THE HOUSEHOLD EXPENDITURE RESPONSE TO A CONSUMPTION TAX RATE INCREASE

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ABSTRACT

This study measures the effect of an increase in Japan’s Value Added Tax rate on the timing of household expenditures. The analysis finds that expenditures on durables and storables surged in the month prior to the tax rate increase, fell sharply upon implementation, but quickly returned to their previous long-run levels. Non-storable non-durable expenditures increased slightly in the month prior to the tax rate increase, but were otherwise unresponsive. A dynamic structural model of household consumption reveals that the expenditure responses were driven by the insensitivity of durable and non-durable consumption, strong complementarities between durables and non-durables, and durable and storable arbitrage behavior that was moderated by significant adjustment frictions. The results suggest that salient intertemporal price variation may have a large, though highly transitory impact on household expenditures.

Keywords: Intertemporal substitution, consumption, fiscal policy, VAT

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1. Introduction

The sensitivity of the timing of household expenditures to intertemporal price variation is of great importance to economic policymakers. For example, the more willing are households to substitute expenditures across time, the greater will be the impact of countercyclical policies that induce intertemporal price variation, such as a temporary reduction in the tax rate on expenditures by the fiscal authority. The magnitude and duration of this substitution behavior depend critically on the ability and willingness of households to change the timing and composition of their consumption, as well as their ability to engage in “arbitrage” behavior such as the accelerated purchase of durables and stockpiling of storables.¹ In this study, I examine these components of substitution behavior by measuring the effect of a two percentage point increase in Japan’s Value Added Tax (VAT) rate on the timing of durable, storable non-durable, and non-storable non-durable household expenditures, and then match the empirical results to expenditure paths generated by a dynamic structural model of household consumption. Japan’s April 1997 VAT rate increase is particularly well-suited to measure substitution behavior because the tax rate increase was the dominant political issue of its time and other factors that affect substitution behavior, such as pre-tax prices, were stable.

Using monthly household-level panel survey data, I find that the timing of expenditures were quite sensitive to the VAT rate increase, but only in the months just prior to and following its implementation, with expenditures quickly returning to their previous long-run levels thereafter. In particular, durable expenditures were 8 and 23 percent greater in the two months before the tax rate increase than they would have been in its absence. Durable spending then dropped sharply following implementation and returned to trend within a few months. Consistent with the predictions of a dynamic model of durable consumption, expenditures on goods with higher levels of durability (e.g. furniture) were more sensitive to the VAT rate increase. Expenditures on storable goods were 9 percent higher in the month before implementation than they otherwise would have been. Like durable spending, expenditures on storables dropped precipitously in the month of implementation and returned to trend in the ensuing months, indicating that households engaged in a significant amount of stockpiling just prior to the price increase. Additional evidence for stockpiling comes from the observation that

¹ Barrell and Weale (2009) refer to the short-lived shifting behavior immediately before and after a price change as an arbitrage effect.
spending on goods with higher levels of storability (e.g. toilet paper) was more sensitive to the tax rate increase. Finally, non-storable non-durable expenditures (e.g. perishables, dining out), for which the timing of consumption roughly coincides with the timing of expenditure, were 1.5 percent higher in the month prior to the tax rate increase than they would have been in its absence. However, they exhibited little variation in other months prior to and following its implementation.

The empirical results demonstrate that households respond to modest but salient intertemporal substitution incentives over a broad swath of goods and services. Previous studies have examined the sensitivity of the timing of expenditures to substitution incentives for a narrower set of goods and services or for policies that are less commonplace than a VAT rate increase. Mian and Sufi (2012) estimate the impact of the 2009 Cash for Clunkers program on short- and medium-run auto purchases. Hendel and Nevo (2004, 2006) estimate own-price and cross-price elasticities for brands within a few storable product categories such as laundry detergent. And of course there exists a large literature on intertemporal substitution in consumption that examines the sensitivity of non-durable consumption expenditures to variation in the real interest rate (see Attanasio and Weber, 2010). The empirical results also corroborate recent studies (e.g. D’Acunto, Hoang, and Weber, 2015) that find increased inflation expectations have a positive effect on a household’s reported willingness to spend on durables.

In order to better understand the link between the preference parameters governing household consumption and the observed expenditure responses to Japan’s VAT rate increase, I develop a dynamic structural model of household consumption of durables, storables, and non-storable non-durables. The model is subjected to a two percentage point increase in the tax rate on expenditures that the representative household becomes aware of several periods prior to its implementation, as was the case in Japan. The expenditure paths generated by the model are governed by preference parameters such as the intertemporal elasticity of substitution in consumption (IES), the intratemporal elasticity of substitution between durables and non-durables, and adjustment cost parameters associated with the purchase of durables and stockpiling of storables. The remaining model parameters, such as the nominal interest rate, are set to match Japanese economic data from the time period surrounding the VAT rate increase. To estimate the IES, intratemporal elasticity of substitution, and adjustment cost parameters, I find the set of parameters that generates the time path of durable, storable, and non-storable non-
durable expenditures that is most “similar” to the observed expenditure response in that it minimizes a weighted sum of the squared deviations between the output generated by the model and the empirical estimates.

The methodological approach described above has several advantages with regards to estimation of the IES. First, the IES point estimate is derived from the observed expenditure responses of the three composite goods rather than non-durable consumption expenditures only. As such, it should more accurately reflect the sensitivity of overall household consumption to intertemporal price variation. While a few studies have accounted for intratemporal substitution between durables and non-durables when estimating the IES (e.g. Ogaki and Reinhart, 1998; Pakos, 2011; Cashin and Unayama, 2016), they do not directly incorporate the response of durables into the IES estimate. Second, the approach allows me to assess whether durable consumption is more sensitive to intertemporal price variation than non-durable consumption. Whether durable consumption is more sensitive is important in light of the fact that previous studies show employment in the durables sector is particularly sensitive to cyclical fluctuations.2 Consequently, economic policymakers should be interested in the responsiveness of durable consumption to intertemporal price variation, since countercyclical policy often involves such a change. Third, the additional information provided by the sensitivity of the timing of durable and storable expenditures to the tax rate increase should yield a more precise IES estimate than those in the extant literature.

I find that the observed expenditure responses to the VAT rate increase were driven by the insensitivity of durable and non-durable consumption, strong complementarities between durables and non-durables, and adjustment costs that significantly reduced the magnitude and duration of durable and storable arbitrage behavior relative to what would have been observed in the absence of frictions (even for a small IES). The point estimate for the IES is just 0.13, with a 95 percent confidence interval given by [0.05, 0.20]. In contrast, Cashin and Unayama (2016), who examine the same VAT rate increase but do not use the durable or storable responses to estimate the IES, obtain an estimate of 0.21 with a 95 percent confidence interval of [-0.48, 0.90]. The smaller and more precise point estimate in this study is primarily attributable to the

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2 Loungani and Rogerson (1989) find that worker flows into durables are procyclical, while worker flows out of durables into other sectors is countercyclical. Barlevy (1999) argues that the durable good sector accounts for a disproportionate share of the increase in unemployment during recessions.
fact that a larger IES estimate is very inconsistent with the observed response of durable expenditures. Specifically, a larger IES would imply a much sharper decline in durable expenditures upon implementation of the VAT rate increase and more gradual return to a new and lower trend. It follows that durable consumption is not necessarily more sensitive to a change in the real interest rate than non-durable consumption, as was found in Mankiw (1985).

Finally, to assess the external validity and generalizability of the parameter estimates in other contexts where there is salient intertemporal price variation, I use the model and parameter estimates derived from the Japanese data to predict the response of durable and non-durable (storable and non-storable) expenditures to the July 1989 increase in New Zealand’s Goods and Services Tax (GST) rate, which is examined in Cashin (2011). This particular tax rate increase was also announced prior to its implementation, and it featured an increase in the tax rate from 10 to 12.5 percent. The time paths of durable and non-durable expenditures generated by the model are similar to the observed responses, which suggests the estimates presented in this paper are applicable in other contexts.

In sum, the key results of this study – that the timing of expenditures were highly sensitive to Japan’s VAT rate increase, but only in the months immediately surrounding its implementation; the observed expenditure responses were due to the insensitivity of durable and non-durable consumption, strong complementarities between durables and non-durables, and significant adjustment costs for durables and storables; and the model exhibits external validity - suggest that salient intertemporal price variation like Japan’s pre-announced VAT rate increase may have a large, though highly transitory impact on household expenditures.

2. Japan’s April 1997 VAT Rate Increase: An Ideal Natural Experiment to Examine Household Substitution Behavior

2.1. Japan’s VAT and the April 1997 Rate Increase

Japan’s ‘Consumption Tax’ is a VAT. Unlike VAT in many other countries, it has a single flat rate with relatively few exemptions. The VAT was introduced in 1989 at a rate of

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3 Exemptions included transfer of lease or land, transfer of securities and transfer of means of payment, interest on loans and insurance premiums, transfer of postal and revenue stamps, fees for government services, international postal money orders, foreign exchange, medical care under the Medical Insurance Law, social welfare services
three percent, and the rate was increased from three to five percent in April 1997. The 1997 VAT rate increase, which is the focus of this study, was originally proposed as a part of the Murayama Tax Reform, which passed through the Japanese Diet in late 1994. Because the primary purpose of the reform was to continue the shift from direct to indirect taxation, the future VAT rate increase was coupled with immediate cuts in income tax rates.

 Although the Murayama reform package set a target date of April 1997 for the VAT rate increase, it was unclear whether the increase would actually be implemented then. This is because the reform legislation also stated that the rate increase would be imposed only if the economy had sufficiently recovered from a prolonged recession from 1991 to 1993, and feeble growth thereafter. Having judged the economy to have sufficiently recovered, the ruling Liberal Democratic Party (LDP) decided to raise the tax rate as scheduled. The bill to raise the VAT rate passed through the Upper House on June 25, 1996, and the tax rate increase was scheduled to take effect on April 1, 1997.

 Even after this passage, the LDP stated that they would revisit the issue of the tax rate increase when they submitted the fiscal year 1997 budget. The VAT rate increase was the central issue in October 1996 elections to the Lower House of the Diet, with the LDP’s opposition promising to postpone the tax rate increase if elected. The LDP narrowly won the election, and on December 26, 1996, the government submitted the fiscal year 1997 budget, which officially increased the VAT rate to five percent on April 1, 1997.

 2.2. The VAT Rate Increase as a Natural Experiment to Measure Substitution Behavior

Estimation of intertemporal substitution behavior requires variation in the real interest rate, which is the price of current consumption relative to future consumption. Because the real interest rate is defined as the nominal interest rate minus the expected inflation rate, a change in expected inflation will induce the necessary intertemporal price variation. As a result, the April 1997 VAT rate increase, which represented an exogenous and anticipated increase in the future price level during a period in which nominal interest rates, pre-tax prices, and income were stable, presents an ideal natural experiment to estimate intertemporal substitution behavior.

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specified by the Social Welfare Services Law, midwifery service, burial and crematory service, transfer or lease of goods for physically handicapped persons, tuition, entrance fees, facilities fees, and examinations fees of schools designated by the Articles of the School Education Law, transfer of school textbooks, and the lease of housing units. For additional information on Japan’s Consumption tax base and its administration, see Beyer (2000).

For further discussion of the political process, see Ishi (2001) and Takahashi (1999).
First of all, the tax rate increase can be regarded as an exogenous change in the future price level. Not only is it the case that the tax system is exogenous from the perspective of individual households, but it is also true that the impact of the tax rate increase is largely independent of consumer behavior. This is because the VAT by and large applies to expenditures regardless of the characteristics of the consumer, the point of purchase, or the type of goods purchased.

While exogenous intertemporal price variation is a necessary condition for estimating intertemporal substitution behavior, it must also be the case that households were aware of the change, and expected to bear the burden of the tax rate increase in the form of higher prices upon implementation. Indirect evidence of this awareness is available in the form of news coverage of the VAT rate increase prior to its implementation. Figure 1 reports the number of articles per month that mention the phrase ‘Consumption Tax’ in the Nihon Keizai Shimbun, Japan’s leading business newspaper with a circulation of over three million (in 2010), and the Yomiuri Shimbun, a leading non-business newspaper with a circulation of over 10 million (in 2010). There was a steady upward trend in coverage of the ‘Consumption Tax’ that began just prior to enactment of the June 1996 legislation. Coverage peaked in the Yomiuri Shimbun in October 1996, which coincided with elections to the Lower House of the Diet. Overall coverage in both papers was consistently high in the months following the election but prior to the tax change, with nearly 300 articles in the Nihon Keizai Shimbun mentioning the ‘Consumption Tax’ in March 1997. This evidence strongly suggests that households were aware of the tax rate increase and might therefore engage in intertemporal substitution behavior.

The news coverage also suggests that households were aware of the effects of the Murayama reform package as a whole. Figure 1 shows that coverage initially peaked in September 1994, which coincided with the passage of the Murayama reform package. Accordingly, households may have known about the VAT rate increase well in advance of its implementation, and furthermore, that it was intended to be compensated in the form of the front-loaded income tax cuts. As a result, one might expect any income effect associated with the VAT rate increase to have been small, and to have taken effect well in advance of its
implementation. This conjecture is important because it suggests that deviations in expenditure around the time of the VAT rate increase were due solely to substitution effects.5

In addition to public awareness of the VAT rate increase, it seems likely that households expected to bear the full burden of the tax rate increase in the form of higher prices at the time of implementation. For one, when the VAT was imposed in April 1989 at a rate of three percent, the prices of goods and services that had not previously been subject to tax increased by just under three percent upon implementation.6 Furthermore, the Japanese government made it clear that they expected consumers to bear the full burden of the VAT increase upon implementation.7

In fact, households did bear the burden of the tax rate increase. The bottom right graph in Figure 2 shows the seasonally-adjusted month-to-month percentage change in the consumer price index for all goods and services subject to the VAT. While inflation was negligible in most months prior to and following implementation of the tax rate increase, the price level increased by just under two percent between March and April 1997, which is consistent with full forward shifting of the two percentage point tax rate increase onto consumers at the time of implementation. As a result, focus can be placed on a one-time price change, and I can ignore the influence of pre-tax price variation.

Also note that the VAT rate increase was pushed onto consumers in full for all three of the composite goods and services with which this study is concerned. Prices increased by 2.11, 1.67, and 1.82 percent in April 1997 for non-storable non-durables, storables, and durables, respectively. And as Figure 3 demonstrates, relative prices exhibited little variation around time of the VAT increase, which will further simplify the analysis below.

Any influence of the nominal interest rate on the real interest rate around the time of the VAT increase can also be ruled out. Figure 4 presents the average contracted interest rate on short-term loans, as well as the six-month Treasury bill rate (at auction). The former rate is the

5 Nevertheless, the empirical estimates presented in Section 4.4 will be robust to any income effect that became evident upon ‘announcement’ of the VAT rate increase, which at the latest would be December 1996, when the tax rate increase became a certainty.

6 Carroll et al. (2011) find that full forward shifting at the time of a consumption tax rate increase is the norm across most countries, likely as a result of factor price rigidities.

7 When the VAT was introduced in 1989, the government took several steps to ensure this outcome. First, a Special Council on the Transition was formed to promote enforcement of the VAT across agencies. Second, the government carried out an extensive advertising campaign to allay the public’s fear of price hikes and to restrain overcharging by traders. A telephone service was also set up so consumers could report complaints about prices. Finally, the Economic Planning Agency increased the budget for the price monitoring system. The situation was nearly identical in 1997.
average interest rate applied to a contract of less than one year between a commercial bank and lender. Both interest rates fell precipitously throughout 1995, but remained relatively constant thereafter. This suggests that households would not have changed their nominal interest rate expectations in the months surrounding implementation of the VAT rate increase.\(^8\)

Finally, one might worry that household income changed concurrently with the VAT rate, perhaps because employers offset the burden of the tax rate increase with wage increases. In this case, estimates of the IES could be biased downwards to the extent the income increase was not expected. Figure 5 displays the seasonally-adjusted percentage deviation in disposable household income from the sample average. While the data is somewhat noisy, there does not appear to have been any major change in household income upon implementation of the VAT rate increase. Furthermore, income was relatively stable in the years leading up to and the months following the tax rate increase. Consequently, it seems unlikely that income changes would significantly bias the results upwards or downwards.

The facts presented above imply that the tax rate increase can be regarded as an exogenous change in the real interest rate, which allows for consistent estimation of the intertemporal substitution response using ordinary least squares. Accordingly, I do not have to rely on a lagged IV approach to overcome endogeneity concerns, nor do I need to worry about issues associated with this approach, such as weak instruments and aggregation bias.\(^9\)

3. Estimates of the Expenditure Response to Japan’s VAT Rate Increase

3.1. Data

The Japanese Family Income and Expenditure Survey (JFIES) is used to estimate the expenditure response to the VAT rate increase.\(^10\) The JFIES is a rotating panel survey in which households are interviewed for six consecutive months, and approximately 8,000 households are interviewed each month.\(^11\)

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8 Alternatively, the figure suggests that households did not demand higher nominal interest rates in response to the one-off increase in the price level.
10 See Stephens and Unayama (2011, 2012) for more information regarding the JFIES design and content.
11 Until 2002, single-person and agricultural households were excluded from the JFIES. As of the 2009 JFIES, single-person households comprised 11.8 percent of the population and were responsible for 18.1 percent of
The estimates make use of JFIES data from the period between April 1992 and March 2002, a symmetric five-year window around the April 1997 tax rate increase. Data from the “bubble” period (before April 1992) are excluded because household expenditures grew at a much faster pace than they did after the bursting of the economic bubble in 1991, while remaining more or less flat after that. The sample period ends in March 2002, which coincided with the beginning of another boom.

The sample is limited to households who complete all six interviews, but nearly all households are used, as the response rate of the JFIES is quite high. Although data for agricultural households is available in the JFIES after 1999, they are excluded from the analysis to maintain consistency over the sample period. Also, the analysis restricts the sample to male-headed households and those whose head does not change his job. The latter restriction is imposed because March is the end of the fiscal year in Japan. As a result, several job changes are observed, which may cause systematic changes in consumption around the time of the VAT rate increase. After imposing the sample restrictions, the dataset includes 646,900 observations from 129,380 households. Table 1 presents summary statistics for the sample.

The JFIES expenditure data is highly disaggregated by item type, which allows for an accurate categorization of goods and services. For the purposes of this study, it is critical to distinguish not only between taxable and tax-exempt goods and services, but also between durables, storables, and non-storable non-durables.

To construct the expenditure data, expenditures on goods and services that are exempt from the VAT are separated from expenditures on taxable items. While tax-exempt expenditure will not formally be used in the analysis below, I will examine it to determine whether there might be aggregate factors other than the VAT rate increase affecting expenditure around the time of the tax rate increase. As shown in Table 1, expenditure on taxable items comprised 70% of total expenditure, while most tax-exempt expenditure consists of rent for housing and school tuition.

The second step is to divide goods and services that are subject to the VAT into three sub-categories: durables \((D)\), storables \((S)\), and non-storable non-durables \((N)\). \(N\) are defined as goods and services which are neither storable nor durable. That is, they depreciate relatively

\[\text{expenditures, while agricultural households accounted for 2.0 percent of the population, and 2.1 percent of expenditures.}\]
quickly over time when not in use, and when in use, are fully consumed. For example, fresh fruit, if not eaten, will spoil, and is fully consumed with use. This category also includes services such as taxi fare and dining out, which are consumed at the point of purchase. It follows that monthly expenditure on $N$ should approximately coincide with monthly consumption of $N$.

$S$ are defined as goods and services that depreciate slowly over time if not used and fully if used. For example, laundry detergent can be stored for long periods of time with little to no effect on its ability to clean clothing, but once it is put into use, it is fully consumed. More generally, the characteristics that define $S$ allow for stockpiling during low price periods in order to consume in relatively high price periods. As a result, monthly expenditure on $S$ does not necessarily coincide with consumption, and expenditure on $S$ should be more sensitive to price changes than $N$.

Finally, $D$ are defined as goods and services which depreciate relatively slowly over time if not used and do not depreciate fully with use. This category includes traditional durables such as refrigerators and automobiles, as well as goods such as clothing and footwear that are classified as semi-durables in the JFIES. In addition, this category includes a select group of services such as home repair and tailoring, which consumers derive benefits from long after the service is provided. Like $S$, expenditure on $D$ should be more sensitive to changes in intertemporal prices than $N$. This is because $D$ can be purchased during a low price period, with most of its service flow consumed during a relatively high price period. See Appendix Table A.1 for a complete categorization of the composite goods and services.

Monthly expenditures are deflated using tax-inclusive consumer price indices specific to each good and service category. The analysis thus makes use of real monthly expenditures for Japanese households from April 1992 through March 2002. Table 1 shows that more than half of taxable expenditure is on $N$, while expenditure on $S$ and $D$ is similar.

3.2. Empirical Model and Specification

Suppose that the logarithm of real monthly expenditures by household $h$ on good-type $j \in \{D, S, N\}$ in year $y$ and month $m$ can be expressed as

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12 Laspeyres price indices are constructed for each of the four categories using item-specific price indices and expenditure shares in 1990 for each of these items as the weights.
\[
\ln E_{h,y,m}^j = \mu_h^j + \delta_m^j z_m + \phi^j x_{h,y,m} + \gamma_{y,m}^j d_{y,m} + \epsilon_{h,y,m}^j
\]  

(1)

$\mu_h^j$ is a household fixed effect, which I remove by taking the first difference of (1). $Z_m$ is a vector of month dummies with a corresponding vector of coefficients, $\delta_m^j$, that capture seasonality effects over the sample period. $X_{h,y,m}$ is a vector of potentially time-varying household characteristics that affect expenditures, such as the number of household members.\(^{13}\) $D_{y,m}$ contains the regressors of interest. It includes a vector of month dummies for the months surrounding the VAT rate increase, when substitution behavior may be observed. The corresponding vector of coefficients, $\gamma_{y,m}^j$, capture the percentage deviation in expenditures on good $j$ relative to the month that precedes the first $D_{y,m}$ dummy.\(^ {14}\) These deviations will inform us of the nature of household substitution behavior, such as whether it is driven by changes in the timing of consumption (i.e. intertemporal substitution in consumption), arbitrage (e.g. the stockpiling of storables), changes in the composition of consumption (i.e. intratemporal substitution), or all of the above, and will be used in conjunction with the dynamic structural model of household consumption introduced in Section 4.1 to estimate the parameters of the model related to substitution effects. Finally, $\epsilon_{h,y,m}^j$ accounts for unobservables that impact monthly household expenditures on good-type $j$.

To transition from the empirical model to the baseline regression specification, I must specify the period covered by $D_{y,m}$. Because I am interested in substitution behavior, the period chosen should be devoid of any income effect associated with the VAT rate increase. For this reason, I choose January 1997 as the first month included in $D_{y,m}$. The VAT rate increase became a certainty during the fourth quarter of 1996, and thus any income effect associated with it should have appeared by then. Nonetheless, it is worth noting that the empirical estimates presented in the next subsection and the structural parameter estimates presented in Section 5.3

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\(^{13}\) $X_{h,y,m}$ includes the number of household members, the number of workers, the number of household members under the age of 18, the number of household members over age 65, and interview dummies, which control for “survey fatigue”, the tendency of households to report lower expenditure in later interviews.

\(^{14}\) Halvorsen and Palmquist (1980) demonstrate that in regressions with a logarithmic dependent variable, it is incorrect to interpret the coefficient on a dummy variable multiplied by 100 as the percentage effect of that variable on the variable being explained. Nonetheless, when the coefficients on the dummy variables are close to zero, as is the case in this study, multiplying the coefficient by 100 provides a good approximation to the actual percentage effect of the independent variable on the dependent variable.
are robust to the choice of earlier base months as well. This result is likely due to the fact that the VAT rate increase was intended to be compensated, and consequently, the income effect was small. The final month I include in $D_{y,m}$ is December 1997, though for reasons discussed below, the structural estimation only makes use of the empirical estimates through July 1997. It follows that the baseline specification I use to generate the empirical estimates of the expenditure response to the VAT rate increase is

$$\Delta \ln E_{h,y,m}^j = c^j + \Delta (\delta^j_m z_m) + \phi^j \Delta X_{h,y,m} + \sum_{y=1997, m=1}^{y=1997, m=12} \Delta (y^j_{y,m} D_{y,m}) + \Delta \epsilon^j_{h,y,m} \tag{2}$$

where a constant, $c^j$, is added to capture any linear trend in household expenditure growth over the sample period. Standard errors are clustered by household, and are thus robust to heteroskedasticity and serial correlation of unknown form within households.

In the structural estimation procedure described in Section 4, I do not utilize the estimates of $y^j_{y,m}$ beyond July 1997 for two reasons. First, identification of the structural parameters I am concerned with requires only the few months prior to and following the VAT rate increase. In fact, I obtain similar estimates of the structural parameters presented in Section 4.3 using only the estimates of $y^j_{y,m}$ from February to May 1997 because nearly all of the observed variation in expenditures on $D$, $S$, and $N$ resulting from the VAT rate increase occurred during these four months. Second, estimates of $y^j_{y,m}$ beyond July are likely biased due to unseasonably warm weather in the second quarter of 1997 (which significantly increased electricity costs in the third quarter) and the onset of the Japanese banking crisis in November 1997.

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15 In particular, I chose October 1996, the election month that made the tax rate increase a relative certainty, as an alternative base month.
16 To avoid taking the logarithm of zero, monthly durable expenditure is set to ¥100, or approximately U.S. $1, in months that a household reported zero expenditure on durables. The results are robust to different choices of minimum durable expenditure values (e.g. ¥1 or ¥1000). Overall, 94 percent of the monthly observations report positive durable expenditures.
17 As a robustness check, year dummies were also added to the empirical specification given in Equation (4). These dummies capture average monthly growth rates in household spending within a year relative to the omitted year (captured by $c$), and are important if growth rates varied considerably over the sample period. I find that inclusion of year dummies does not significantly impact the results of the baseline specification, which is not surprising given that real household expenditures were roughly flat over the sample period.
Related to the last point, a possible concern with the specification given in (2) is that the estimates of \( \gamma_{y,m} \) from January to July 1997 are biased due to confounding factors. However, it is worth reiterating that

1) Neither income, nominal interest rates, nor pre-tax prices changed much around the time of the tax rate increase.
2) The empirical analysis excludes households whose head changed jobs.
3) The VAT rate increase was the dominant political and economic news story at the time.

One can also examine expenditures on tax-exempt goods and services to assess whether aggregate factors other than the VAT rate increase had an impact during the period of interest, since tax-exempt expenditures were not affected by intertemporal substitution incentives, but would be affected by the same aggregate factors as \( D \), \( S \), and \( N \) if these different categories of expenditures share a common component. This seems likely given that monthly changes in tax-exempt expenditures are positively correlated with expenditures on \( D \), \( S \), and \( N \) throughout the sample period.\(^{18}\) Figure 6 shows that tax-exempt expenditures did not change significantly during the period surrounding the VAT rate increase, suggesting that any bias in the estimates of \( \gamma_{y,m} \) for \( D \), \( S \), and \( N \) due to confounding factors were also small.\(^{19}\)

3.3. Empirical Estimates

Figure 7 presents estimates of \( \gamma_{y,m} \) for \( D \), \( S \), and \( N \) for January to July 1997, along with the corresponding 95 percent confidence intervals. The bottom left graph depicts the durable response to the tax rate increase. Expenditures in the final two months prior to implementation were 8 and 23 percent higher than they otherwise would have been. Expenditures in April 1997 were 13 percent lower than it would have been in the absence of the VAT rate increase, and gradually returned to trend over the next few months. This pattern is consistent with accelerated purchases of durables that would have otherwise been bought after the tax rate increase.

\(^{18}\) The correlations between the residuals from (2) for tax-exempt expenditures and \( D \), \( S \), and \( N \) (excluding observations from 1997 when intertemporal substitution incentives were present for \( D \), \( S \), and \( N \)) are 0.03 (0.00), 0.03 (0.00), and 0.09 (0.00), respectively. P-values are listed in parentheses.

\(^{19}\) Tax-exempt expenditures fell between March and April 1997 by nearly four percent. While not statistically significant, if this decline is indicative of an aggregate factor that negatively impacted expenditures at the time of the tax rate increase, then the estimate of the IES in Section 5.3 could suffer from an upward bias.
The bottom four plots in Figure 8 examine the expenditure response to the VAT rate increase for several types of durable goods and services. Note that the response in March 1997 was largest for furniture and household appliances, followed by consumer electronics, with almost no response for automobiles.\textsuperscript{20} Perhaps not coincidentally, there is some evidence of an inverse relationship between the expenditure response observed in March 1997 and the depreciation rate estimates associated with each good type in Fraumeni (1997).\textsuperscript{21} All else equal, a dynamic model of durable consumption would predict that expenditure in the month prior to implementation would be more sensitive for goods with lower depreciation rates, and the estimates in Figure 8 are generally consistent with this prediction. Finally, it is interesting to note that the dip in durable expenditures in July 1997 is due primarily to a reduction in expenditures on household appliances, and specifically, air conditioners. This suggests that households were forward looking enough to purchase air conditioners in March 1997 that would not be used until later in the year.

The top right graph in Figure 7 shows the expenditure response for storables. Expenditure in March 1997 was nine percent higher than it otherwise would have been. In April 1997, storable non-durable expenditure was 7 percent lower than it would have been in the absence of the VAT rate increase, and gradually increased over the next few months. This pattern suggests that households stockpiled goods just prior to implementation, and then consumed from their storable inventory over the next few months.

This explanation is further reinforced by comparing the response for storable non-durables that possess different levels of storability. The top four plots in Figure 8 examine the response of domestic household goods (e.g. laundry detergent, toilet paper), personal care items (e.g. medicine, shaving cream), beverages (alcoholic and non-alcoholic), and storable foods (e.g. butter, noodles, yogurt) to the VAT rate increase. Domestic household goods and personal care items are storable for long periods of time, while beverages and foods are storable for a relatively shorter period. As Hendel and Nevo (2006) note, this is at least in part because storability for the latter groups decreases once the container or packaging is opened. I find that expenditure is more

\textsuperscript{20} The lack of response for automobiles is surprising, especially given that automobile prices rose approximately two percent between March and April 1997. One possibility is that high transaction costs associated with the sale or disposal of an existing automobile dissuaded households from making a new automobile purchase.

\textsuperscript{21} The annual rates of depreciation given in Fraumeni (1997) are the following: furniture (0.12), household appliances (0.15), home electronic equipment (0.18), and motor vehicles (0.17).
sensitive for goods with higher levels of storability, which is consistent with the consumer inventory model of stockpiling behavior (see Hendel and Nevo, 2004 and 2006).

On average, there was little change in non-storable non-durable expenditures prior to and following implementation of the VAT rate increase. Note, however, that non-storable non-durable expenditures were 1.51 percent higher in March 1997 than they otherwise would have been, which is significant at the ten percent level. One might suspect that this increase is due to a limited amount of storability for the goods and services that make up $N$. However, if this were the case, we would expect to observe an equal and offsetting decrease in expenditure on $N$ in April 1997, which is not apparent. Instead, given that the user cost of durables fell in March 1997, the result suggests that durables and non-durables are complements, while the lack of variation in expenditures in all months excluding March suggests the IES estimate will be small. The intuition for both conjectures is discussed further in Section 4 with the aid of the model.

In summary, the empirical results suggest that the timing of expenditure was sensitive to the VAT rate increase, but the timing of consumption was relatively insensitive. And while expenditures did respond to the price change, the response was confined to the months immediately preceding and following implementation. That said, it is possible that the response is biased downward to some extent by borrowing constrained households. To explore this possibility, Figure 9 displays point estimates of the expenditure response for households with incomes above and below the median, under the assumption that households with incomes below the median are more likely to be constrained. The responses are remarkably similar, which suggests that borrowing constraints are not biasing the point estimates for the overall sample.

4. Characterizing the Substitution Response to Japan’s VAT Rate Increase

4.1. The Representative Agent’s Problem

In this subsection, I introduce a representative agent model of durable, storable, and non-storable non-durable consumption whose expenditure paths will be matched to the empirical estimates presented in Section 3.3. The purpose of this exercise is to understand the link between the preference parameters governing household consumption and the observed expenditure responses to Japan’s VAT rate increase.
In each period $t$, taken here to be one month, a representative household chooses non-storable non-durable consumption, $C_t^N$; storable consumption, $C_t^S$; storable expenditure, $X_t^S$; the stock of storables, $S_t$, that will be carried over into period $t+1$; the durable stock, $D_t$, which provides a flow of consumption services; durable expenditure, $X_t^D$; and financial assets, $A_t$, to maximize the present value of lifetime utility, $U$, subject to the budget constraint, laws of motion for $S_t$ and $D_t$, and stochastic processes for the tax rate on expenditure, $\tau_t$, and income, $Y_t$.

Formally, the representative agent solves

$$U = \sum_{t=0}^{\infty} \beta^t \left( \frac{\sigma}{\sigma - 1} \right) u_t^{\frac{1}{\sigma}}$$

subject to

1) $A_t = (1 + i)A_{t-1} + Y_t - (1 + \tau_t)(C_t^N + X_t^S + X_t^D) - \frac{\zeta^D}{2} (D_t - D_{t-1})^2 - \frac{\zeta^S}{2} (S_t - S^*)^2$
2) $S_t = S_{t-1} + X_t^S - C_t^S$
3) $D_t = (1 - \delta)D_{t-1} + X_t^D$
4) $\tau_t = \tau_{t-1} + \epsilon^\tau_{t-1}$
5) $Y_t = Y_{t-1} + \epsilon^\gamma_{t-1}$

where

$$u_t = \left(1 - \psi^D \epsilon^D\right)^{\frac{1}{\epsilon^D}} \left\{ \left(1 - \psi^S \epsilon^S\right)^{\frac{1}{\epsilon^S}} C_t^N \epsilon^N + \psi^S \epsilon^S C_t^S \left(1 - \frac{1}{\epsilon^S}\right) \right\}^{\frac{1}{\epsilon^S}} + \psi^D \epsilon^D D_t \left(1 - \frac{1}{\epsilon^D}\right)$$

Intertemporal preferences are assumed to be iso-elastic and governed by the IES, $\sigma$, which is one of the parameters to be estimated. Note that the value of $\sigma$ will be determined by changes in $C_t^N$, $C_t^S$, and $D_t$ in response to the VAT rate increase. As noted earlier, a potential advantage of the approach employed in this study is that information from consumption of all three goods will be used to determine $\sigma$. While $C_t^S$ and $D_t$ are not directly observable, their values can be inferred using observable expenditures, $X_t^S$ and $X_t^D$, in conjunction with the laws of motion for $S_t$ and $D_t$. 

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The intratemporal preference specification is assumed to take a nested constant elasticity of substitution (CES) form. Under this specification, preferences over the durable stock and the non-durable composite good are homothetic. As mentioned earlier, Pakos (2011) provides evidence that durables are luxuries and non-durables are necessities, and that the assumption of homotheticity when preferences are actually non-homothetic biases estimates of $\sigma$ upward. However, given that the VAT rate increase was intended to be compensated and that any income effect associated with the tax reform should have occurred prior to the period from which the empirical estimates are drawn, the simplifying assumption of homotheticity seems reasonable.

Preferences over the durable stock and a non-durable composite good are governed by $\psi^D$, a parameter measuring the overall importance of the durable stock in generating utility, and $\epsilon^D$, the elasticity of substitution between durables and non-durables.\(^{22}\) The value of $\epsilon^D$ will also be estimated using the expenditure response to the VAT rate increase. In particular, it is identified off of the change in the durable to non-durable consumption ratio resulting from the reduction in the user cost of durables prior to the VAT rate increase. It is worth noting that because I allow for non-separable preferences over durables and non-durables (i.e. $\epsilon^D$ is not restricted to be equal to $\sigma$), the estimate of $\sigma$ should be free from intratemporal substitution bias.\(^{23}\) Preferences over the non-durable composite good are also assumed to be of the CES form, where $\psi^S$ is the share of storables in non-durable consumption, and $\epsilon^S$ is the elasticity of substitution between storable and non-storable non-durables.\(^{24, 25}\)

The following assumptions are made with respect to prices. The nominal interest rate, $i$, is constant, since Japan’s benchmark nominal interest rate was constant in the years prior to and following the VAT rate increase. The model abstracts from time-varying pre-tax prices on the three composite goods, as the price ratios for these goods were stable during the period of interest. Finally, the burden of the VAT is assumed to fall entirely on the representative

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\(^{22}\) As in many previous studies of durable goods, such as Bernanke (1985), I assume that the service flow derived from durables is proportional to the durable stock.

\(^{23}\) See Ogaki and Reinhart (1998) for an explanation of intratemporal substitution bias.

\(^{24}\) $\psi^S$ represents a share only if preferences are Cobb-Douglas, or if prices are assumed to be one, as in this study.

\(^{25}\) The nested CES form restricts the intratemporal elasticity of substitution between storables and durables and non-storables and durables to be equal. To test the validity of this assumption, I used a quadratic specification that allowed for an interaction term between storables and durables, as well as non-storables and durables. After doing so, the implied intratemporal elasticities of substitution between storables and durables and non-storables and durables were similar.
consumer in the form of higher prices, which is consistent with Japan’s experience and the experience of other countries with a VAT.

The budget constraint also includes durable and storable convex adjustment cost functions, both of which take quadratic functional forms.\textsuperscript{26} While studies (e.g. Caballero, 1993) have shown that durable adjustment is lumpy and infrequent at the microeconomic level and therefore inconsistent with convex adjustment costs, a number of more recent studies (e.g. House, 2014) argue that models based on convex adjustment costs are adequate for understanding the behavior of long-lived goods such as consumer durables in the aggregate. The parameter associated with the durable adjustment cost function, $\zeta^D$, is another parameter which I will estimate based on the expenditure response to the VAT rate increase. It is identified by the difference between the durable expenditure response that would be observed in the months preceding and following implementation (holding the other parameters fixed) in the absence of frictions and the observed response, and is increasing in that difference.

The adjustment cost function for storables depends on two parameters, $S^*$ and $\zeta^S$. $S^*$, which is assumed to be greater than zero, is a storable inventory bliss point. $S_t > S^*$ generates a cost to the household due to space constraints and the time cost associated with stockpiling. $S_t < S^*$ also generates a cost due to the inconvenience of holding too few storables. $\zeta^S$ is a parameter to be estimated in the same manner as $\zeta^D$.

The law of motion for the stock of storables is the same as that used by Hendel and Nevo (2006). Note that the stock of storables carried over from one period to the next does not depreciate. In effect, I assume that these foods are fully consumed before they perish. The law of motion for durables is standard and depends on the durable depreciation rate, $\delta$.

The tax rate on expenditure in month $t$, $\tau_t$, is set equal to last month’s tax rate, plus any shock to the tax rate, $\epsilon_{t-l}^T$, that was announced $l$ months prior. So long as $l \geq 4$, where $l = 4$ corresponds to announcement of the tax rate increase in December 1996 and implementation ($t^*$) in April 1997, the lag chosen has little impact on the time path of expenditures generated by the model in the months immediately surrounding the tax rate increase.

\textsuperscript{26} Including the adjustment cost functions in the budget constraint rather than the preference specification does not significantly impact the time path of expenditures generated by the model, though it will impact the point estimate for the adjustment cost parameters, $\zeta^D$ and $\zeta^S$. 
The model abstracts from a formal income process since there was little change in household income around the time of the VAT rate increase. To account for the income tax cuts that took effect immediately upon passage of the reform package, households are compensated for the two percentage point increase in the tax rate on expenditure with a 1.94 percent increase in income in the month which the tax rate increase is announced, $t^* - l$. The tax rate increase and the offsetting compensation are known to the representative household well before $t^*$, and thus any change in expenditure around the time of the tax rate increase that is generated by the model will be attributable to substitution effects.

Finally, the model does not incorporate borrowing constraints or the labor/leisure decision. Recall from Section 3.4 that the expenditure responses to the VAT rate increase for households above and below median income were similar. Consequently, it seems unlikely that borrowing constraints played a significant role in generating the response observed for the entire sample. I ignore the labor/leisure decision because the empirical estimates to which the model is matched are restricted to households whose job status did not change while in the sample.

4.2. Econometric Methodology

To estimate the parameters that characterize the substitution response to Japan’s VAT rate increase – $\sigma, \epsilon^D, \zeta^D,$ and $\zeta^S$ – I first solve for the model’s steady state. I then log-linearize the model around its steady state. The parameters of the model that are not estimated - $i, \beta, \psi^S, \psi^D, \delta, \epsilon^S,$ and $S^*$ - are chosen based on available data from Japan around the time of the VAT rate increase.

The value of $i$ is set to 0.0015, which compounded monthly corresponds to an annual interest rate of 0.018. This was the average annual interest rate on short-term loans and discounts prior to and following the VAT rate increase (see Figure 4). $\beta$ is set such that $\beta(1 + i) = 1$, since the model begins in steady state. $\psi^S$ is set to 0.29, which was storable non-durable expenditure as a share of non-durable expenditure in the JFIES in 1996. The value of $\delta$ is set to 0.022, which corresponds to an annual depreciation rate of 0.23. This value was computed by combining good-specific annual depreciation rates from Fraumeni (1997) with good-specific expenditure shares on durables from the JFIES. The value of $\epsilon^S$ has no impact on the time path of expenditures generated by the model, because there is no change in the price of storables relative to non-storable non-durables. The output generated by the model is also completely
insensitive to the choice of $S^*$.\textsuperscript{27} $\psi^D$ is expressed as a function of the other parameters from the model and the ratio of steady-state durable expenditure to non-storable non-durable expenditure, $\frac{X^D}{CN}$. I set this ratio to 0.42, which was the ratio in the JFIES in 1996.\textsuperscript{28} Finally, initial income, $Y_0 + iA_0$, is normalized to 1, and the initial tax rate on expenditure, $\tau_0$, is set to 0.03.

Let $\theta = [\sigma \ e^D \ \zeta^D \ \zeta^S]^T$, where $T$ is the transpose operator. Given a $\theta$, the parameters of the model are fully defined. To generate a time path of expenditures from the model, the shocks to the tax rate on expenditure and income are introduced. They propagate through the system of log-linearized equations, generating a time path of percentage deviations in durable, storable, and non-storable non-durable expenditures from their steady-state values.\textsuperscript{29} For the purposes of estimating $\theta$, I am interested in the expenditure output from $t^* - 3$ to $t^* + 3$, which corresponds to the empirical estimates from January to July 1997.

To estimate $\theta$, I use an econometric procedure similar to that employed by Christiano et al. (2005). Intuitively, $\hat{\theta}$ is the set of substitution parameters that minimizes a measure of the distance between the time path of expenditures generated by the model and the empirical estimates presented in Section 3.3. More specifically, I conduct a grid search over combinations of $\theta$. $\hat{\theta}$ is the vector of parameter values that minimizes a weighted sum of the squared deviations between the $M \times 1$ vector of time-series output generated by the model, $\gamma(\theta)$, and the $M \times 1$ vector of empirical estimates, $\hat{\gamma}$, depicted in Figure 7. The estimates are chosen to match the durable, storable, and non-storable non-durable empirical estimates from January through July 1997. Thus, $M = 7$ months $\times$ 3 goods $= 21$. Formally,

$$\hat{\theta} = \arg \min_{\theta} [\hat{\gamma} - \gamma(\theta)]^T W^{-1} [\hat{\gamma} - \gamma(\theta)] \quad (3),$$

\textsuperscript{27}The difference between $S_t$ and $S^*$, which is what generates the storage cost, is independent of the magnitude of $S^*$. Rather, this difference is a function of $\tau_t$, $i$, and $\zeta^S$. For example, in steady state, $S - S^* = -(1 + \tau) \left( \frac{1}{1 + r} \right) \left( \frac{1}{\zeta^S} \right)$.

\textsuperscript{28}Specifically, $\psi^D = \frac{(1 - \psi^S)(1 + \tau)}{1 + (1 - \psi^S) \left( \frac{r + \zeta^S}{1 + r} \right) \left( \frac{X^D}{CN} \right)}$. This expression is obtained by solving for steady state durable expenditure in terms of non-storable non-durable expenditure, and then rearranging and solving for $\psi^D$.

\textsuperscript{29}Recall that the $\hat{y}_{j,m}$'s are percentage deviations in expenditure relative to December 1996, four months prior to implementation of the VAT increase. In order to make the output generated by the model consistent with the empirical estimates, I convert the output from percentage deviations in expenditure relative to the steady state to percentage deviations in expenditure relative to expenditure four periods prior to the tax rate increase. The conversion makes little difference for the model's estimated parameters.
where $W$ is an $M \times M$ weighting matrix that contains the sample variances and covariances of the $\hat{y}_{y,m}^j$'s. Standard errors for the structural parameters are estimated using the delta method, which is documented in Appendix B.

4.3. Parameter Estimates

Table 2 presents point estimates and 95 percent confidence intervals for the model parameters that comprise $\theta$. The estimate of $\sigma$ is 0.13, with a 95 percent confidence interval given by [0.05, 0.20]. The point estimate is similar to, though slightly lower than, the estimate in Cashin and Unayama (2016). However, the confidence interval is much tighter, as expected. The null hypothesis that the IES is zero can be rejected at conventional levels of significance. Nevertheless, the fact that the estimate is close to zero should mitigate concern over the potential for an upward bias in the estimate of $\sigma$ discussed in Section 3.3. The result implies that consumption growth was relatively insensitive to the VAT rate increase, and further that durable good consumption is not necessarily more sensitive to a change in the real interest rate than non-durable consumption, as found in Mankiw (1985).

The point estimate of $\epsilon^D$ is small and negative. The value is -0.03, with a 95 percent confidence interval given by [-0.05, -0.01]. The null hypothesis that durables and non-durables are perfect complements (i.e. $\epsilon^D = 0$) is rejected at conventional significance levels. However, the small point estimate suggests that the assumption of Leontief preferences, where durables and non-durables are consumed in fixed proportions, is a good starting point when jointly modeling durable and non-durable consumption.

The null hypothesis that preferences over durables and non-durables are separable is rejected at all conventional significance levels.31 This finding suggests that the assumption of separability in previous studies may have induced bias in estimates of $\sigma$. In particular, given that I find a high level of complementarity between durable and non-durable consumption, previous estimates of $\sigma$ that assumed separable preferences and were derived from non-durable consumption expenditure data may suffer from an upward bias.

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30 The covariances for different goods are assumed to be zero. That is, $\text{cov}(\hat{y}_{y,m}^j, \hat{y}_{y,m}^{k,c}) = 0, j \neq k$.
31 The Wald test statistic, $W$, for the null hypothesis $H_0: \sigma = \epsilon^D$ is $W = 9.64$, which exceeds the critical value of 6.63 when the degrees of freedom equals one, and the significance level is one percent.
How do I obtain the results that $\sigma > \epsilon^D$ and $\sigma$ is small? Recall that the user cost of durables fell relative to non-durables prior to implementation of the VAT rate increase. As illustrated in Figure 10, if $\sigma < \epsilon^D$, one would observe non-storable non-durable expenditures trending downwards in the months prior to implementation, as households substitute away from non-durables to durables, followed by an upward trend thereafter. In addition, Figure 11 demonstrates that durable expenditures would fall far more drastically upon implementation, and return to trend much more gradually, than was observed. This is because a larger value of $\epsilon^D$ will require a greater fall in durable expenditure after implementation in order to restore the original durable to non-durable consumption ratio. If instead preferences are separable over durables and non-durables, one would observe no change in non-storable non-durable expenditures in the months prior to implementation, and a fall in expenditures upon implementation that remains constant thereafter. Durable expenditures would exhibit a similar, albeit less pronounced, pattern as was the case when $\sigma < \epsilon^D$. Finally, if $\sigma > \epsilon^D$, one would observe an upward trend in non-storable non-durable expenditures in the months prior to implementation (as households increase non-durable consumption to complement durable consumption), a fall in expenditures upon implementation, and a slight decline thereafter. This pattern is largely consistent with the empirical estimates. Durable expenditures would exhibit a far less pronounced decline upon implementation, and a quicker return to trend, which is also consistent with the empirical estimates. Consequently, I find that $\sigma > \epsilon^D$.

Given that $\sigma > \epsilon^D$, why is $\sigma$ small? Figure 12 illustrates the non-storable non-durable expenditure response for $\sigma = 0.8, 0.4,$ and $0.13$ when the other parameters are set to their baseline estimates. Note that the larger is $\sigma$, the larger is the fall in non-storable non-durable expenditures following implementation. In addition, Figure 13 demonstrates that larger values of $\sigma$ imply that the fall in durable expenditures upon implementation greatly exceeds the spike in expenditures in the month prior to implementation. The empirical estimates in Figure 12 show that non-storable non-durable expenditures were not noticeably lower following implementation than they were before. The estimates in Figure 13 show that durable expenditures fell upon implementation, but not by an amount greater than the spike in expenditures in the month prior to implementation. Together, these facts imply that $\sigma$ is small.

The point estimates for $\zeta^D$ and $\zeta^S$ are 0.09 and 1.30, respectively. Each is statistically significant at the one percent level. Taking as given that $\sigma > \epsilon^D$ and that $\sigma$ and $\epsilon^D$ are small,
Figure 14 demonstrates that $\zeta^D$ is identified primarily by the spike and trough in durable expenditures in the months prior to and including the VAT rate increase. Simply, the larger is $\zeta^D$, the smaller will be the spike and trough in durable expenditure in March and April 1997, respectively. Similarly, the larger is $\zeta^S$, the lower will be the amount of stockpiling in the month prior to implementation, and consequently, the lower will be the jump in storable expenditures.

For durables, the marginal adjustment cost is given by $\zeta^D(D_t - D_{t-1})$. Plugging in the relevant values for March 1997, the month in which durable expenditure was most sensitive to the VAT rate increase, I find that the marginal adjustment cost for the last unit of durables purchased was 0.28 percent of monthly income. Similarly for storables, the marginal storage cost is expressed as $\zeta^S(S_t - S^*)$. Given that the marginal benefit of stockpiling in March 1997 was constant at approximately 0.02 (the amount of the tax rate increase), it follows that stockpiling continued until the marginal cost of storing an additional unit of the storable good was 0.02. For the baseline value of $S^*$, the implication is that households’ inventory in March 1997 exceeded $S^*$ by $X\%$ (RERUN WITH NEW BASELINE $S^*$).

In contrast to $\sigma$ and $\epsilon^D$, which I interpret as structural parameters, it is more appropriate to treat $\zeta^D$ and $\zeta^S$ as reduced form parameters. As Caballero (1994) notes, the aggregate durable response to a shock such as a VAT rate increase depends on the cross-sectional distribution of the deviation between households’ actual and desired durable stocks. Because this distribution varies over time, the estimate of $\zeta^D$ is likely a function of the distribution in early 1997. Hendel and Nevo (2004) make a similar point for the cross-sectional distribution of storable inventories. As such, policy analysis based on these adjustment and storage cost estimates may be flawed.

Recall that several of the model parameters were chosen prior to estimation. It seems reasonable to test the sensitivity of the elements of $\hat{\theta}$ to different values of these parameters. Table 3 presents the results of the sensitivity analysis. In general, the elements of $\hat{\theta}$ are robust to the alternative parameter values. Given a change in one of the fixed parameters, each new parameter estimate lies within its confidence interval shown in Table 2.

Figure 7 plots the time path of expenditures generated by the model against the empirical estimates. Overall, the model matches the empirical estimates reasonably well. Despite the reasonable fit, the validity of the model is rejected by the test of overidentifying restrictions, as the $J$-statistic equals 71.84, which exceeds the critical value of 33.41 ($\alpha = 0.01$).
non-durable expenditures lie entirely within the confidence interval. The model closely matches storable expenditure in the months prior to and including implementation, but the match is rather poor in the months following implementation. This is because the representative household will fully consume the stockpiled storable good in the month of implementation before making any additional purchases. In reality, only some households stockpile, and among those that do, the amount stockpiled varies. As a result, there is a gradual return to trend in storable expenditures following implementation that the model is not flexible enough to match. The time path of durable expenditures generated by the model stays within the 95 percent confidence interval in most months. However, the model under predicts the sensitivity of durable expenditure to the VAT rate increase in February and March 1997, and over predicts the decline in expenditure in April 1997. This is a result of the choice of a quadratic adjustment cost specification, which requires a symmetric expenditure response on either side of the tax rate increase.

5.4. External validity check

One might question the external validity of the parameter estimates, given that the estimates are derived from one event in one country. To address this concern, I use the model and the baseline parameter estimates to predict the expenditure response to New Zealand’s Goods and Services Tax (GST) rate increase from 10 to 12.5 percent, which was announced in March 1989 and implemented in July 1989. Figure 16 compares the time path of expenditures generated by the model to the durable and non-durable retail sales response to the GST rate increase (see Cashin, 2011). The empirical estimates provide seasonally-adjusted percentage deviations in retail sales for May through October 1989 relative to April 1989. The expenditure patterns generated by the model match up quite well with the point estimates for non-durable retail sales (storables and non-storables) before and after implementation, and durable retail sales prior to implementation. The model over predicts the fall in durable retail sales upon implementation, and the recovery of durable expenditure following implementation is quicker than what was observed in New Zealand. This is likely the result of the choice of a quadratic adjustment cost function for durables. Overall, however, this exercise lends additional support to the main finding in this paper, which is that expenditure is sensitive to a change in the future price level, albeit over a short period preceding and following the price change, while consumption growth is not.
5. Conclusion

This study demonstrates that the timing of household expenditures are highly sensitive to salient intertemporal price variation such as a VAT rate increase due to accelerated purchases of durables and the stockpiling of storables. However, durable and non-durable consumption are relatively insensitive. As a result, the household expenditure response is large, but short-lived, confined to the months immediately prior to and following the anticipated price change. For economic policymakers considering the use of countercyclical policies that induce intertemporal price variation, such as a temporary VAT cut, these findings suggest that the substitution channel alone is not sufficiently strong to stimulate aggregate demand and employment due to the lack of persistence in the expenditure response.

The methodological approach taken in this study seems like a promising one for future research on intertemporal substitution behavior. By decomposing the household expenditure response to a consumption tax rate increase into durables, storables, and non-storable non-durables, and then controlling for durable and storable arbitrage effects via the structural model, I am able to map observable expenditures into unobservable consumption. This approach has two advantages relative to the traditional approach of examining non-durable consumption expenditures only. First, it incorporates durable consumption, which is more sensitive to cyclical fluctuations and may respond differently to intertemporal price variation. Second, by examining total consumption, the precision of the IES estimate can be significantly improved.
References


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<td></td>
<td>129,380</td>
</tr>
</tbody>
</table>

Note: Yearly household income and monthly household expenditures are listed in thousands of yen, with 2005 serving as the base year.
Table 2. Baseline Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>95 percent confidence intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>0.13***</td>
<td>[0.05, 0.20]</td>
</tr>
<tr>
<td>$\varepsilon^D$</td>
<td>-0.03***</td>
<td>[-0.05, -0.01]</td>
</tr>
<tr>
<td>$\zeta^D$</td>
<td>0.09***</td>
<td>[0.01, 0.18]</td>
</tr>
<tr>
<td>$\zeta^S$</td>
<td>1.30***</td>
<td>[1.12, 1.47]</td>
</tr>
</tbody>
</table>

95 percent confidence intervals for the structural parameter estimates above are listed in brackets, and are computed using the delta method (see Appendix B for a full explanation). *** implies significance at the one percent level.
<table>
<thead>
<tr>
<th>Fixed Parameter</th>
<th>Value</th>
<th>( \sigma )</th>
<th>( \varepsilon^D )</th>
<th>( \zeta^D )</th>
<th>( \zeta^S )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>See Section 5.2</td>
<td>0.13</td>
<td>-0.03</td>
<td>0.09</td>
<td>1.30</td>
</tr>
<tr>
<td>( i )</td>
<td>0.0008 (0.01)</td>
<td>0.13</td>
<td>-0.03</td>
<td>0.09</td>
<td>1.30</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.0025 (0.03)</td>
<td>0.13</td>
<td>-0.03</td>
<td>0.10</td>
<td>1.28</td>
</tr>
<tr>
<td>( \psi^S )</td>
<td>0.01 (0.15)</td>
<td>0.08</td>
<td>-0.02</td>
<td>0.11</td>
<td>1.27</td>
</tr>
<tr>
<td>( \chi^D )</td>
<td>0.03 (0.30)</td>
<td>0.19</td>
<td>-0.04</td>
<td>0.09</td>
<td>1.31</td>
</tr>
<tr>
<td>( \xi^N )</td>
<td>0.25</td>
<td>0.12</td>
<td>-0.03</td>
<td>0.08</td>
<td>1.54</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>0.14</td>
<td>-0.03</td>
<td>0.10</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>0.14</td>
<td>-0.03</td>
<td>0.12</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td>0.50</td>
<td>0.11</td>
<td>-0.03</td>
<td>0.07</td>
<td>1.37</td>
</tr>
</tbody>
</table>

Numbers listed in parentheses represent annual values for the fixed parameters.
Figure 1 presents the number of articles per month that mention the phrase ‘Consumption Tax’ in the Nihon Keizai Shimbun and the Yomiuri Shimbun. Source: Author’s calculations.
Figure 2 presents seasonally-adjusted month-to-month percentage changes in the price of goods and services that were subject to the VAT. The vertical dashed line represents April 1997, the month of the VAT rate increase from three to five percent. To generate these estimates, the month-to-month percentage change in price is regressed on month dummies. The residuals from this regression yield the seasonally-adjusted month-to-month percentage change in the price indices.
Figure 3 presents the ratio of seasonally-adjusted durable and storable non-durable prices to non-storable non-durable prices. To remove seasonality, the consumer price indices for durables, storable non-durable, and non-storable non-durable goods and services are regressed on month dummies. The residuals are added to the constant in the regression to obtain seasonally-adjusted price indices. To calculate the ratios, I divide the seasonally-adjusted durable and storable non-durable price by the seasonally-adjusted non-storable non-durable price in each month. The dashed vertical line in the figure is April 1997, the month of implementation.
Figure 4 presents two interest rates. The higher rate is the average contracted interest rate applied to a contract of
less than one year between commercial banks and lenders (“Commercial banks”). This data comes from the Bank of
Japan. The lower rate is the rate at the monthly auction of the 6-month Treasury bill (“6-month T-bill”). This data
comes from Japan’s Ministry of Finance
Figure 5 presents the percentage deviation in disposable household income relative to the sample average after controlling for household fixed effects, seasonality, and time-varying household characteristics. To generate the plot, the logarithm of disposable income is regressed on a constant, a household fixed effect, month dummies, and time-varying household characteristics. The residuals from the regression are then averaged in each month and plotted.
Figure 6 presents the percentage deviation in tax-exempt expenditures relative to expenditure in this category in December 1996, controlling for household fixed effects, a linear trend in consumption growth, seasonality, and time-varying household characteristics. The solid red line gives the point estimates for each month, which are derived from the regression specification in Equation (2). The dashed blue lines give the 95 percent confidence intervals. Standard errors are panel-robust. The dashed vertical line represents April 1997, the month of implementation.
Figure 7 presents the percentage deviation in durable, storable, and non-storable non-durable expenditures relative to expenditure in these categories in December 1996, controlling for household fixed effects, a linear trend in consumption growth, seasonality, and time-varying household characteristics. I interpret the results as yielding the substitution effects associated with the VAT rate increase. The solid red line gives the point estimates for each month, which are derived from the regression specification in Equation (2). The dashed blue lines give the 95 percent confidence intervals. Standard errors are panel-robust. The time path of expenditures generated by the model is given by the dashed-dotted green line. The dashed vertical line represents April 1997, the month of implementation. If monthly durable expenditure for a household is reported as zero, it is set to ¥100 to avoid taking the logarithm of zero.
Figure 8. Storable and durable response to the VAT rate increase

See Figure 7 for an explanation of how these plots were generated.
Figure 9 presents the percentage deviation in income relative to December 1996 for households above and below the median income. See Figure 7 for an explanation of how these plots were generated.
Figure 10 presents non-storable non-durable expenditure patterns in the months prior to and following implementation of the VAT increase for different values of $e^D$ when $\sigma = 0.13$, which is the baseline estimate for $\sigma$. The squared green, circled cyan, and black diamond lines show expenditure when $e^D = 0.5, 0.13, \text{ and } -0.03$, respectively. The figure also displays the empirical estimates from the top left plot in Figure 7, where the solid red line represents the point estimates, and the dashed blue lines the 95 percent confidence intervals for the point estimates.
Figure 11 presents durable expenditure patterns in the months prior to and following implementation of the VAT increase for different values of $e^D$ when $\sigma = 0.13$, which is the baseline estimate for $\sigma$. The squared green, circled cyan, and black diamond lines show expenditure when $e^D = 0.5, 0.13, \text{ and } -0.03$, respectively. The figure also displays the empirical estimates from the bottom left plot in Figure 7, where the solid red line represents the point estimates, and the dashed blue lines the 95 percent confidence intervals for the point estimates.
Figure 12 presents non-storable non-durable expenditure patterns in the months prior to and following implementation of the VAT increase for different values of $\sigma$ when the other structural parameters are set to their baseline values. The squared green, circled cyan, and black diamond lines show expenditure when $\sigma = 0.80$, $0.40$, and $0.13$, respectively. The figure also displays the empirical estimates from the top left plot in Figure 7, where the solid red line represents the point estimates, and the dashed blue lines the 95 percent confidence intervals for the point estimates.
Figure 13 presents durable expenditure patterns in the months prior to and following implementation of the VAT increase for different values of \( \sigma \) when the other structural parameters are set to their baseline values. The squared green, circled cyan, and black diamond lines show expenditure when \( \sigma = 0.80, 0.40, \) and \( 0.13 \), respectively. The figure also displays the empirical estimates from the bottom left plot in Figure 7, where the solid red line represents the point estimates, and the dashed blue lines the 95 percent confidence intervals for the point estimates.
Figure 14 presents durable expenditure patterns in the months prior to and following implementation of the VAT increase for different values of $\zeta^D$ when the other structural parameters are set to their baseline values. The squared green, circled cyan, and black diamond lines show expenditure when $\zeta^D = 0.09$, $0.20$, and $0.30$, respectively. The figure also displays the empirical estimates from the bottom left plot in Figure 7, where the solid red line represents the point estimates, and the dashed blue lines the 95 percent confidence intervals for the point estimates.
Figure 15 compares non-storable non-durable and durable expenditure patterns in the months surrounding the VAT rate increase when $\sigma = \varepsilon^D = 0.8$, and $\xi^D$ and $\xi^S$ are chosen to minimize Equation (5). The dashed green line shows the time path generated by the model, while the solid red line shows the empirical estimates of the expenditure response to the VAT rate increase based on the JFIES survey data and the specification in Equation (4). The dashed blue lines are 95 percent confidence intervals for the empirical estimates. The dashed vertical line represents April 1997, the month the VAT rate increase was implemented.
Figure 16 compares the time path of expenditures generated by the model using the structural parameter estimates in Table 2 to empirical estimates of the seasonally-adjusted percentage deviation (relative to April 1989) in retail sales in the months surrounding New Zealand’s July 1989 Goods and Services Tax (GST) rate increase from 10 to 12.5 percent, which is documented in Cashin (2011). The rate increase was announced in March 1989 and was uncompensated.
<table>
<thead>
<tr>
<th><strong>Durable (D)</strong></th>
<th><strong>Durable (cont.)</strong></th>
<th><strong>Non-storable non-durable (cont.)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tools</td>
<td>Accessories (e.g. watch)</td>
<td>Water</td>
</tr>
<tr>
<td>Cooking appliance</td>
<td>Other personal effects (e.g. cane)</td>
<td>Flowers</td>
</tr>
<tr>
<td>Refrigerator</td>
<td>Home repair (e.g. plumbing)</td>
<td>Newspaper</td>
</tr>
<tr>
<td>Vacuum</td>
<td>Clothing services (e.g. tailoring)</td>
<td>Eating out</td>
</tr>
<tr>
<td>Washing machine/dryer</td>
<td>Auto repair</td>
<td>Domestic services</td>
</tr>
<tr>
<td>Other household durables (e.g. microwave)</td>
<td>Personal care services (e.g. haircut)</td>
<td>Bus fare</td>
</tr>
<tr>
<td>Air conditioner</td>
<td>Personal effect services (e.g. watch repair)</td>
<td>Taxi fare</td>
</tr>
<tr>
<td>Fan heaters</td>
<td>Personal care item (e.g. hair dryer)</td>
<td>Airfare</td>
</tr>
<tr>
<td>Stove</td>
<td></td>
<td>Other public transit</td>
</tr>
<tr>
<td>Other heating and cooling appliances</td>
<td></td>
<td></td>
</tr>
<tr>
<td>General furniture</td>
<td>Storable (S)</td>
<td></td>
</tr>
<tr>
<td>Clock</td>
<td>Grains (e.g. noodles)</td>
<td></td>
</tr>
<tr>
<td>Lighting</td>
<td>Fish (dried, fish paste)</td>
<td></td>
</tr>
<tr>
<td>Floor coverings and curtains</td>
<td>Meat (processed)</td>
<td></td>
</tr>
<tr>
<td>Other interior furnishings</td>
<td>Dairy (e.g. butter)</td>
<td></td>
</tr>
<tr>
<td>Bedding</td>
<td>Vegetable (e.g. beans)</td>
<td></td>
</tr>
<tr>
<td>Utensils</td>
<td>Fruit (canned)</td>
<td></td>
</tr>
<tr>
<td>Japanese clothing</td>
<td>Oils, spices, and seasonings</td>
<td></td>
</tr>
<tr>
<td>Western clothing</td>
<td>Sugar</td>
<td></td>
</tr>
<tr>
<td>Women’s coats</td>
<td>Sweets (e.g. chocolate)</td>
<td></td>
</tr>
<tr>
<td>Shirts</td>
<td>Cooked food</td>
<td></td>
</tr>
<tr>
<td>Underwear</td>
<td>Beverages (e.g. tea)</td>
<td></td>
</tr>
<tr>
<td>Other clothing</td>
<td>Alcoholic beverages</td>
<td></td>
</tr>
<tr>
<td>Footwear</td>
<td>Light bulbs</td>
<td></td>
</tr>
<tr>
<td>Automobile</td>
<td>Domestic goods (e.g. laundry detergent)</td>
<td></td>
</tr>
<tr>
<td>Other vehicle</td>
<td>Cloth</td>
<td></td>
</tr>
<tr>
<td>Bicycle</td>
<td>Medicine</td>
<td></td>
</tr>
<tr>
<td>Auto parts</td>
<td>Medical supplies (e.g. bandages)</td>
<td></td>
</tr>
<tr>
<td>Telephone</td>
<td>Stationery</td>
<td></td>
</tr>
<tr>
<td>Textbook</td>
<td>Film</td>
<td></td>
</tr>
<tr>
<td>Television</td>
<td>Recording media (e.g. CD)</td>
<td></td>
</tr>
<tr>
<td>Stereo</td>
<td>Pet food</td>
<td></td>
</tr>
<tr>
<td>Portable audio equipment</td>
<td>Personal care items (e.g. shaving cream)</td>
<td></td>
</tr>
<tr>
<td>Video recorder</td>
<td>Tobacco</td>
<td></td>
</tr>
<tr>
<td>Camera</td>
<td>Rail service</td>
<td></td>
</tr>
<tr>
<td>Computer</td>
<td>Gasoline</td>
<td></td>
</tr>
<tr>
<td>Musical instrument</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Desk</td>
<td>Non-storable non-durable (N)</td>
<td></td>
</tr>
<tr>
<td>Other recreational durable goods</td>
<td>Bread</td>
<td></td>
</tr>
<tr>
<td>Golf equipment</td>
<td>Fish (fresh)</td>
<td></td>
</tr>
<tr>
<td>Other sporting goods</td>
<td>Meat (raw)</td>
<td></td>
</tr>
<tr>
<td>Sport outfits</td>
<td>Dairy (e.g. milk)</td>
<td></td>
</tr>
<tr>
<td>Toys</td>
<td>Vegetable (fresh)</td>
<td></td>
</tr>
<tr>
<td>Other recreational goods</td>
<td>Fruit (fresh)</td>
<td></td>
</tr>
<tr>
<td>Books</td>
<td>Cake</td>
<td></td>
</tr>
<tr>
<td>Personal effects (e.g. umbrella)</td>
<td>Cooked food (e.g. sushi)</td>
<td></td>
</tr>
<tr>
<td>Handbag</td>
<td>Electricity</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Natural gas</td>
<td></td>
</tr>
</tbody>
</table>
Appendix B: Computing the Standard Errors

Denote the mapping in Equation (3) as
\[ \hat{\theta} = f(\hat{\gamma}). \]

Provided that
\[ \sqrt{n}(\hat{\gamma} - \gamma_0) \xrightarrow{d} N(0,W), \]
by the delta method, it can be shown that
\[ \sqrt{n}(\hat{\theta} - \theta_0) \xrightarrow{d} N(0,f'(\gamma_0)W f'(\gamma_0)^T), \]
where \( n \) is the number of observations used in the regressions that yield the \( \hat{\gamma}' \)s, \( f'(\gamma_0) \) is a \( P \times M \) matrix of derivatives (\( P = \# \) of estimated model parameters; \( M = \# \) of moments), and \( W \) is the asymptotic variance-covariance matrix of \( \sqrt{n}(\hat{\gamma} - \gamma_0) \). In practice, \( W \) is replaced by its sample estimate, \( \hat{W} \).

To compute \( f'(\gamma_0) \), let
\[ L(\hat{\gamma}, \gamma(\theta)) = (\hat{\gamma} - \gamma(\theta))^T W^{-1}(\hat{\gamma} - \gamma(\theta)). \]

Then
\[ \frac{\partial L(\hat{\gamma}, \theta)}{\partial \theta} = L_\theta(\hat{\gamma}, \theta) \]
is a \( 1 \times P \) vector, where \( L_\theta(\hat{\gamma}, \theta) = 0 \). By the implicit function theorem,
\[ L_{\theta \hat{\gamma}}(\hat{\gamma}, \theta) + L_{\theta \theta}(\hat{\gamma}, \theta) f'(\gamma) = 0, \]
and it follows that $f'(\gamma)$ can be approximated as

$$f'(\gamma) = -L_{\theta,\theta}(\hat{\gamma}, \hat{\theta})^{-1}L_{\theta,\gamma}(\hat{\gamma}, \hat{\theta}).$$

where $f'(\gamma)$ is a $P \times M$ matrix, $L_{\theta,\theta}(\hat{\gamma}, \hat{\theta})$ is a $P \times P$ matrix, and $L_{\theta,\gamma}(\hat{\gamma}, \hat{\theta})$ is a $P \times M$ matrix. Each element of $L_{\theta,\theta}(\hat{\gamma}, \hat{\theta})$ can be expressed as

$$L_{\theta_p,\theta_q} = \sum_{i=1}^{M} \sum_{j=1}^{M} \omega_{j,i} \left[ \frac{\partial \gamma_{l}(\theta)}{\partial \theta_p} \frac{\partial \gamma_{j}(\theta)}{\partial \theta_q} + \frac{\partial \gamma_{j}(\theta)}{\partial \theta_p} \frac{\partial \gamma_{l}(\theta)}{\partial \theta_q} \right] \forall \ p = 1, ..., P; q = 1, ..., P$$

where $\omega_{j,i}$ is the element of $W^{-1}$ found in row $j$ and column $i$. Each element of $L_{\theta,\gamma}(\hat{\gamma}, \hat{\theta})$ can be expressed as

$$L_{\theta_p,\gamma_m} = -2 \sum_{i=1}^{M} \omega_{i,m} \frac{\partial \gamma_{l}(\theta)}{\partial \theta_p} \forall \ m = 1, ..., M; \ p = 1, ..., P.$$  

$\frac{\partial \gamma_m(\theta)}{\partial \theta_p}$ is computed numerically as

$$\frac{\partial \gamma_m(\theta)}{\partial \theta_p} = \frac{\gamma_{m}(\hat{\theta} + he_p) - \gamma_{m}(\hat{\theta} - he_p)}{2h} \forall \ m = 1, ..., M; \ p = 1, ..., P,$$

where $h$ is small and $e_p$ is a $P \times 1$ vector with a one in the $p^{th}$ row and a zero in all others.\(^{34}\)

\(^{34}\) In practice, $h = 1 \times 10^{-6}$. The standard error estimates are robust to larger choices of $h$. 

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