[25 Points, a-g 3 points each, h 4 points] 1. Simplify each of the following expressions to minimum SOP form.

a. \( f(a, b, c, d) = (b' + c' + d)(a' + b' + c')(a + b + c)(b + c + d) \)

b. \( f(a, b, c) = x'y'z + yz \)

c. \( f(w, x, y, z) = w'x'y' + w'xz' + (x + y + w')z(x' + z' + wy') \)

d. \( f(w, x, y, z) = wxy' + (w'y' = x) + (y \oplus wz). \) Recall \( a \oplus b = a'b + ab' \), and \( a \equiv b = (a \oplus b)' \)

e. \( f(u, v, w, x, y, z) = (v' + u + w)(wx + y + uz') + (wx + uz' + y) \)

f. \( f(w, x, y, z) = (w + x')(y + z')(w + y)(x + y)(w + z)(x + z) \)

g. Use QM to simplify 1b. Show your work for full credit.

\[ f(a, b, c, d) = a'bc'd + a'b'd + a'cd + abd + abc. \]

h. Assume that ABCD = 0101, ABCD = 1001, ABCD = 1011 never occur.
\[
E = (b' + c' + d) \cdot (b' + c' + a') \cdot (b + c + a) \cdot (b + c + d) \\
= (b' + c' + a + d) \cdot (b + c + d) = b'c + b'c'd + c'b + c'd + c'ad + d + a'dc + c'da
\]

\[
\text{ECE 273 Midterm Exam}
\]
[26 Points] 2. A combinational network has four inputs (A, B, C, and D), which represent a binary coded decimal digit. The network has two groups of outputs – S, T, U, V, and W, X, Y, Z. Each group represents a BCD digit. The output digits represent a decimal number which is 5 times the input number. For example, if ABCD = 0111, the outputs are 0011 0101. Assume that invalid BCD digits do not occur as inputs (this assumption ought to be reflected in the truth table).

[10] (a) Construct the truth table.

This is problem 5.27 from the homework.

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(b) From inspection:

- S = 0'
- T = A
- U = B
- V = C
- W = 0'
- X = D
- Y = D'

(c) (can do Kmap instead)
[24 Points] 3. Complete the following conversion problems.
   a. Write $wxy' + w'x'z + wy'z + w'yz'$ in minimal POS form.
   b. Given $f'(a, b, c, d) = \prod M(0, 1, 2, 3, 7, 9, 15)$, find the minterm expansion for $f$ (algebraic form)
   c. Given $f(a, b, c, d) = \sum m(0, 1, 4, 5, 10, 12) + \sum d(1, 2, 3, 4)$, write $f$ in minimum POS form.
   d. Given $f(a, b, c, d) = \prod M(0, 1, 2, 3, 7, 9, 15)$, write $f$ in minimum SOP form.
   e. Draw $wxy' + w'x'z + wy'z'$ using only NAND gates.
   f. Draw $wxy' + w'x'z + wy'z'$ using only NOR gates.
   g. Convert $f(a, b, c, d) = \sum m(0, 1, 4, 5, 10, 12) + \sum d(1, 2, 3, 4)$ to minimum SOP form.
   h. Convert $f$ into POS form.
(a) \[ f' = w'y + xyz' + w'x'y + x'y'z' \]
\[ f = (f')' = \frac{(w' + y')(x' + y'z')(w' + x'y')(x + y'z')}{(w' + y')(x' + y'z')(w' + x'y')(x + y'z')} \]

(b) \[ f' = \bar{a}m(4, 5, 6, 9, 13, 14) \]
\[ f = \bar{a}m(0, 1, 2, 3, 7, 9, 15) \]
\[ = a'b'c'd + a'b'c'd' + a'b'cd + ab'cd' + ab'c'd' + abcd' + \]
\[ a'b'c'd + a'b'c'd + abcd' + abcd \]

(c) \[ f = \bar{a}m(0, 5, 10, 12) \]
\[ f' = bc + ad + ab'c' \]
\[ f = (b' + c')(a' + d')(a' + b + c) \]

(d) \[ f = \bar{a}m(4, 5, 6, 8, 10, 11, 12, 13, 14) \]
\[ \rightarrow bc' + bd' + a' + bd' + ab'c' \]

(e) \[ \text{This is the same as (c).} \]

(f) \[ f = b'cd' + a'c' + bc'd' \]

\[ f' = wx + wy + wx'z + w'x'z' \]
\[ f = (w'x')(w'y')(w'x'z')(w'x'z') \]
[25 Points] 4. A combinational network has four input bits A, B, C, and D. You are to design a network with an output Z (which consists of multiple bits). Z is a binary number whose value represents the number of 1's in the input. For example, if ABCD = 1001, the bits of Z represent the number 2.

(a) How many bits must Z have?

3 bits

(b) Construct a truth table for the bits of Z.

\[
\begin{array}{cccc|ccc}
A & B & C & D & Z_1 & Z_2 & Z_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 \\
\end{array}
\]

(b) \( Z_1 = abcd \)

\[
Z_2 = abc' + a'b'd + a'bd + a'b'c
\]

\[
+ a'cd + acd'
\]

(c) \( Z_3 = a'b'c'd + a'b'cd' + a'bcd' + a'bc'd + ab'cd + abc'd + a'bc'd + abcd' \)