1. The first part of the assignment asks you to solve and simulate a basic neoclassical growth model. Consumers solve

\[ \max E_0 \left[ \sum_{t=0}^{\infty} \beta^t \ln C_t \right] \]

subject to:

\[ K_{t+1} = K_t (1 - \delta_t) + K_t^\alpha - C_t \]

\[ \delta_t = (1 - \rho) \bar{\delta} + \rho \delta_{t-1} + \varepsilon_t \]

where \( \varepsilon_t \) is distributed according to a log normal distribution.

(a) Solve for the steady state equilibrium for this economy.

(b) The dynamics of the system will be governed by three equations (these will be the Euler equation, the capital accumulation equation and the equation for the law of motion of depreciation). Log linearize these equations around the steady state.

(c) Write a MATLAB program to solve the system. Construct \( B_1 \) and \( B_2 \) so that:

\[ B_1 E_t \begin{bmatrix} \tilde{C}_{t+1} \\ \tilde{K}_{t+1} \\ \tilde{\delta}_{t+1} \end{bmatrix} = B_2 \begin{bmatrix} \tilde{C}_t \\ \tilde{K}_t \\ \tilde{\delta}_t \end{bmatrix} \]

and construct \( M = B_1^{-1}B_2 \). Use the eigenvalue/eigenvector decomposition (for \( M \)) to decouple the system. Solve for the policy function

\[ \tilde{C}_t = P \begin{bmatrix} \tilde{K}_t \\ \tilde{\delta}_t \end{bmatrix} \]

and then return the VAR solution: \( Y_{t+1} = AY_t + B\varepsilon_t \) (here \( Y_t \) is the vector \([C_t, K_t, \delta_t]\)). Use parameters \( \beta = .99, \bar{\delta} = .025, \alpha = .35 \) (these conform to a quarterly model). Set the autoregressive root on the depreciation shocks to \( \rho = .95 \).

(d) Set the standard deviation of \( \varepsilon_t \) to .01 (i.e. 1%). Write a MATLAB program to find the variance of the vector \( Y_t \). What are the volatilities of each variable?

(e) Subject the economy to a 1% shock to depreciation. Plot out \( \tilde{C}, \tilde{K}, \) and \( \tilde{\delta} \).

(f) Start the economy off at \( \tilde{K}_0 = -.5 \) (50% below steady state). Be careful to start \( C_0 \) at the appropriate level – use the policy function you solved for before. Simulate the convergence path of the economy.

(g) Simulate the exact convergence path. To do this try the following algorithm: Guess \( C_0 \), this implies \( K_1 \) and the future marginal product of capital \((\alpha K_1^\alpha - 1)\). Solve for the implied value of \( C_1 \) and iterate into the future. The path you get will “explode” either up or down. As soon as \( C \) goes above the steady state \( C^* \) or starts to decline, reset your guess (either up or down). Iterate this to convergence. Compare the actual path to the linear approximation.
2. Consider the following version of Hansen’s indivisible labor model:

Consumers maximize:

$$\max E_0 \left[ \sum_{t=0}^{\infty} \beta^t (\ln C_t - BN_t) \right]$$

subject to:

$$K_{t+1} = K_t (1 - \delta_t) + I_t$$

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha}$$

$$A_t = (1 - \rho) + \rho A_{t-1} + \varepsilon_t$$

(a) Solve for the steady state values of $N, K, Y, C, w$, and $r$, where $w$ and $r$ are the real wage and the real rental price of capital given by:

$$w_t = (1 - \alpha) A_t K_t^\alpha N_t^{-\alpha}$$

$$r_t = \alpha A_t K_t^{\alpha - 1} N_t^{1-\alpha}$$

(b) Log linearize the equations describing the dynamics of the system.

(c) Let’s declare $K_t, A_t$ as the state variables and $C_t$ as the co-state. The remaining variables, $Y, I, w, N$ and $r$ are “redundant” variables. Write the system as

$$AE_t [Y_{t+1}] = BY_t$$

using:

$$Y_t = \begin{pmatrix} y_t \\ x_t \end{pmatrix}$$

and

$$y_t = \begin{pmatrix} \tilde{C}_t \\ \tilde{K}_t \\ \tilde{A}_t \end{pmatrix}, \quad x_t = \begin{pmatrix} \tilde{r}_t \\ \tilde{w}_t \\ \tilde{N}_t \\ \tilde{Y}_t \\ \tilde{I}_t \end{pmatrix}$$

Reduce the system by solving for $x_t = F y_t$. Write the reduced system in “standard form”

$$B_1 E_t [y_{t+1}] = B_2 y_t$$

and solve the system.

(d) Reconstruct the VAR form for the complete system and find the variance matrix for $Y$.

(e) Simulate the economy for 1000 periods. Use the HP filter program from last time to get the HP filtered data. Compare the simulated moments to the analytical ones. What are the main differences?