

Multi-valued Treatment Effects (1,000 words)

The term *multi-valued treatment effects* broadly refers to a collection of population parameters that capture the impact of a given treatment assigned to each observational unit, when this treatment status takes multiple values. In general, treatment levels may be finite or infinite as well as ordinal or cardinal, leading to a large collection of possible treatment effects to be studied in applications. When the treatment effect of interest is the mean outcome for each treatment level, the resulting population parameter is typically called the *Dose-Response Function* in the statistical literature, regardless of whether the treatment levels are finite or infinite. The analysis of multi-valued treatment effects has several distinct features when compared to the analysis of binary treatment effects, including: (i) a comparison or control group is not always clearly defined, (ii) new parameters of interest arise capturing distinct phenomena such as non-linearities or tipping points, (iii) in most cases correct statistical inferences require the joint estimation of all treatment effects (as opposed to the estimation of each treatment effect at a time), and (iv) efficiency gains in statistical inferences may be obtained by exploiting known restrictions among the multi-valued treatment effects.

Treatment Effect Model and Population Parameters

A general statistical treatment effect model with multi-valued treatment assignments is easily described in the context of the classical potential outcomes model. This model assumes that each unit i in a population has an underlying collection of potential outcome random variables $\{Y_i(t): t \in \mathcal{T}\}$, where \mathcal{T} denotes the collection of possible treatment assignments. The random variables $Y_i(t)$ are usually called potential outcomes because

they represent the random outcome that unit i would have under treatment regime $t \in \mathcal{T}$. For each unit i and for any two treatment levels t_1 and t_2 , it is always possible to define the *individual treatment effect* given by $Y_i(t_1) - Y_i(t_2)$, which may or may not be a degenerate random variable. However, because units are not observed under different treatment regimes simultaneously, such comparisons are not feasible. This idea, known as the *fundamental problem of causal inference*, is formalized in the model by assuming that for each unit i only (Y_i, T_i) is observed, where $Y_i = Y_i(T_i)$ and $T_i \in \mathcal{T}$. In words, for each unit i only the potential outcome for treatment level $T_i = t$ is observed while all other (counterfactual) outcomes are missing. Of course, in most applications, which treatment each unit has taken up is not random and hence further assumptions would be needed to identify the treatment effect of interest.

A binary treatment effect model has $\mathcal{T} = \{0,1\}$, a finite multi-valued treatment effect model has $\mathcal{T} = \{0,1, \dots, J\}$ for some positive integer J , and a continuous treatment effect model has $\mathcal{T} = [0,1]$. (Note that the values in \mathcal{T} are ordinal, that is, they may be seen just as normalizations of the underlying real treatment levels in a given application.) Many applications focus on a binary treatment effects model and base the analysis on the comparison of two groups, usually called *treatment group* ($T_i = 1$) and *control group* ($T_i = 0$). A multi-valued treatment may be collapsed into a binary treatment, but this procedure usually would imply some important loss of information in the analysis. Important phenomena such as non-linearities, differential effects across treatment levels or tipping points cannot be captured by a binary treatment effect model.

Typical examples of multi-valued treatment effects are comparisons between some characteristic of the distributions of the potential outcomes. Well-known examples

are mean and quantiles comparisons, although in many applications other features of these distributions may be of interest. For example, assuming that the random potential outcomes are equal for all units to simplify the discussion (this holds, for instance, in the context of random sampling), the mean of the potential outcome under treatment regime $t \in \mathcal{T}$ is given by $\mu(t) = \mathbb{E}[Y_i(t)]$. The collection of these means is the so-called *Dose Response Function*. Using this estimand, it is possible to construct different multi-valued treatment effects of interest such as pair-wise comparisons (e.g., $\mu(t_2) - \mu(t_1)$) or differences in pair-wise comparisons, which would capture the idea of non-linear treatment effects. (In the particular case of binary treatment effects, the only possible pair-wise comparison is $\mu(1) - \mu(0)$, which is called the *Average Treatment Effect*.) Using the Dose Response Function, it is also possible to consider other treatment effects that arise as non-linear transformations of $\mu(t)$, such as ratios, incremental changes, tipping points, or the maximal treatment effect ($\mu^* = \max_{t \in \mathcal{T}} \mu(t)$), among many other possibilities. All these multi-valued treatment effects are constructed based on the mean of the potential outcomes, but similar estimands may be considered based on quantiles, dispersion measures or other characteristics of the underlying potential outcome distribution. Conducting valid hypothesis testing about these treatment effects requires in most cases the joint estimation of the underlying multi-valued treatment effects.

Statistical Inference

There exists a vast theoretical literature proposing and analyzing different statistical inference procedures for multi-valued treatment effects. This large literature may be characterized in terms of the key identifying assumption underlying the treatment effect

model. This key assumption usually takes the form of a (local) independence or orthogonality condition such as (i) a conditional independence assumption, which assumes that conditional on a set of observable characteristics selection into treatment is random, or (ii) an instrumental variables assumption, which assumes the existence of variables that induce exogenous changes in the treatment assignment. Using an identifying assumption (together with other standard model assumptions), it has been shown in the statistical and econometrics literatures that several parametric, semiparametric and nonparametric procedures allow for optimal joint inference in the context of multi-valued treatments. These results are typically obtained using large sample theory and justify (asymptotically) the use of classical statistical inference procedures involving multiple treatment levels.

Matias D. Cattaneo

See also Multiple Treatment Interference, Observational Study, Propensity Score Analysis, Treatment(s), Selection

Further Readings

Heckman, J.J. & Vytlacil, E.J. (2007). Econometric Evaluation of Social Programs, Part I: Causal Models, Structural Models and Econometric Policy Evaluation. In J.J. Heckman and E.E. Leamer (Eds.), *Handbook of Econometrics*, vol. 6B (pp. 4779–4874). Amsterdam, Netherlands: North-Holland.

Imbens, G.W. & Wooldridge, J.M. (2009). Recent Developments in the Econometrics of

Program Evaluation. *Journal of Economic Literature*, 47, 5–86.

Imai, K. & van Dyk, D.A. (2004). Causal Inference With General Treatment Regimes:

Generalizing the Propensity Score. *Journal of the American Statistical*

Association, 99, 854–866.

Rosembaum, P. (2002). *Observational Studies*. New York, NY: Springer.