

Introduction

Sullivan's Dictionary

Dictionary Entries or "Translation" Examples

"Language"	Complex dynamics	Hyperbolic geometry
Object acting on Riemann Sphere	Möbius transformations	Kleinian groups
Parameter Space	Mandelbrot set	Bers Slice
Object of Interest	Julia sets	Limit sets
Missing Entry?	Degeneracy of Julia sets	Degeneracy of quasi-circle limit sets

Background

Complex Dynamics

**Definition.** The filled Julia set of a degree  $d \geq 2$  polynomial  $f : \widehat{\mathbb{C}} \rightarrow \widehat{\mathbb{C}}$  is the set of points whose orbit under  $f$  is bounded.

The Mandelbrot set and Julia set examples

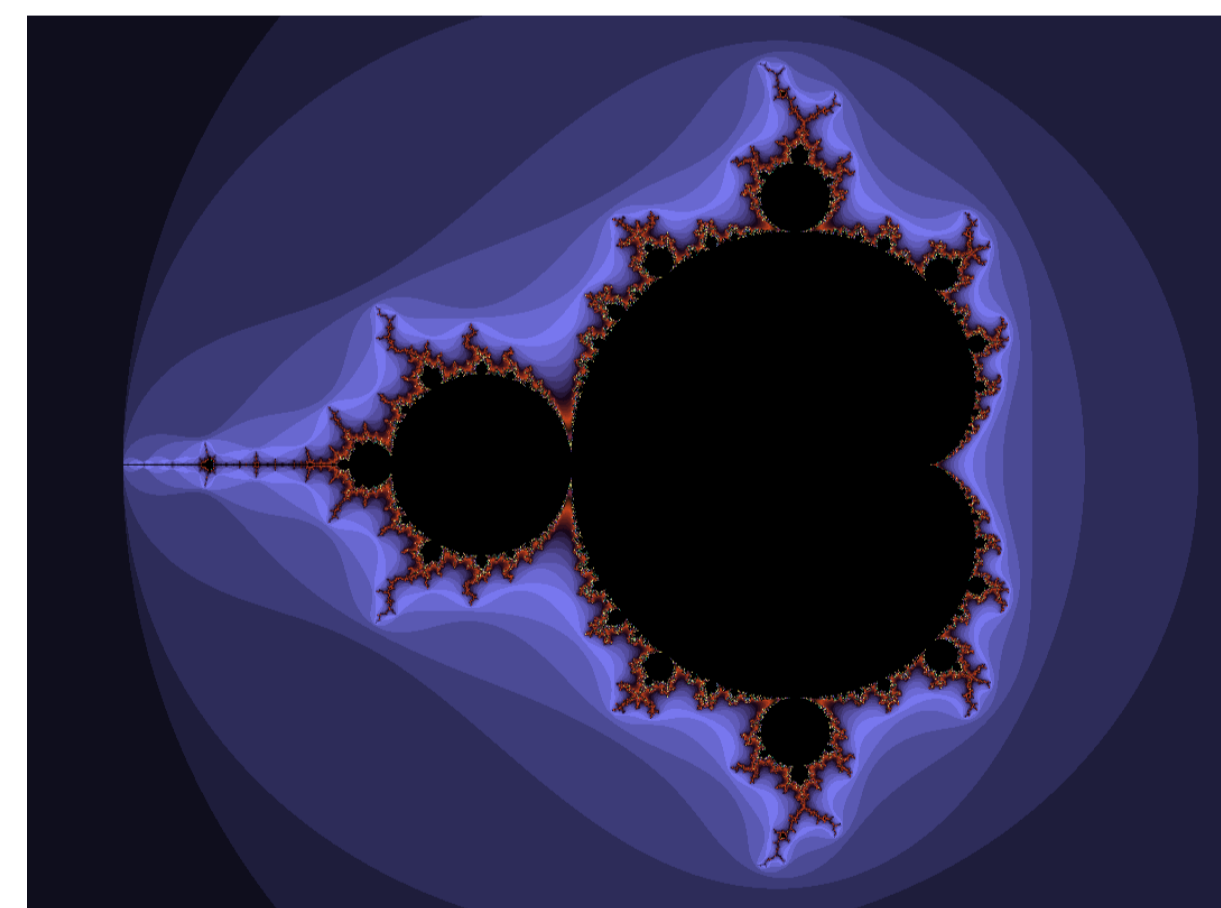


Figure 2: The Mandelbrot set:  $\{c \in \mathbb{C} \text{ s.t. } f^{(k)}(0) \text{ is bounded for } f(z) = z^2 + c\}$

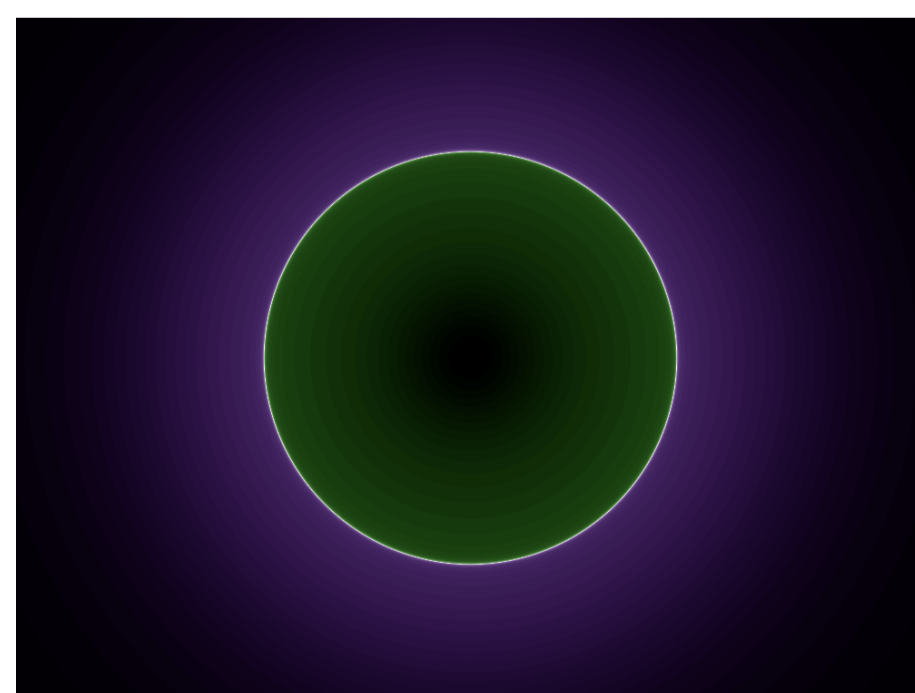


Figure 3: Julia Set of  $f(z) = z^2$

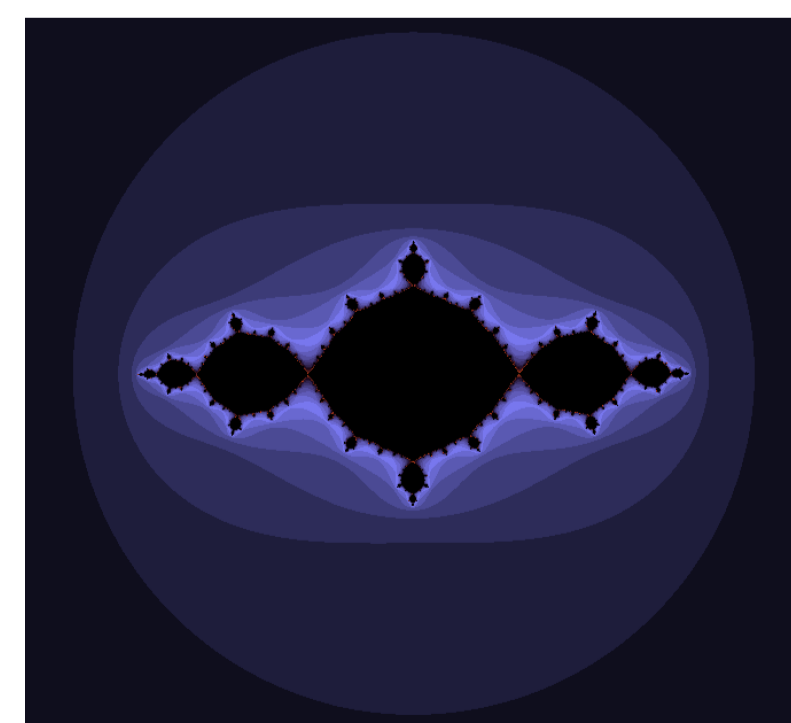


Figure 4: Julia Set of  $f(z) = z^2 - 1$

Hyperbolic Geometry

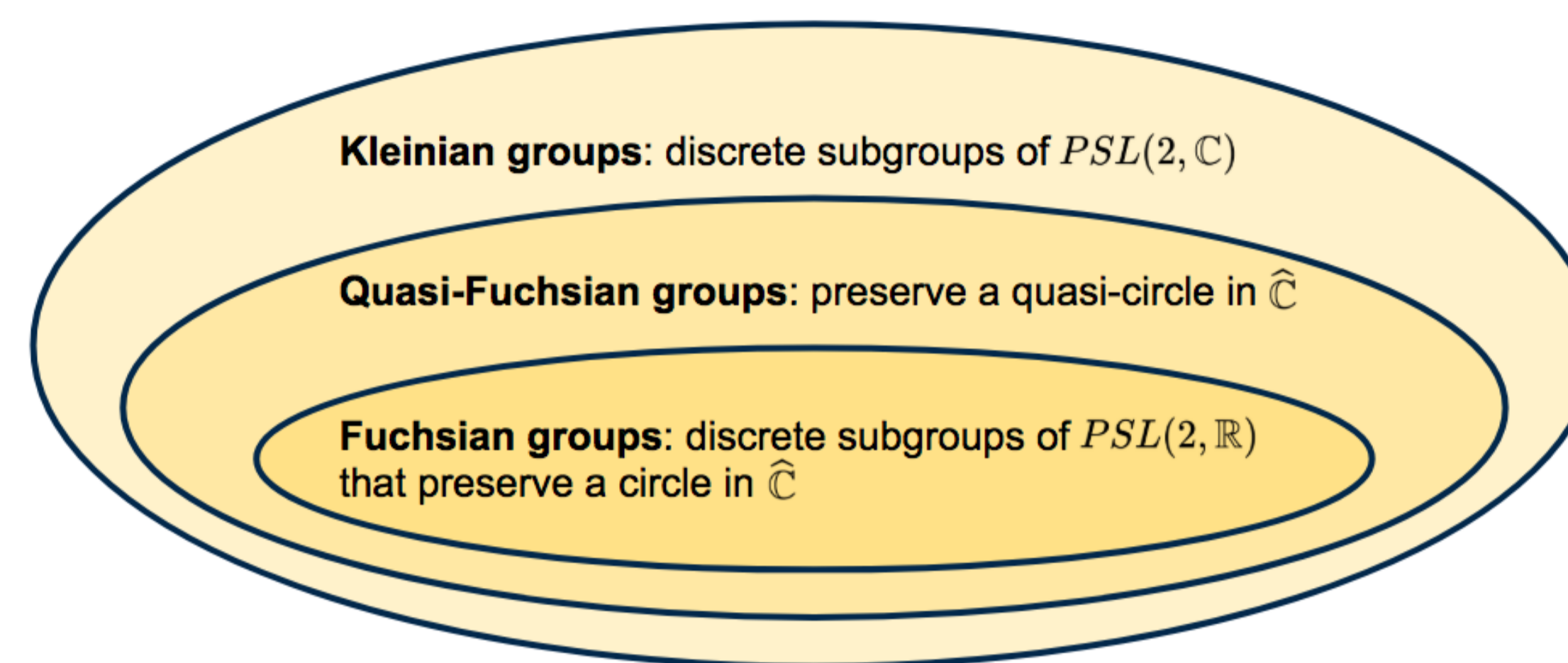


Figure 5: Groups acting on  $\widehat{\mathbb{C}}$  or, by extension, on  $\mathbb{H}^3$ .

**Definition.** The Limit Set  $\Lambda(\Gamma)$  of a Kleinian Group  $\Gamma$  is the smallest closed  $\Gamma$ -invariant subset of  $\widehat{\mathbb{C}}$ , or, similarly, the set of accumulation points of  $\Gamma x$  in  $\partial\mathbb{H}^3$  for  $x \in \mathbb{H}^3$ .

Examples of limit sets

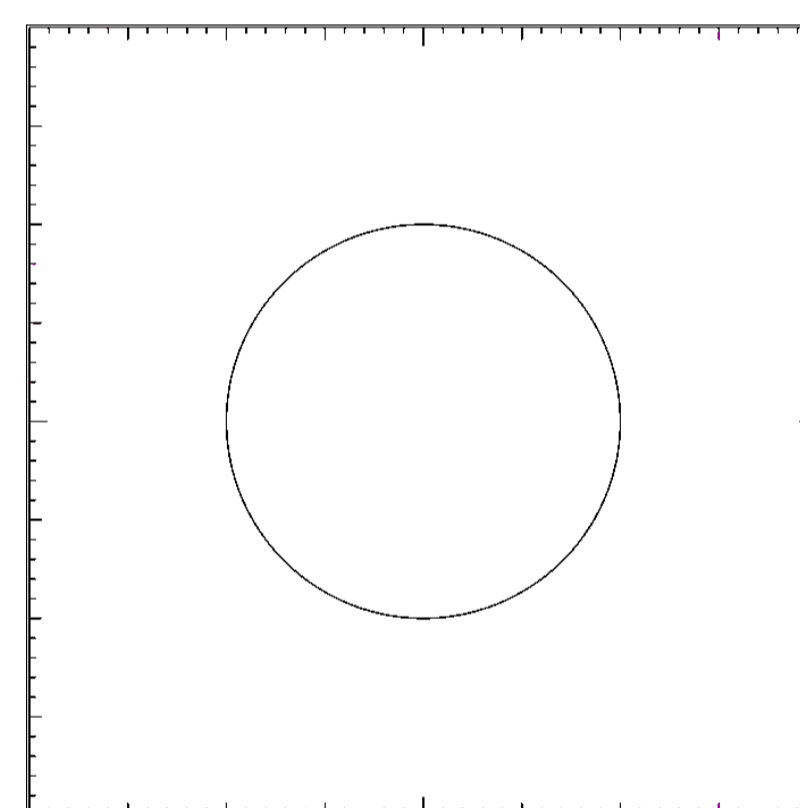


Figure 6: Circle limit set of a Fuchsian group.

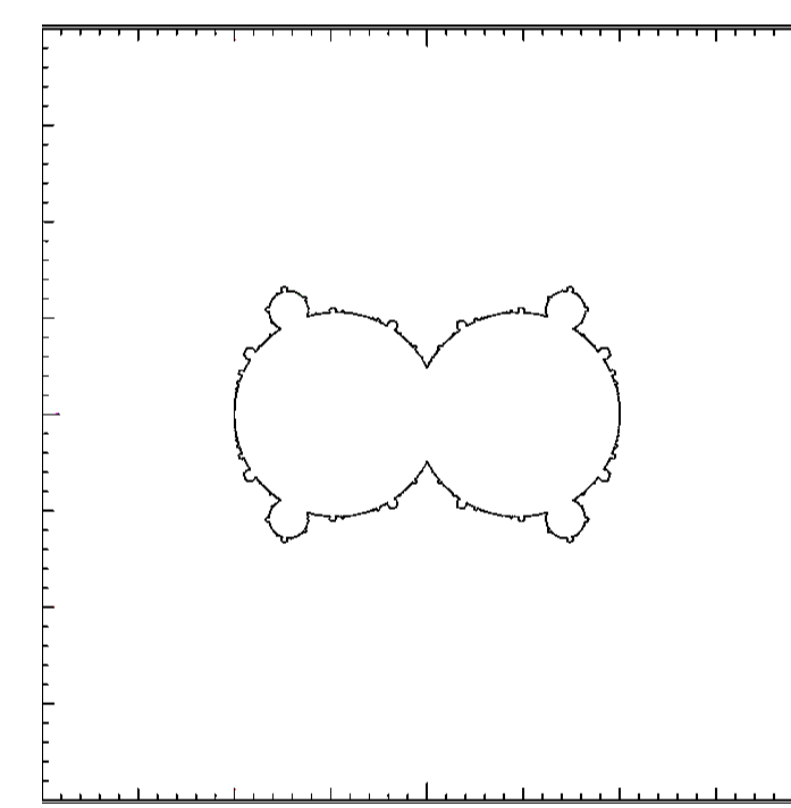


Figure 7: Quasi-circle limit set of a quasi-Fuchsian group.

A Missing Entry?

Degenerate behavior of the cauliflower Julia set



Figure 8: Julia Set of  $f(z) = z^2 + \frac{1}{4}$

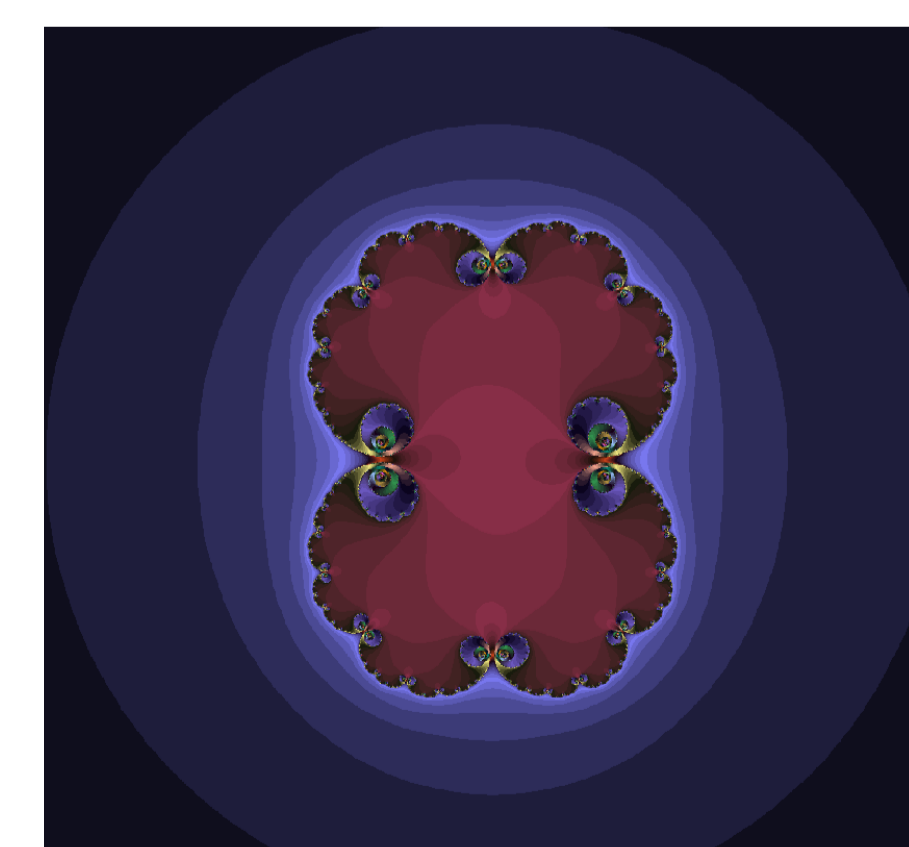


Figure 9: Julia Set of  $f(z) = z^2 + \frac{1}{4} + \epsilon$

Degenerate behavior of limit sets

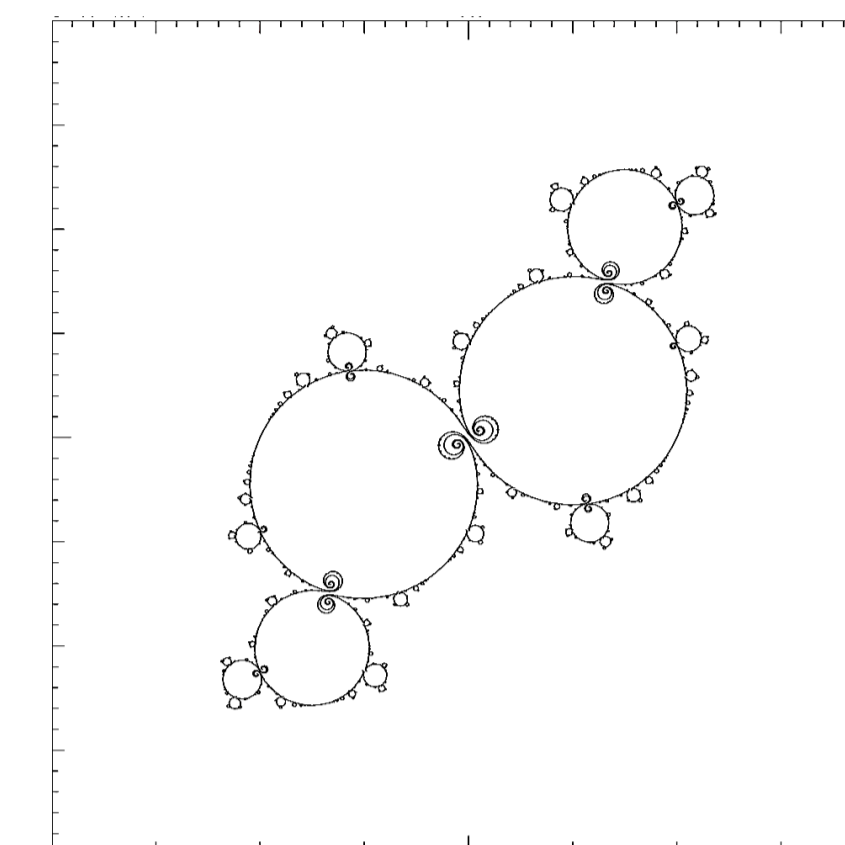


Figure 10: Quasi-circle limit set of a quasi-Fuchsian group near a cusp of the Bers slice

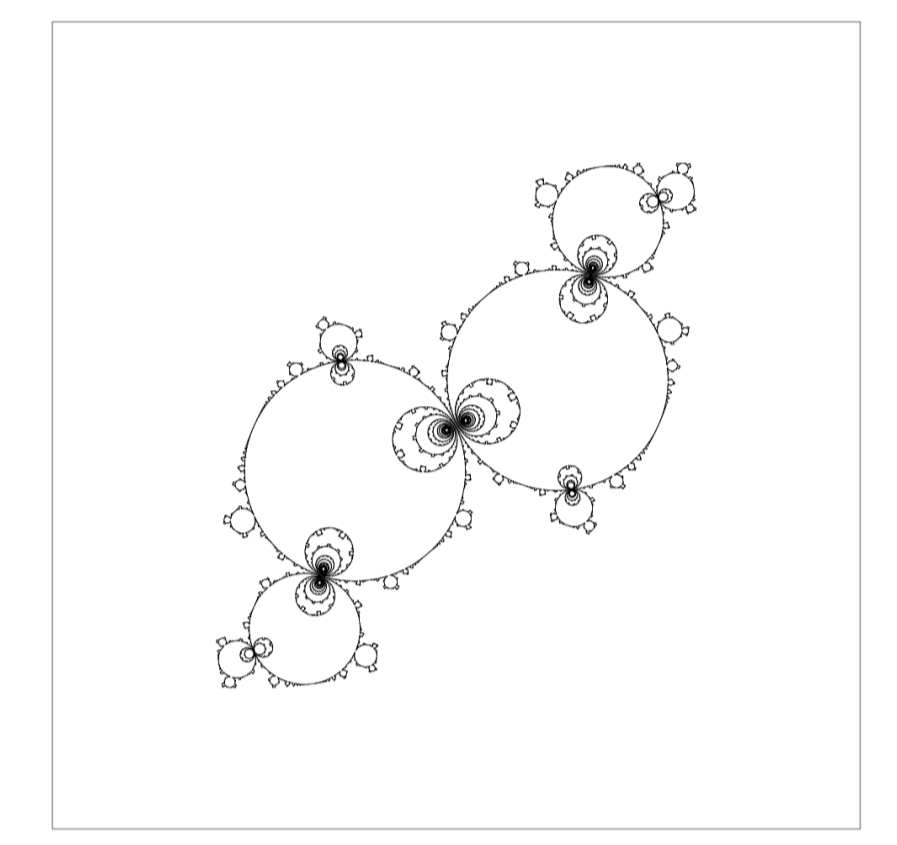


Figure 11: Quasi-circle limit set of a quasi-Fuchsian group even nearer a cusp of the Bers slice

Similar degenerate behaviors of limit sets and Julia sets

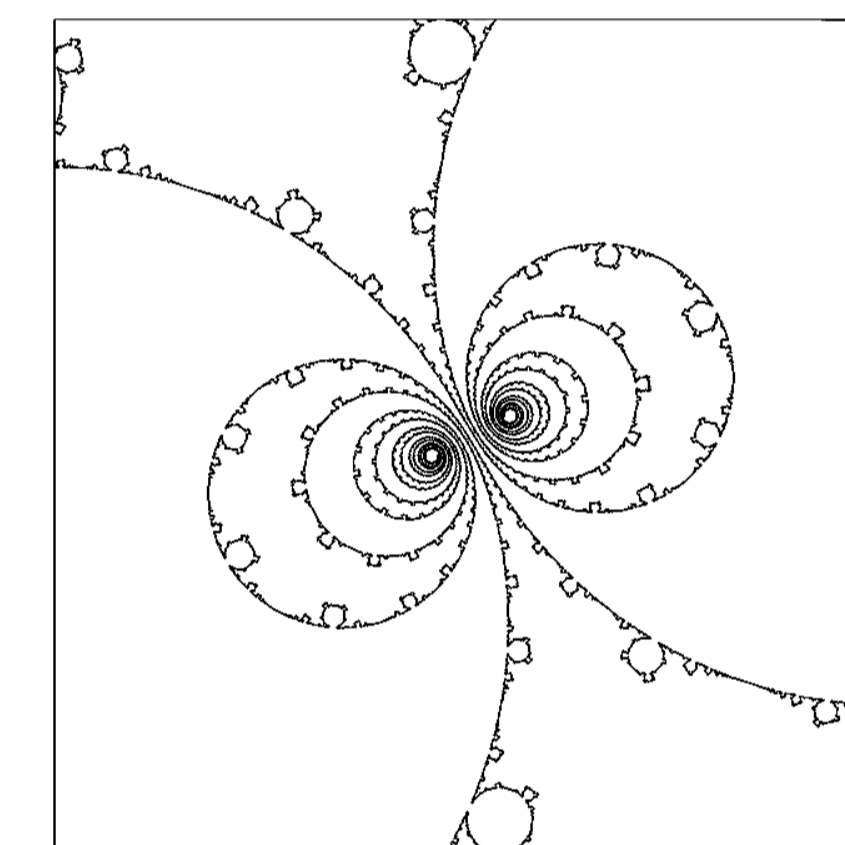


Figure 12: Part of the quasi-circle limit set in figure 11

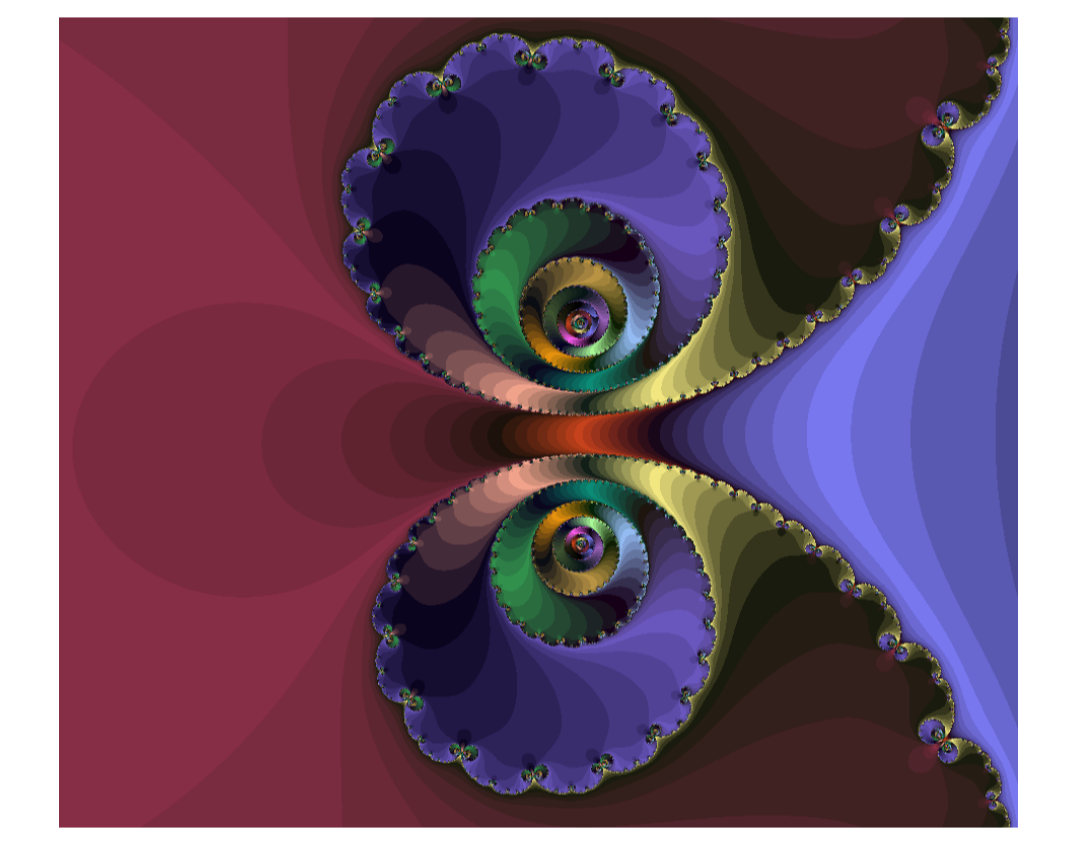


Figure 13: East 'cusp' of the Julia Set of  $f(z) = z^2 + \frac{1}{4} + \epsilon$

Future Directions

New code implementation

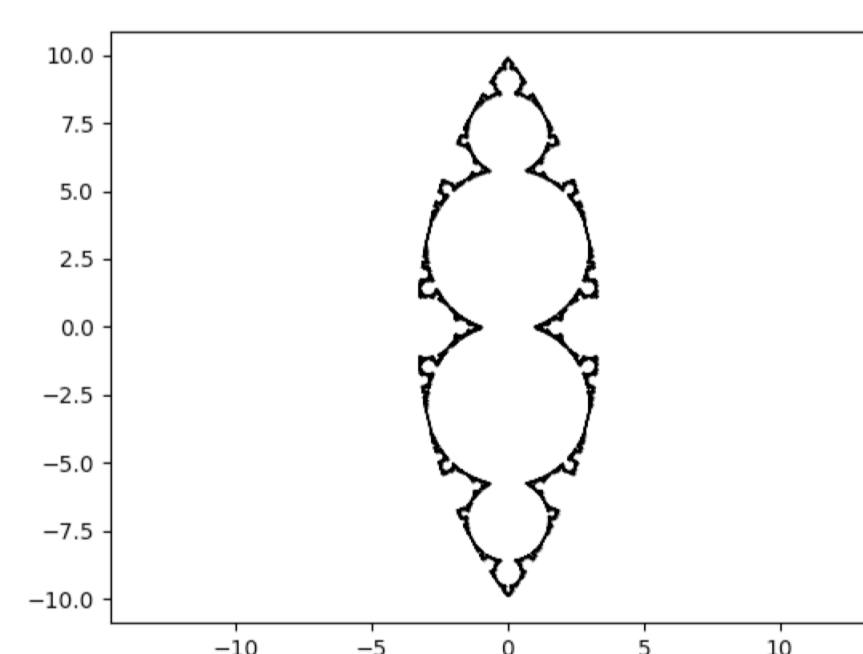


Figure 14: Sample output: quasi-circle limit set

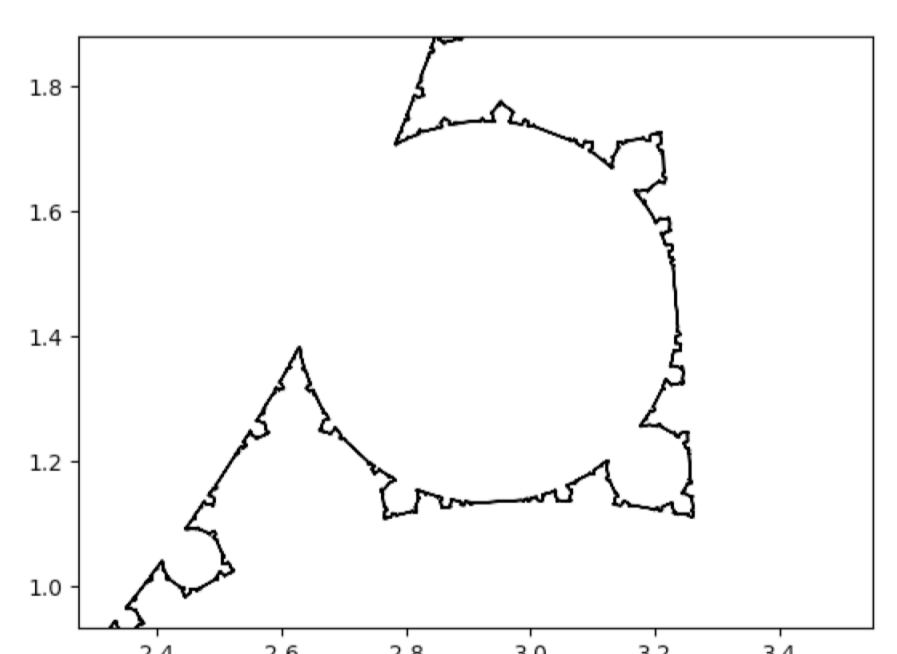


Figure 15: Blown up portion of figure 14

References

- [1] John Hubbard. *Teichmüller Theory and Applications to Geometry, Topology, and Dynamics Volume 2: Surface Homeomorphisms and Rational Functions*.
- [2] David Mumford, Caroline Series and David Wright. *Indra's Pearl: The Vision of Felix Klein*.
- [3] Albert Marden. *Hyperbolic Manifolds*.
- [4] All complex dynamics images were generated using the program XaoS, and unless stated otherwise, limit set images were generated using the program Kleinian.