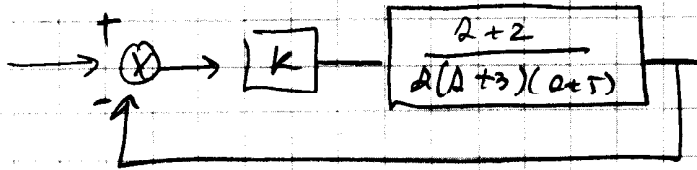


QUIZ 9 - KEY

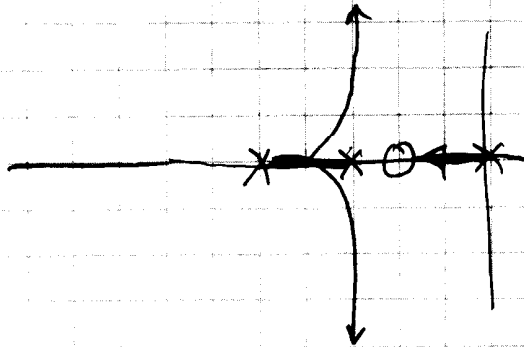


- a) i) $F_z @ -2$ 2 Infinite Zeros
 ii) $F_p @ 0, -3, -5$ 0 Infinite Poles
 iii)

$$\sigma_0 = \frac{(0 - 3 - 5) - (-2)}{3 - 1} = -3$$

$$\theta = \frac{(2k+1)\pi}{3-1} = 90^\circ, 270^\circ$$

Sketch:



b) i) $\frac{1}{s+2} = \frac{1}{s} + \frac{1}{s+3} + \frac{1}{s+5}$ or $\frac{1}{s+2} = \frac{(s+3)(s+5) + 0(s+5) + 0(s+3)}{s(s+3)(s+5)}$

$$\Rightarrow (s+2)[s^2 + 8s + 15 + s^2 + 5s + 0 + s^2 + 3s] = s[s^2 + 8s + 15]$$

$$(s+2)[3s^2 + 16s + 15] = s^3 + 8s^2 + 15s$$

$$3s^3 + 22s^2 + 47s + 30 = s^3 + 8s^2 + 15s$$

$$2s^3 + 14s^2 + 32s + 30 = 0$$

$$s = -3.86, -1.56 \pm j1.19$$

Breakaway must be -3.86

b-ii Routh table:

$$\frac{G}{1+GH} = \frac{K(s+2)}{s(s+3)(s+5) + (s+2)K} = \frac{s+2}{s^3 + 8s^2 + (15+K)s + 2K}$$

s^3	1	15+K
s^2	8	2K
s^1	$\frac{3}{4}(20+K)$	
s^0	2K	

So $K = -20$ (Not possible) or $K = 0$

When $K=0$, we have $s^3 + 8s^2 + 15s = 0$
or $s = 0$

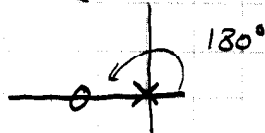
This is the open loop pole we already knew about

b-iii angle of departure/arrival

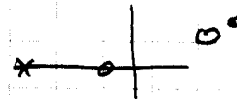
- pole @ $s = 0$ 180°
- $s = -3$ 180°
- $s = -5$ 0°
- $s = -2$ 0°

Example - Zero at $s = -2$

angle from $s = 0$



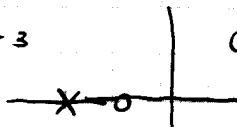
angle from $s = -5$



angle from

$s = -3$

0°



$$180 + \sum \text{Z's fp} = 360^\circ = 0^\circ$$