

# **Multi-valued Treatment Effects (1,000 words)**

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## **2. Key Words**

Causal Inference, Generalized Propensity Score, Identification, Matching Estimators, Program Evaluation, Semiparametric Estimation, Semiparametric Efficiency, Treatment Effects, Unconfoundedness.

## **3. Article**

The term *multi-valued treatment effects* refers to a collection of population parameters capturing the impact of a treatment variable on an outcome variable, when the treatment takes multiple values. For example, in labor training programs participants receive different hours of training or in anti-poverty programs households receive different levels of transfers. Multi-valued treatments may be finite or infinite as well as ordinal or cardinal, and naturally extend the idea of binary treatment effects, leading to a large collection of treatment effects of interest in applications. The analysis of multi-valued treatment effects has several distinct features when compared to the analysis of binary treatment effects, including: (i) a comparison or control group is not always clearly defined, (ii) new parameters of interest arise that capture distinct phenomena such as non-linearities or tipping points, (iii) in general, correct statistical inference requires the joint

estimation of all treatment effects (as opposed to the estimation of each treatment effect separately), and (iv) efficiency gains in statistical inference may be obtained by exploiting known restrictions among the multi-valued treatment effects.

## **Treatment Effect Model and Population Parameters**

A general statistical treatment effect model with multi-valued treatment assignments is typically described in the context of the classical potential outcomes model. Heckman and Vytlačil (2007) and Imbens and Wooldridge (2009) provide recent surveys, with particular emphasis on causal inference in program evaluation. The model assumes that each unit  $i$  in a population has an underlying collection of potential outcome random variables  $\{Y_i(t): t \in \mathcal{T}\}$ , where  $\mathcal{T}$  denotes the collection of possible treatment assignments. The random variables  $Y_i(t)$  are usually called potential outcomes because they represent the random outcome that unit  $i$  would have under treatment regime  $t \in \mathcal{T}$ . Each unit is not observed under different treatment regimes simultaneously, which leads to the *fundamental problem of causal inference* (Holland (1986)). This idea is formalized in the model by assuming that for each unit  $i$  only  $(Y_i, T_i)$  is observed, where  $Y_i = Y_i(T_i)$  and  $T_i \in \mathcal{T}$ . In words, for each unit  $i$  only the potential outcome for treatment level  $T_i = t$  is observed while all other (counterfactual) outcomes are missing. Of course, in most applications, which treatment each unit has taken up is not random and hence further assumptions would be needed to identify the treatment effect of interest.

When  $\mathcal{T} = \{0,1\}$ , the model reduces to the classical binary treatment effect model. A finite multi-valued treatment effect model is given by  $\mathcal{T} = \{0,1, \dots, J\}$ , for some positive integer  $J$ , while  $\mathcal{T} = [0,1]$  leads to a continuous treatment effect model. (The

values in  $\mathcal{T}$  are ordinal, and may be interpreted as normalizations of the underlying real treatment levels in a given application.) Many applications focus on the classical binary treatment effects model, which has only two groups: *treatment group* ( $T_i = 1$ ) and *control group* ( $T_i = 0$ ). A multi-valued treatment may be collapsed into a binary treatment, which permits the use of classical binary treatment effect (semiparametric) econometric techniques, but this procedure usually would imply an important loss of information in the analysis.

Multi-valued treatment effects are comparisons between some characteristic of the (conditional) distributions of the potential outcomes. Typical examples are mean and quantile comparisons, although in many applications other features of these distributions may be of interest. For example, making the simplifying assumption that the random potential outcomes are equal for all units, the mean of the potential outcome under treatment regime  $t \in \mathcal{T}$  is given by  $\mu(t) = \mathbb{E}[Y_i(t)]$ . The collection of these means is the so-called *Dose Response Function* in the statistical literature and the *Average Structural Function* in the econometrics literature. Using this estimand, it is possible to construct different multi-valued treatment effects such as pair-wise comparisons ( $\mu(t_2) - \mu(t_1)$ ) or differences in pair-wise comparisons, which captures the idea of non-linear treatment effects. (In the particular case of binary treatment effects, the only possible pair-wise comparison is  $\mu(1) - \mu(0)$ , which is called the *Average Treatment Effect*.) It is also possible to consider other treatment effects that arise as non-linear transformations, such as ratios, incremental changes, tipping points, or the maximal treatment effect ( $\max_{t \in \mathcal{T}} \mu(t)$ ), among many other possibilities. All these multi-valued treatment effects are constructed based on the mean of the potential outcomes, but similar estimands may

be considered based on quantiles, dispersion measures or other characteristics of the underlying potential outcome distribution.

## **Statistical Inference**

Identification of (multi-valued) treatment effects is typically achieved by imposing some form of (local) independence or orthogonality condition together with other model assumptions. A typical identifying assumption is the so-called conditional independence assumption, which assumes that treatment is randomly assigned conditional on a set of observable characteristics. For example, using this assumption, identification is discussed in Imbens (2000) and Lechner (2001) for finite multiple treatments, and in Hirano and Imbens (2004) and Imai and van Dyk (2004) for continuous treatments, while efficient semiparametric estimation of finite multi-valued treatments is studied in Cattaneo (2010). An alternative identifying assumption is an instrumental variables assumption, which assumes the existence of variables that induce exogenous changes in the treatment assignment. For example, using this assumption, Nekipelov (2008) discusses identification and efficient semiparametric estimation of finite multi-valued treatments, while the case of continuous treatments is studied in Florens, Heckman, Meghir and Vytlačil (2008). See Heckman and Vytlačil (2007) and Imbens and Wooldridge (2009) for comprehensive recent reviews on these and other related results.

## **See also**

Causal inference, program evaluation, propensity score, selection bias and self-selection, semiparametric estimation, treatment effect

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