required -6 if the compensator pole is at -9 as shown below.



Adding the compensator pole and zero to the system poles, the gain at the design point is found to be 29.16.

Summarizing the results: $G_c(s) = \frac{s+5}{s+9}$ with K = 29.16.

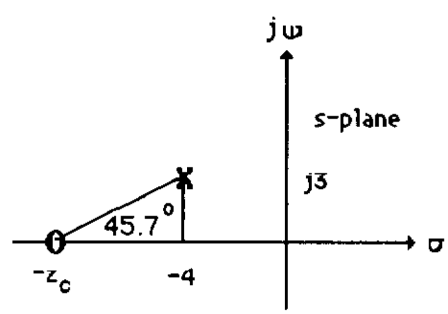
21. Since $T_p = 1.047$, the imaginary part of the compensated closed-loop poles will be $\frac{\pi}{1.047} = 3$.

Since $\frac{Im}{Re} = \tan(\cos^{-1}\zeta)$, the magnitude of the real part will be $\frac{Im}{\tan(\cos^{-1}\zeta)} = 4$. Hence, the design point is

$-4+j3$.

Assume an PI controller, $G_c(s) = \frac{s+0.1}{s}$, to reduce the steady-state error to zero.

Using the system's poles and the pole and zero of the ideal integral compensator, the summation of angles to the design point is $-225.7°$. Hence, the ideal derivative compensator must contribute $225.7°-180° = 45.7°$. Using the geometry below, $z_c = 6.93$. The PID controller is thus $\frac{(s+6.93)(s+0.1)}{s}$. Using all poles and zeros of the system



and PID controller, the gain at the design point is $K = 3.08$. Searching the real axis segment, a higher-order poles is found at -0.085. A simulation of the system shows the requirements are met.