1. Consider the following model:

The economy consists of households and firms. There is a continuum of firms on \([0, 1]\).

**I. Consumers**

Consumers seek to maximize their utility:

\[
E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \ln C_t + \gamma \ln \left( \frac{M_t}{P_t} \right) - \phi \frac{N_t^{1+\eta}}{1+\eta} \right) \right]
\]

subject to the flow constraints:

\[
M_t + P_tC_t = W_tN_t + M_{t-1} + T_t + \Pi_t
\]

Here \(T_t\) are monetary transfers (“helicopter drops” of money). \(\Pi_t\) are profits returned to the household from the firms.

**II. Intermediate Goods Producing Firms**

The \(i^{th}\) firm produces output according to:

\[
c_{it} = A_t n_{it}^\alpha
\]

Prices are sticky in this environment. Specifically, these monopolistic firms set price \(p_{it}\) at time \(t-1\) to maximize:

\[
E_{t-1} [p_{it}c_{it} - W_t n_{it}]
\]

Given \(p_{it}\), firms set \(n_{it}\) to meet demand at that price.

**III. Final Goods Producers:**

The final consumption good is produced from the intermediate consumption goods:

\[
C_t = \left[ \int_0^1 c_{it}^{\frac{1}{\epsilon-1}} \, di \right]^{\frac{\epsilon-1}{\epsilon}}
\]

These firms choose the \(c_{it}\)’s to maximize profits:

\[
P_tC_t - \int_0^1 p_{it}c_{it} di
\]

**IV. Shocks**

Technology \((A_t)\) and the money supply \((M_t)\) obey the following processes:

\[
A_t = A_{t-1} \exp \{ \eta_t \}, \eta_t \sim N(0, \sigma_A^2)
\]

\[
\frac{M_{t+1}}{M_t} = \exp \{ \xi_{t+1} + \theta \eta_{t+1} \}, \xi_t \sim N(0, \sigma_A^2)
\]


(a) Consider the final goods producers. Show that the demand for $c_{it}$ is:

$$c_{it} = C_t \left( \frac{p_{it}}{P_t} \right)^{-\varepsilon}$$

Write down an expression for the aggregate price level $P_t$ in terms of the $p_{it}$.

(b) Now consider the intermediate goods producers. Recall that $n_{it}$ will be set to meet demand at the price they set. Show that profit maximization by firms implies that:

$$E_{t-1} \left[ p_{it} c_{it} - \frac{\mu}{\alpha} W_t n_{it} \right] = 0$$

where $\mu = \frac{\varepsilon}{\varepsilon - 1}$. Interpret this condition.

(c) Show that the solution to the households problem implies the following two conditions:

$$\frac{1}{C_t} = \gamma \frac{P_t}{M_t} + E_t \left[ \beta \frac{1}{C_{t+1}} \frac{P_t}{P_{t+1}} \right]$$

$$\phi N^\frac{1}{\eta} = \frac{1}{C_t} \frac{W_t}{P_t}$$

Interpret these conditions.

(d) Assume a symmetric equilibrium; show that the economy satisfies a quantity equation of the form:

$$C_t = \Phi \frac{M_t}{P_t}$$

for some constant $\Phi$.

(e) How does this economy respond to a shock to technology? Interpret this result. If you were the monetary authority and you wanted to stabilize employment in response to technology shocks, how would you set the money supply growth process?
Consider the following model:

I. Households

A representative household maximizes utility given by:

\[ U = \sum_{t=0}^{\infty} \beta^t [\ln C_t + \phi \ln(1 - N_t)] \]

subject to the following nominal flow constraint:

\[ P^c_t C_t + P^I_t I_t \leq R_t K_t + W_t N_t + \Pi_t \]

\[ K_{t+1} = K_t (1 - \delta) + I_t \]

Here \( \Pi_t \) are nominal profits transferred back to the household - lump sum.

(a.) What are the first order conditions for this consumer?

(b.) Write down the Euler equation and the labor supply condition. Interpret these conditions in English.

Production of consumption and investment take place in separate industries.

II. Consumption Goods Sector

Final consumption goods are produced from a mix of intermediate consumption goods with the technology:

\[ C_t = \left[ \int_0^1 c(z) \frac{z^{e+1}}{\epsilon} dz \right]^{\frac{\epsilon}{\epsilon + 1}} \]

Assume that these final goods producers behave to maximize profits:

\[ P^c_t C_t - \int_0^1 p^c_t(z) c_t(z) dz \]

(c.) Show that the demand for any intermediate good \( z \) is given by:

\[ c_t(z) = C_t \left( \frac{p^c_t(z)}{P^c_t} \right)^{-\epsilon} \]

If there is free entry into the production of final consumption goods, what is \( P^c_t \)?

Assume that each producer of an intermediate good is a local monopolist who takes the demand for their product as given. They produce intermediate goods with a Cobb-Douglas technology:

\[ c_t(z) = A [k^c_t(z)]^\alpha [n^c_t(z)]^{1-\alpha} \]

They behave competitively in the input markets taking the nominal wage \( W_t \) and the nominal rental price \( R_t \) as given.
(d.) Suppose that one such monopolist has decided upon a level of production \( \bar{c} \) and that she wants to minimize nominal costs. What are the first order conditions for \( k^c_t(z) \) and \( n^c_t(z) \)?

(e.) Write down an expression for nominal marginal cost \( MC^c_t \) in terms of \( W_t \) and \( R_t \) (use your class notes).

The monopolist wants to choose \( p^c_t(z) \) to maximize profits

\[
p^c_t(z)c_t(z) - MC^c_t c_t(z)
\]

(note \( c_t(z) \) is a function of \( p^c_t(z) \)).

(f.) If there are no sticky prices, what is the profit maximizing choice of \( p^c_t(z) \) in terms of \( MC^c_t \)? What is the price of the consumption good \( P^c_t \)?

III. Investment Goods Sector

Production of the investment good is similar. Final investment goods are produced from a mix of intermediate investment goods via:

\[
I_t = \left[ \int_0^1 i_t(z) \frac{\psi-1}{\psi} dz \right]^{\frac{\psi}{\psi-1}}
\]

(note: \( \psi \neq \varepsilon \)) and the final goods producers behave to maximize profits:

\[
P^I_t I_t - \int_0^1 p^I_t(z)i_t(z)dz
\]

(g.) Derive the demand function for any intermediate good \( z \).

(h.) What is the aggregate price of investment?

The production of intermediates is also similar. Each monopolist has a production function

\[
i_t(z) = B \left[ k^I_t(z) \right]^\theta \left[ n^I_t(z) \right]^{1-\theta}
\]

They take their demand curves as given and maximize profits.

(i.) Use the first order conditions for cost minimization to derive an expression for nominal marginal cost \( MC_t \).

(j.) What is the profit maximizing price \( p^I_t(z) \)? What is the aggregate price of investment goods?

IV. Steady State:

(k.) What are the market clearing conditions for capital and labor?

(l.) Assume that \( \varepsilon = \psi \), \( A = B \), and \( \alpha = \theta \). Solve for the steady state for this economy.

(m.) What if these numbers are not equal? Can you solve for the steady state?