Nonparametric Structural Analysis of Asymmetric Auctions and Implications for Merger Analysis: the Case of NJ School Bus Route Auctions

Preliminary and incomplete
comments welcome

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Abstract

In this essay we apply recent advances in econometric methods to study firm asymmetry in a procurement auction setting. In particular we identify the impact that allowing cost asymmetries can have on traditional merger analysis. To our knowledge, this is the first application of these tools in such a setting.

We begin by nonparametrically estimating underlying cost distributions using observed bids and theoretical implications of equilibrium bidding behavior. We modify existing estimation techniques to allow for variation in bidder participation without a reserve price via the introduction of bid preparation costs. Costs are then estimated using a unique data set of bids from bus route auctions in the state of New Jersey from 2000 to 2006.

By utilizing these estimated cost distributions we conduct a series of counter factual merger simulations. We find that 59% of simulated auctions have an increase in the state's procurement costs due to the merger, while the other 41% yield a decrease in associated procurement costs. The reduction in the total

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number of potential bidders in the post-merger environment cannot explain this outcome. Rather, the change in the expected number of bidders – sometimes negative, and in other cases close to zero and even positive – can better account for this pattern. This result suggests that anti-trust authorities may need to adopt a more nuanced stance when evaluating proposed mergers in settings where entry is likely in the short run.

JEL Classifications: C14, C53, H75, L41, L9

1. Introduction

The recent merger of the two largest private student transportation firms in the United States, and the resulting investigation by several State Attorneys General provide an interesting setting for analysis. As part of the investigation the state of New Jersey undertook a comprehensive data collection effort. Using this data set and advances in the nonparametric estimation of cost distributions in asymmetric auctions (Guerre et al. [2000], Flambard et al. [2007], etc.) we are able to evaluate the impact of potential mergers in the marketplace. This is the first such application of nonparametrically derived asymmetric cost estimates to merger simulation. Indeed, previous analyses of mergers in auction markets in the antitrust literature generally assume some cost distribution and then proceed to numerically simulate equilibrium bids.

The data set contains 950 bids on 304 student bus route auctions occurring between 2000 and 2006. This bidding information was supplemented with school and bus depot locations, as well as auction specific information, such as the number of students transported on a particular route. We first show that significant cost asymmetries exist between groups of firms in this market. Interestingly the asymmetries identified when firms are grouped by depot location, relative to the school being served, are larger than the asymmetries identified based on firm size. We then identify all possible mergers between firms that bid against each other in the data. In each case firms were categorized based on depot locations and submitted bids were simulated using the estimated cost distributions, allowing for the endogenous participation decisions of firms. Our results indicate that 59% of simulated auctions produced an increase in procurement costs for school districts. The other 41% involve auctions where the simulations indicate a price decrease after the merger. In these instances the effects of having fewer potential bidders in an auction is outweighed by an increase in the empirically derived probability of participation by the remaining firms. Where traditional antitrust regulation often concerns itself with the overall size of the firm resulting from a merger, our results show that other sources of asymmetry, e.g. depot locations, or the impact of a merger on bidder participation rates should not be overlooked in the analysis.

The economic literature on the theory and econometrics concerning auctions is extensive.. Recent review articles of interest here include Athey and Haile [2007] considers nonparametric approaches to auctions, and Hendricks and Porter [2007], which offers an overview of the empirical literature.

Although a general discussion of the empirical auction literature is outside the scope of this paper, targeted discussion is warranted. In all but the simplest settings auction theory is inadequate for empirical applications.

For example, in settings where independent private values (IPV) and symmetry among bidders is reasonable there is no guarantee that an analytical solution exists. Often, profit maximizing behavior among bidders implies an intractable system of first order conditions. Economists have developed a variety of techniques to overcome these difficulties. Some numerical methods useful for estimating equilibrium bidding are discussed in Bajari [2001], as well as Gayle and Richard [2008].[2][3]

In addition to the numerical methods described in the papers above, other researchers have used interesting tricks to sidestep computational issues. Laffont et al. [1995], use the Revenue Equivalence Theorem (Myerson [1981], Riley and Samuelson [1981]) to transform a computationally difficult first price auction to a more tractable second price auction. In the most basic case,[4] the expected revenue is the same whether the auction mechanism is a first price or second price. This result is a due to bid shading by bidders in the first price auction. The equilibrium bid is the expected value of the bidder with the second highest valuation. In second price auctions the equilibrium bid is to simply bid your valuation, and to pay the valuation of the second highest bidder. This is the result that allows Laffont et al. [1995] to reduce a potentially complicated first price auction analysis by looking at the second price analogue. Rather than use numerical approximations to the structural equations, they instead simulate the winner in a second price auction with the same underlying valuations. The resulting estimator, called a simulated non-linear least squares estimator, is one that can be easily implemented by simple repeated sampling from candidate distributions.

When moving to a more robust setting where the distributions of values are no longer assumed to be the same for all bidders researchers encounter more difficulties. Of particular note is that the Revenue Equivalence Theorem no longer holds, which makes simplifications of the type above not possible. Cantillon [2008] compares the revenue results of first and second price auctions when bidders are symmetric and when bidders have asymmetric valuation distributions. Of interest here is the result that increasing[5] asymmetry reduces competition among bidders. Fibich et al. [2004] show that when asymmetries exist, but are small, it is possible to analyze the bidding behavior of the "average" type

[2]Gayle and Richard are developing a suite of tools to simulate asymmetric auctions with parametrically specified bidder valuations.

[3]For example, when there are multiple items with interdependent values it can be impossible for bidders to even know their valuations for various combinations of items, let alone possible for them to express their preferences to the auctioneer.[For a good introduction into the many complexities of combinatorial auctions see Cramton et al. [2006].] Even if we assume bidders are sufficiently sophisticated, and the bidding language robust enough to express and transmit valuations over all possible combinations of items, there are still complications. Indeed selecting the revenue maximizing combination of winning bids is a variant of the knapsack problem, which is NP-Complete.

[4]The basic case involves risk-neutral bidders, and independent valuations whose distributions are common knowledge. In addition the bidders are symmetric in that they draw from the same underlying value distribution.

[5]It is useful to think an individual bidder's valuation function comprising of a term common to all bidders plus a term (potentially) unique to each bidder. In the notation Fibich et al. [2004] $F_i\left(v\right) = F\left(v\right) + \epsilon_i H\left(v\right)$for $i = 1, \ldots, n$ Where $F\left(\underline{v}\right) = 0,\ F\left(\overline{v}\right) = 1,\ H\left(\underline{v}\right) = H\left(\overline{v}\right) = 0$ and $H(v) \leq 1$ in $[\underline{v}, \overline{v}]$ As this idiosyncratic term approaches zero ($\epsilon \to 0$) the bidders approach symmetry. Or, conversely as ϵ increases the firms become more asymmetric.

and then proceed with a symmetric auction setting comprised of bidders with this "average" value distribution. Maskin and Riley [2000] also study revenue equivalence in the presence of strong and weak bidders

A major step forward in the analysis of asymmetric auctions was made by Guerre et al. [2000], where the intuitive link between underlying valuation distributions and observed bid distributions is formalized. Assuming we observe auction characteristics, as well as the number and identity of bidders we can do away with any distributional assumptions as well as sidestep any need for numerical methods. This advancement allows researchers to re-write the system of first order conditions in terms of observed bid distributions as opposed to unobserved cost distributions. In the same paper Guerre et. al. extend their results to consider observable auction heterogeneity and non-participation of some bidders, due to reservation prices. Campo et al. [2003] push this estimator further by introducing asymmetries in the form of two (or more) types of bidders. Flambard and Perrigne [2006] also adapt Guerre et al. [2000] to allow for asymmetric bidders, only this time working in a procurement auction setting. Flambard and Perrigne [2006] then use the estimator to study snow removal auctions in Montréal where firms are either 'on the island' or 'off the island.' After showing that cost asymmetries do exist between the two groups of firms, they are able to perform a policy experiment where subsidies are used to reduce bidder asymmetry, which fits nicely with the results of Cantillon [2008].

Obviously there are a multitude of potential complications that need to be addressed. Among several dealt with in recent empirical literature are dynamic aspects due to auctioning goods over time. This could potentially introduce asymmetry in later auctions when ex-ante there was none. De Silvaa et al. [2005] examine this possibility using Oklahoma Department of Transportation auction data where asymmetries are introduced by the timing of auctions. Contracts with potential synergies are auctioned at different times during a single day. Jofre-Bonet and Pesendorfer [2003] introduce time in a different manner. By allowing for differing project lengths and firm capacities, they construct a dynamic optimization problem that firms solve when competing in California road construction contracts. The capacities are endogenous and determined by a firm's history of winning. This creates an interesting dynamic where firms must weigh the option value of waiting against bidding in the current set of auctions. Withholding some capacity potentially allows a firm to avoid increased costs in future periods due to equipment rental. We feel comfortable not directly accounting for these possibilities in our estimation due to the nature of the bus routes up for auction as well as several institutional details, which will be discussed later.

Finally we would be remiss to not mention an interesting paper that considers bus auctions. Cantillon and Pesendorfer [2004] examine London's effort to privatize bus service. There the auction design explicitly allowed for package bids, allowing firms to express bids that encompass potential economies of scale. In addition, smaller more peripheral areas of bus service were auctioned off first to allow for learning by firms before placing bids on larger more centralized areas of service.

2. Description of the Market

Compared to its origins in the late 19th and early 20th centuries, when most students walked to school, publicly funded student transportation today in the United States touches the lives of a substantial number of students. During the 1919-20 school year, only 1.7% of students were transported to school at public expense [Bryans, 1986]. During the 1998-99 school year, 58% of students rode a bus.[6] Over the past century, these transportation services have been offered by private contractors and a variety of public entities, including school districts and regional agencies (e.g. in New Jersey, some school districts "outsource" these services to CTSAs or Coordinated Transportation Services Agencies that provide services across district lines). According to sources cited by Lazarus [2004], in the U.S. private contractors were generally relied upon into the 1940s; afterward, the role of district-owned and managed school buses grew in importance. However, during the 1990s, some states passed legislation that encouraged school districts to outsource student transportation [Lazarus, 2004]. During the 1998-99 school year, private contractors owned 29.5% of the school buses in the U.S. (School Bus Fleet Magazine [2002]).[7]

Currently there are a small number of very large national busing firms, operating thousands of buses and transporting tens of thousands of students, but the vast majority of busing firms own a few dozen buses or less.[8] This aspect of market structure has been affected by consolidation since at least the early 1990s [Lazarus, 2004]. The largest U.S. contractor in 1993, Laidlaw, transported 883,000 students each day; by 2002 this number had more than doubled to 1.9 million. The second largest contractor, Ryder, and its later incarnation, First Student, also grew quickly over this time period. In 1993, Ryder transported 450,000 students; by 2002, First Student was transporting 1 million students daily. In early 2007, these two firms, Laidlaw and First Student, announced a merger. Later that year, following an antitrust investigation by the U.S. Department of Justice and a number of U.S. States, the merger was approved.

At a more micro level, i.e. the school district, there exists considerable variation in the relative mix of public and private provision of student transportation services. Within New Jersey itself, some districts rely completely on private contractors, others provide the services themselves or outsource this responsibility to a CTSA, and still others rely on some combination of these approaches.[9]

[6][National Center for Education Statistics, Digest of Education Statistics 2001, 2002]

[7]By comparison, Lazarus [2004] cites sources indicating that over 80% of students in Canada are transported by private contractors; this fraction reaches 99% in Ontario.

[8]According to School Bus Fleet Magazine (2010) the largest busing firm in the US, First Student Inc. operates 60,000 buses, and transports 4,000,000 students in the US. The tenth largest firm operates 1103 buses, and the 50th largest, 150 buses. By comparison, the largest district-owned and operated bus fleet, in Gwinnett County, Georgia, numbers 1564 buses, and transports 122,000. In NJ, during our sample period almost 300 contractors were active, most of them operating far fewer than 150 buses. To some extent this reflects state-specific bus specifications that prevent buses from operating in more than one state. That is, entry is limited in the short run at least by the existing supply of buses in a particular state.

[9]According to Lazarus [2004], there also exists a fourth approach: district ownership of

An interesting research question outside the scope of this project would be to explore the factors that explain this heterogeneity.

Our focus of course is on the bidding behavior of private contractors, and the potential impact of mergers on outcomes.[10] School busing auctions typically occur at the district level, i.e. bids are for contracts that cover all routes operated by an individual district. However, in some states, such as New Jersey, auctions most often occur at the route level, so depending on the circumstances, it is possible for multiple busing firms to operate routes within the same district.[11] There are multiple instances in our data set of a single school being served by multiple private contractors in a given school year. Auctions in New Jersey are sealed-bid, with districts required to select the lowest bid, subject to certain quality considerations (N.J.A.C. 6A:27).The state also tries to encourage efficiency by requiring that (contractors') buses on average achieve greater than 100% daily utilization.[12] That is, the average bus must transport enough students to exceed 100% of its capacity, to and from school. This can be accomplished by designing longer bus routes, and deploying individual buses on multiple routes. Contractors, of course have an incentive to deploy their buses in an efficient fashion, and they presumably bid on routes that allow them to deploy their buses on multiple routes. The rational for school districts to encourage greater utilization is due to how student transportation is funded. The state reimburses school districts for the cost of transporting students, and in the event that a district doesn't meet the mandated efficiency numbers they could see increased scrutiny of payment requests from the state. Some school districts stagger school start times to facilitate this practice, and may solicit bids for a package of such tiered routes. The impact these regulations have on auction outcomes is questionable. Firstly, even though the state has efficiency requirements, the districts cannot take them into account when awarding routes. Secondly, no direct financial penalties were implemented in the corresponding legislation (known as CEIFA, for Comprehensive Education Improvement and Funding Act).

School districts have some discretion in the choice of contracts they may offer contractors: renewable multiyear or single year contracts. State regulations govern the rate of cost escalation during the multiyear contracts, as well as

buses that are operated by private contractors. This option is apparently not observed in New Jersey [McCabe, 2008, Interview of Jerry Ford].

[10] Our interest in this subject arose after participation by one of this paper's authors (McCabe) in the aforementioned 2007 investigation by twelve U.S. States of the proposed merger between Laidlaw and First Student. In addition to New Jersey, these included Alaska, California, Connecticut, Illinois, Maine, Massachusetts, Minnesota, Missouri, New York, Rhode Island and Washington.

[11] There is some limited "bulk bidding" observed in New Jersey, i.e. contractors may bid on a set of routes simultaneously. In these cases, districts may permit contractors to offer a discount on the set of routes in combination, relative to the prices offered individually for routes in the set. See N.J.A.C. 6A:27.

[12] Lazarus [2004], cites evidence that this efficiency criteria can also have negative consequences, namely reductions in test scores due to longer commuting time for some students.

the price increases permitted under renewals.[13] Our data, which reveals the identity of route operators (public and private) during the period 2000-2006, indicates that most (80-90%) existing contracts are renewed. When auctions are observed, there are several possible explanations: (1) State regulations require rebidding when changes in existing routes occur, e.g. a change in the school destination, or the type of bus used, (2) a new route is needed to accommodate demographic changes in a district, (3) either the incumbent contractor is not satisfied by the terms of the existing contract (i.e. the price is too low given economic conditions), the district is unhappy with its contractor's performance and/or it believes the economic environment favors rebidding (i.e. the price is too high). It is this lack of fixed contract end dates combined with firms' difficulty in predicting future changes in demand, which drives our assumption that firms don't take into account the type of dynamics studied by Jofre-Bonet and Pesendorfer [2003].

3. Description of the Data

3.1. DRTRS

While the state leaves specifics of transportation decisions to individual districts, the state provides funding for all student transportation. In order to accurately determine the allocation of the transportation budget each district is required to register a list of their transported students. This information is collected and stored in the District Report of Transported Resident Students (DRTRS). This data is based on a "snapshot" of the state of transported students in mid October of each year. Responsibility for reporting is assigned to the district in which the student *resides*.[14] The DRTRS comprises two related data sets.

The first, called the "DRTRS-Students" file, contains one entry for every student transported, or given aid in lieu of transportation. A complete list of all information collected in this data set can be found in AppendixB. The fields of primary interest for this study are: district of residence, school ID number, as well as bus route and license number. In Table 1 we summarize the number of students transported by bus in NJ during the period 1999-2006 by school type (in this paper we consider only public student transportation).

[13] Specifically, the school districts cannot grant annual percentage price increases greater than the rate implied by a NJ-specific consumer price index.

[14] If a student is transported to a different district, the resident district is still responsible for submitting correct information. So what matters is where the student is from, not where they are going.

Table 1: Transported students by School Type and Year

Transported students by School Type and Year ('000s)									
School Type	1999	2000	2001	2002	2003	2004	2005	2006	Total
Public	631	646	668	686	695	702	710	714	5,450
Non-public	110	111	113	114	112	111	110	109	891
Private school (handicapped)	11	11	12	12	12	13	13	13	98
Charter School	3	4	4	4	4	5	5	6	35
Early childhood	0	0	0	0	0	0.96	0.946	0.904	3
Total	755	773	798	817	823	832	838	842	6,478

The "DRTRS-Route" file is the second component of the DRTRS data set and contains one entry per route.[15] In this file we can obtain the name of the operator, how much they were paid, as well as the route number. See AppendixC for a list of all of the data collected by the state in this file. In Table 2 we summarize the number of routes operated in NJ during the period 1999-2006 by operator type, routes operated by private contractors were awarded via auctions.

Table 2: Routes by Operator Type ('00s)

Routes Operated*	1999	2000	2001	2002	2003	2004	2005	2006	Total
District Owned	119	125	129	140	141	139	147	144	1,085
Contracted	121	163	171	174	128	128	127	127	1,139
Host District	29	18	18	18	28	29	31	30	199
Host CTSA	40	16	18	17	46	51	47	53	289
Total	309	322	336	348	344	346	352	354	2,711

* The values in this table represent individual routes as identified by unique license plate and route identification information found in the DRTRS-Routes file. Duplicate entries as described in footnote 15 have been removed to the best of our abilities.

3.2. District Surveys

To supplement the information obtained from the DRTRS, surveys were sent to individual school districts. As is often the case in survey research, the response rate to the survey was generally low. However, the districts that did participate provided a sampling[16] of the auctions conducted during the study period. For each auction reported we know the route number, the incumbent

[15]Since each district must report information for all of its resident students it is common to have routes appear multiple times in the data set. For example if student A rides a bus to school that is not in her district, then the route may be included in our data twice. first by the "Host district" and then by student A's own district. Another instance arises from the rule of thumb provided by the state to "Follow the Money" when entering data into the DRTRS. Each time money is exchanged between entities an entry into DRTRS is required. So, if a district pays a CTSA to operate the route, we would observe one entry. If, however, the CTSA then subcontracts the operation of the route to a private contractor we would observe a second entry due to the payment from the CTSA to the private firm.

[16]While the districts were asked to provide information on all auctions held during the study period, we believe only a subset of the auctions were included in the responses. Characteristics and outcomes for the reported auctions are not significantly different than those for the entire district, and so we treat the survey responses as being representative of auctions in general for each district.

Table 3: District Surveys

District	Auctions Observed	Firms	Average Distance
Cranford Township	4	4	3.59
Hillside Township	4	2	1.79
Lakewood Township	23	5	9.98
Matawan-Aberdeen Regional	5	4	7
Monroe Township	35	8	10.53
Paterson City	71	9	4.95
Princeton Regional	18	8	7.19
Ramsey Borough	2	5	10.55
Salem City	10	11	16.81
South Brunswick Township	59	6	11.24
Tenafly Borough	1	3	3.75
Vernon Township	34	7	10.39
Warren County Vocational	9	5	17.13
Westfield Township	2	3	16.2
Total	280	2142	12.08

Table 4: District Surveys

District	Bids Observed	BidsAuction	Average Bid
Cranford Township	11	2.75	$203.59
Hillside Township	8	2	$224.38
Lakewood Township	41	1.78	$120.11
Matawan-Aberdeen Regional	11	2.2	$175.16
Monroe Township	127	3.63	$188.38
Paterson City	251	3.54	$177.16
Princeton Regional	97	5.39	$131.22
Ramsey Borough	2	1	$186.00
Salem City	25	2.5	$190.49
South Brunswick Township	220	3.73	$185.12
Tenafly Borough	4	4	$132.74
Vernon Township	78	2.29	$156.36
Warren County Vocational	25	2.78	$217.00
Westfield Township	4	2	$142.65
Total	907	2.46	$177.18

firm (if there was one) and as well as the amounts and identities of all participating firms. Tables 3 and 4 summarize the survey results.[17]

3.3. GPS of Schools/Depots

Due to the nature of student busing, a primary component of operating costs comes from the distance traveled by the bus. An ideal data set would contain not only the start (bus depot) and endpoint (school) of every route, but also

[17]The average is calculated using only bids that were used in our analysis. Some bids listed in column 3 were observed (and therefore used to inform entry probabilities as well as sets of competitors), but were dropped at a later point due to various data issues: e.g.: obvious typos in data entered or prices are 'too low' based on discussions with Jerry Ford. For instance the bid might be for $5000 on a route that operates only over the summer. In some instances it was impossible to identify these nonstandard routes and construct an appropriate daily rate.

the path traveled in between. This would allow for an exact calculation of the distance required to service each route.

The first two components of the distance are readily available. Depot inspection locations, which are carried out at the bus' depot, are available online from the New Jersey Motor Vehicle Commission website. School address information is likewise available from the New Jersey Department of Education website. The addresses returned from each of these sites are the current location of the depot or school. The static nature of school and depot locations allows us to use the current location data as the location for all years of the study.[18]

The path connecting the start and endpoints (the route itself) is not readily available, as this information is not collected at the state level. This type of information is often only maintained in hard copy form and kept at the district or school level, which made collecting such route specific information not feasible for this study. An alternative approach is to reconstruct route paths from student address information, which is maintained at the state level. If we make the reasonable assumption that a bus stop is located "near" the homes of those students being transported, then we could construct approximate path information for each route. Understandably there were privacy concerns with sharing the addresses of every student in the state over an 8 year period. Offers to provide the state with a program that would convert an address to GPS coordinates and then add noise to the coordinates, before releasing the information were also rejected.

In the absence of supplemental data we constructed a route distance measure using only the DRTRS data provided. The DRTRS students database contains information on the distance from the student's residence to the school they are attending (one way), and is rounded to the nearest mile. We form a lower bound of the number of bus route stops by assuming all students with the same MILES1 value share a common bus stop.[19] This STOPS variable is used as one of the route specific characteristics used in our analysis, a summary of which can be found in Table 5.

One limitation of using only the depot location, school location and our constructed STOPS variable is that we are unable to differentiate two routes going from the same depot to the same school with the same number of stops from one another. A final offer to use only two randomly selected "noisy" student locations per route was also rejected. Had we been provided this information we could have established the direction of travel along the route in relation to the bus depot.

[18] The notable exception is when schools are closed they are removed from the online address database. When possible supplemental information for these schools was used to establish where they were located. In the majority of these cases however, the school and corresponding routes were dropped from the study.

[19] Meaning, for example, students that live 1-2 miles away catch the bus at the same location; likewise students 2-3 miles away share a bus stop at a different location.

Table 5: Estimated Bus Stops

District	Average # of Bus Stops
Cranford Township	2
Hillside Township	3.25
Lakewood Township	2.09
Matawan-Aberdeen Regional	2.2
Monroe Township	2.77
Paterson City	2.42
Princeton Regional	2.39
Ramsey Borough	3
Salem City	4
South Brunswick Township	2.27
Tenafly Borough	1
Vernon Township	4.65
Warren County Vocational	10.22
Westfield Township	2

3.4. District Maps

The maps included below show the school locations for all route auctions observed in the survey data. Bus depot locations are also shown on the map. For each district the operators shown are competitors in that particular district. All firms with observed bids on routes in a given district have been included on the map. Other firms that we consider competitors in that district have also been mapped. For a description of how we identified competitors in a district see §6.1.

PLEASE SEE APPENDIXA FOR MAPS

4. Theoretical Model and Estimation

4.1. Base Model

We begin by making several common assumptions about firm behavior. Firms placing bids are risk neutral as well as profit maximizing. Firms share a common cost distribution with a *cdf* $F(c)$,[20] which is common knowledge and draws are independent. Finally, the marketplace is assumed to be competitive.

We begin with the profit maximizing behavior of the auction participants. Where bidders must balance a higher bid, and therefore larger profit should they win, against the probability of not being the lowest bidder, which is decreasing in their bid. We restrict our analysis to the set of strictly increasing differentiable

[20]Notation adapted from Flambard et al. [2007]

bidding strategies $\sigma(\cdot)$. A firm that bids using the strategy $\sigma(x)$ can expect to win only if all other firms have costs greater than $\sigma^{-1}(\sigma(x)) = x$.

$$E[\pi_i | c_i, x] = (\sigma(x) - c_i) \cdot (1 - F(x))^{\mathcal{N}-1} \tag{1}$$

This yields the following first order condition.[21]

$$\frac{\partial \pi_i}{\partial x} = [1 - F(x)]^{\mathcal{N}-1} - (\sigma(x) - c_i) \cdot (\mathcal{N}-1) \cdot [1 - F(x)]^{\mathcal{N}-2} \cdot f(x) \cdot \frac{1}{\sigma'(x)} = 0 \tag{2}$$

Dividing out common terms and reorganizing we get:

$$(1 - F(x)) - (\sigma(x) - c_i) \cdot (\mathcal{N}-1) \cdot 1 \cdot f(x) \cdot \frac{1}{\sigma'(x)} \tag{3}$$

$$\frac{(1 - F(x))}{(\mathcal{N}-1)} \cdot \frac{\sigma'(x)}{f(x)} = \sigma(x) - c_i \tag{4}$$

In equilibrium we require that bidders bid as if their costs were their true costs, namely we require that $x = c_i$ and,

$$c_i = \sigma(c_i) - \frac{(1 - F(c_i))}{(\mathcal{N}-1)} \frac{\sigma'(c_i)}{f(c_i)} \tag{5}$$

Equation (5) is the starting point for the work in Guerre et al. [2000].[22] The identification results used in this paper starts with the method they propose. Letting G(b) be the population cumulative distribution of bids, Theorem 1 establishes the relationship between the bid distribution and costs (again adjusting their result to a procurement auction setting).

> **Theorem 1** (Guerre, Perrigne, and Vuong [2000, Theorem 1])[23]
> Let $\mathcal{N} \geq 2$. Let $G(\cdot)$ belong to the set $\mathcal{P}^N$ with support $\left[\underline{b}, \overline{b}\right]^{\mathcal{N}}$. There exists a distribution of bidders' private values $F(\cdot) \in \mathcal{P}$ such that $G(\cdot)$ is the distribution of equilibrium bids in a first-price sealed-bid auction with independent private values and a nonbinding reservation price if and only if:
> C1:$(b_1, \ldots, b_{\mathcal{N}}) = \prod_{i=1}^{\mathcal{N}} G(b_i)$
> C2: The function $\xi(\cdot, G, \mathcal{N})$ defined in (9) below is strictly increasing on $\left[\underline{b}, \overline{b}\right]$ and its inverse is differentiable on $[\underline{c}, \overline{c}] \equiv \left[\xi\left(\underline{b}, G, \mathcal{N}\right), \xi\left(\overline{b}, G, \mathcal{N}\right)\right]$. Moreover, when $F(\cdot)$ exists, it is unique with support $[\underline{c}, \overline{c}]$ and satisfies $F(c) = G\left(\xi^{-1}(\cdot, G, \mathcal{N})\right)$ for

[21]Note: $\frac{\partial}{\partial b}\sigma^{-1}(b) = \frac{1}{\sigma'(c)}$

[22]Notice that if were to solve this first order differential equation (5) we would end up with the standard Symmetric Bayes Nash solution for equilibrium bidding. Only slightly adjusted due to our procurement (lowest bidder wins) framework. $\sigma(c_i) = c_i + \frac{\int_{c_i}^{\infty}[1-F(c)]^{\mathcal{N}-1}dc}{[1-F(c_i)]}$

[23]Proposition 1 in Flambard et al. [2007] shows a similar result.

all $[\underline{c}, \overline{c}]$. In addition, $\xi(\cdot, G, \mathcal{N})$ is the quasi inverse of equilibrium strategy in the sense that $\xi(b, G, \mathcal{N}) = \sigma^{-1}(b, F, \mathcal{N})$ for all $b \in [\underline{b}, \overline{b}]$.

Proof: See Guerre, Perrigne, and Vuong [2000]

The first condition requires that bids be IID, and the second condition requires that costs are an increasing function of observed bids.

$$G(b) = Prob[\tilde{b} \leq b] = Prob[\sigma^{-1}(\tilde{b}) \leq c] = F(\sigma^{-1}(b)) = F(c) \tag{6}$$

Taking the derivative to obtain the *pdf* of observed bids

$$\frac{\partial}{\partial b} G(b) = g(b) = \frac{\partial}{\partial b} F\left(\sigma^{-1}(b)\right) = f\left(\sigma'(b)\right) \frac{1}{\sigma'(c)} = \frac{f(c)}{\sigma'(c)} \tag{7}$$

By reshuffling terms in (5) we get (8)

$$c = \sigma(c) - \frac{\sigma'(c)}{f(c)} \left(\frac{1 - F(c)}{\mathcal{N} - 1}\right) \tag{8}$$

From (7) we can extract the equality $f(c) = g(b) \cdot \sigma'(c)$, and from (6) we get $F(c) = G(b)$. Then by substituting out the $F(c)$ and $f(c)$ terms of equation (8) we obtain equation (9). This final equation relates observables to underlying costs.

$$\xi(\cdot, G, \mathcal{N}) \equiv c = b - \frac{\sigma'(c)}{(g(b)\,\sigma'(c))} \left(\frac{1 - G(b)}{\mathcal{N} - 1}\right) = b - \frac{1}{\mathcal{N} - 1} \cdot \frac{1 - G(b)}{g(b)} \tag{9}$$

Now we only need to observe bids (b), the number of bidders ($\mathcal{N}$) as well as estimate $G(b)$ and $g(b)$ and we can solve for the costs associated with a particular bid. The nonparametric estimators for $G(b)$ and $g(b)$ will be discussed in §5.

4.2. Asymmetric Bidders

Modifying the base model to allow for asymmetric bidders is relatively straight forward. The derivation will be shown here with two groups, but expanding this to an arbitrary number of distinct bidder types adds only notational complexity.

We define $G_1(\cdot), g_1(\cdot), G_2(\cdot), g_2(\cdot)$ as the population *cdf*s, and *pdf*s of bids or bidders in groups 1 and 2, respectively. $\mathcal{N}_1$ and $\mathcal{N}_2$ are the sets of bidders in the auction where $\mathcal{N} = \mathcal{N}_1 + \mathcal{N}_2$, Finally let $F_1(\cdot), f_1(\cdot), F_2(\cdot), f_2(\cdot)$ be the population *cdf*s and *pdf*s of firm costs in their respective groups. We can then write the profit maximization problem (10) faced by a bidder i of type 1 in a manner similar to (1).

$$\max_{b_{1i}} (b_{1i} - c_{1i})\, Prob\,[win|b_{1i}] =$$
$$\max_{b_{1i}} (b_{1i} - c_{1i}) \left[1 - F_1\left(\sigma^{-1}(b_{1i})\right)\right]^{\mathcal{N}_1 - 1} \left[1 - F_2\left(\sigma^{-1}(b_{1i})\right)\right]^{\mathcal{N}_2} \tag{10}$$

Differentiation yields the first order condition

$$\begin{aligned}
&0 = \left[1 - F_1\left(\sigma^{-1}\left(b_{1i}\right)\right)\right]^{\mathcal{N}_1-1}\left[1 - F_2\left(\sigma^{-1}\left(b_{1i}\right)\right)\right]^{\mathcal{N}_2}\\
&-b_{1i}\left[1 - F_1\left(\sigma^{-1}(b_{1i})\right)\right]^{\mathcal{N}_1-2}\left[1 - F_2\left(\sigma^{-1}(b_{1i})\right)\right]^{\mathcal{N}_2}\cdot(\mathcal{N}_1-1)\cdot f_1\left(\sigma^{-1}(b_{1i})\right)\cdot\frac{\partial}{\partial b_{1i}}\sigma^{-1}(b_{1i})\\
&-b_{1i}\left[1 - F_1\left(\sigma^{-1}(b_{1i})\right)\right]^{\mathcal{N}_1-1}\left[1 - F_2\left(\sigma^{-1}(b_{1i})\right)\right]^{\mathcal{N}_2-1}\cdot\mathcal{N}_2\cdot f_2\left(\sigma^{-1}(b_{1i})\right)\cdot\frac{\partial}{\partial b_{1i}}\sigma^{-1}(b_{1i})\\
&+c_{1i}\left[1 - F_1\left(\sigma^{-1}(b_{1i})\right)\right]^{\mathcal{N}_1-2}\left[1 - F_2\left(\sigma^{-1}(b_{1i})\right)\right]^{\mathcal{N}_2}\cdot(\mathcal{N}_1-1)\cdot f_1\left(\sigma^{-1}(b_{1i})\right)\cdot\frac{\partial}{\partial b_{1i}}\sigma^{-1}(b_{1i})\\
&+c_{1i}\left[1 - F_1\left(\sigma^{-1}(b_{1i})\right)\right]^{\mathcal{N}_1-1}\left[1 - F_2\left(\sigma^{-1}(b_{1i})\right)\right]^{\mathcal{N}_2-1}\cdot\mathcal{N}_2\cdot f_2\left(\sigma^{-1}(b_{1i})\right)\cdot\frac{\partial}{\partial b_{1i}}\sigma^{-1}(b_{1i})
\end{aligned} \tag{11}$$

Using the fact that $\frac{\partial}{\partial b}\sigma^{-1}\left(b\right) = \frac{1}{\sigma'(c)}$ and factoring out common terms yields

$$\begin{aligned}
0 \quad = \quad & \left[1 - F_1\left(\sigma^{-1}\left(b_{1i}\right)\right)\right]^{\mathcal{N}_1-1}\left[1 - F_2\left(\sigma^{-1}\left(b_{1i}\right)\right)\right]^{\mathcal{N}_2}\\
&\cdot\left[1 - F_1\left(\sigma^{-1}\left(b_{1i}\right)\right)\right]^{\mathcal{N}_1-1}\left[1 - F_2\left(\sigma^{-1}\left(b_{1i}\right)\right)\right]^{\mathcal{N}_2}\\
&\cdot\{1 - b_{1i}\cdot\frac{1}{1 - F_1\left(\sigma^{-1}\left(b_{1i}\right)\right)}\cdot\left(\mathcal{N}_1 - 1\right)\cdot f_1\left(\sigma^{-1}\left(b_{1i}\right)\right)\cdot\frac{1}{\sigma'\left(c_{1i}\right)}\\
&-b_{1i}\cdot\frac{1}{1 - F_2\left(\sigma^{-1}\left(b_{1i}\right)\right)}\cdot\mathcal{N}_2\cdot f_2\left(\sigma^{-1}\left(b_{1i}\right)\right)\cdot\frac{1}{\sigma'\left(c_{1i}\right)}\\
&+c_{1i}\cdot\frac{1}{1 - F_1\left(\sigma^{-1}\left(b_{1i}\right)\right)}\cdot\left(\mathcal{N}_1 - 1\right)\cdot f_1\left(\sigma^{-1}\left(b_{1i}\right)\right)\cdot\frac{1}{\sigma'\left(c_{1i}\right)}\\
&+c_{1i}\cdot\frac{1}{1 - F_2\left(\sigma^{-1}\left(b_{1i}\right)\right)}\cdot\mathcal{N}_2\cdot f_2\left(\sigma^{-1}\left(b_{1i}\right)\right)\cdot\frac{1}{\sigma'\left(c_{1i}\right)}\}
\end{aligned}$$

continuing with simplification,

$$\begin{aligned}
1 \quad = \quad & b_{1i}\cdot\frac{1}{1 - F_1\left(\sigma^{-1}\left(b_{1i}\right)\right)}\cdot\left(\mathcal{N}_1 - 1\right)\cdot f_1\left(\sigma^{-1}\left(b_{1i}\right)\right)\cdot\frac{1}{\sigma'\left(c_{1i}\right)}\\
&+b_{1i}\cdot\frac{1}{1 - F_2\left(\sigma^{-1}\left(b_{1i}\right)\right)}\cdot\mathcal{N}_2\cdot f_2\left(\sigma^{-1}\left(b_{1i}\right)\right)\cdot\frac{1}{\sigma'\left(c_{1i}\right)}\\
&-c_{1i}\cdot\frac{1}{1 - F_1\left(\sigma^{-1}\left(b_{1i}\right)\right)}\cdot\left(\mathcal{N}_1 - 1\right)\cdot f_1\left(\sigma^{-1}\left(b_{1i}\right)\right)\cdot\frac{1}{\sigma'\left(c_{1i}\right)}\\
&-c_{1i}\cdot\frac{1}{1 - F_2\left(\sigma^{-1}\left(b_{1i}\right)\right)}\cdot\mathcal{N}_2\cdot f_2\left(\sigma^{-1}\left(b_{1i}\right)\right)\cdot\frac{1}{\sigma'\left(c_{1i}\right)}
\end{aligned}$$

by collecting terms we obtain,

$$\sigma'\left(c_{1i}\right)=\left[\sigma\left(c_{1i}\right)-c_{1i}\right]\left\{\frac{\left(\mathcal{N}_1-1\right)\cdot f_1\left(\sigma^{-1}\left(b_{1i}\right)\right)}{1-F_1\left(\sigma^{-1}\left(b_{1i}\right)\right)}+\frac{\mathcal{N}_2\cdot f_2\left(\sigma^{-1}\left(b_{1i}\right)\right)}{1-F_2\left(\sigma^{-1}\left(b_{1i}\right)\right)}\right\} \quad (12)$$

Again in equilibrium $\sigma^{-1}\left(b_{1i}\right)=c_{1i}$

$$\sigma'\left(c_{1i}\right)=\left[\sigma\left(c_{1i}\right)-c_{1i}\right]\left\{\frac{\left(\mathcal{N}_1-1\right)\cdot f_1\left(c_{1i}\right)}{1-F_1\left(c_{1i}\right)}+\frac{\mathcal{N}_2\cdot f_2\left(c_{1i}\right)}{1-F_2\left(c_{1i}\right)}\right\} \quad (13)$$

Using $g_1(b)=\frac{f_1(c)}{\sigma_1'(c)}$, $g_2(b)=\frac{f_2(c)}{\sigma_2'(c)}$,$G_1(b)=F_1\left(\sigma^{-1}(b)\right)$,$G_2\left(b\right)=F_2\left(\sigma^{-1}(b)\right)$ we get

$$\sigma'\left(c_{1i}\right)=\left[\sigma\left(c_{1i}\right)-c_{1i}\right]\left\{\frac{\left(\mathcal{N}_1-1\right)\cdot g_1\left(b_{1i}\right)\cdot\sigma'\left(c_{1i}\right)}{1-G_1\left(b_{1i}\right)}+\frac{\mathcal{N}_2\cdot g_2\left(b_{1i}\right)\cdot\sigma'\left(c_{1i}\right)}{1-G_2\left(b_{1i}\right)}\right\} \quad (14)$$

Now we are able to eliminate the $\sigma'\left(c_{1i}\right)$ terms.

$$1=\left[\sigma\left(c_{1i}\right)-c_{1i}\right]\left\{\frac{\left(\mathcal{N}_1-1\right)\cdot g_1\left(b_{1i}\right)}{1-G_1\left(b_{1i}\right)}+\frac{\mathcal{N}_2\cdot g_2\left(b_{1i}\right)}{1-G_2\left(b_{1i}\right)}\right\} \quad (15)$$

$$\frac{1}{\left(\mathcal{N}_1-1\right)\cdot\frac{g_1\left(b_{1i}\right)}{1-G_1\left(b_{1i}\right)}+\mathcal{N}_2\cdot\frac{g_2\left(b_{1i}\right)}{1-G_2\left(b_{1i}\right)}}=b_{1i}-c_{1i} \quad (16)$$

After further simplification we arrive at an equation analogous to (9). When $\mathcal{N}_2=0$ (17) reduces to the equation obtained in the previous section.

$$c_{1i}=b_{1i}-\frac{1}{\left(\mathcal{N}_1-1\right)\cdot\frac{g_1\left(b_{1i}\right)}{1-G_1\left(b_{1i}\right)}+\mathcal{N}_2\cdot\frac{g_2\left(b_{1i}\right)}{1-G_2\left(b_{1i}\right)}} \quad (17)$$

In a similar manner one can derive the equation for bidders of type 2.

$$c_{2i}=b_{2i}-\frac{1}{\mathcal{N}_1\cdot\frac{g_1\left(b_{2i}\right)}{1-G_1\left(b_{2i}\right)}+\left(\mathcal{N}_2-1\right)\cdot\frac{g_2\left(b_{2i}\right)}{1-G_2\left(b_{2i}\right)}} \quad (18)$$

4.3. Heterogeneous Auctions

For notational simplicity we will assume symmetric bidders in the next two sections. If all the items up for auction are exactly the same in each of the T auctions, then no modifications are needed. However, if, as is often the case, the items differ along an observable dimension, we obviously want to allow bids to be conditioned on the characteristics of the item. In this setting, bids, and therefore bid distributions, are conditioned on z_t, $t\in 1,\ldots,T$, and so (9) becomes.

$$c_{it}=b_{it}-\frac{1}{\mathcal{N}-1}\cdot\frac{1-G\left(b_{it}|z_t\right)}{g\left(b_{it}|z_t\right)} \quad (19)$$

This can easily be reduced once again to observables using, $g(b|z) = \frac{g(b)}{h(z)}$ and $G(b|z) = \frac{G(b)}{h(z)}$ where $h(z)$ is the population distribution of auction characteristics.

4.4. Endogenous N

Guerre et al. [2000] extend their result to include situations with binding reserve prices. Aside from capturing an important institutional detail of many auction settings, the reserve price is a mechanism that creates a difference in the number of potential and actual bidders. Without a reserve price, or some other selection mechanism, it would be difficult to explain observed variations in the number of bidders. With binding reserve prices the econometrician does not observe the full population of bids associated with each cost draw. Rather, the only observed bids are those below the reserve price for the auction. This slightly modifies the link between observed bids and underlying cost distributions. Defining p_0 as the reserve price, the distribution of observed bids becomes

$$\begin{aligned} G(b) &= Prob\left[\tilde{b} \leq b | \tilde{b} \leq p_0\right] \\ &= Prob\left[\sigma^{-1}\left(\tilde{b}\right) \leq c | \tilde{b} \leq p_0\right] \\ &= \frac{F\left(\sigma^{-1}(b)\right)}{F(p_0)} \\ &= \frac{F(c)}{F(p_0)} \end{aligned} \tag{20}$$

$$G(b)\,F(p_0) = F(c) \tag{21}$$

$$g(b) = \frac{\partial}{\partial b} G(b) = \frac{\partial}{\partial b}\frac{F\left(\sigma^{-1}(b)\right)}{F(p_0)} = \frac{f\left(\sigma^{-1}(b)\right)}{F(p_0)\,\sigma'(c)} = \frac{f(c)}{F(p_0)\,\sigma'(c)} \tag{22}$$

$$g(b)\,F(p_0)\,\sigma'(c) = f(c) \tag{23}$$

By substituting (21) and (23) into (9) we have,

$$\begin{aligned} c &= b - \frac{\sigma'(c)}{g(b)\,F(p_0)\,\sigma'(c)}\left(\frac{1 - G(b)\,F(p_0)}{\mathcal{N} - 1}\right) \\ &= b - \frac{1}{\mathcal{N}-1} \cdot \frac{1 - G(b)\,F(p_0)}{g(b)\,F(p_0)} \end{aligned}$$

Theorem 2 (Guerre, Perrigne, and Vuong [2000, Theorem 4])

Let $G(\cdot)$ belong to the set $\mathcal{P}^N$with support $\left[p_0, \overline{b}\right]^{\mathcal{N}}$, and $\pi(\cdot)$ be a discrete distribution. There exists a distribution of bidders' private values $F(\cdot) \in \mathcal{P}$ and $\mathcal{N} \geq 2$ of potential bidders such that such that (i) $G(\cdot)$ is the truncated distribution of distribution of equilibrium bids in a first-price sealed-bid auction with reservation price $p_0 \in (\underline{c}, \overline{c})$ and (ii) $\pi(\cdot)$ is the distribution of the number of actual bidders $\mathcal{N}^a$ if and only if the following conditions hold:

C1:$\pi(\cdot)$ is binomial with parameters $(\mathcal{N}, F(p_0))$ where $0 < F(p_0) < 1$.

C2:The observed bids are i.i.d as $G(\cdot)$ conditionally upon $\mathcal{N}^a$ and $\lim_{b \downarrow p_0} g(b) = +\infty$

C3: The function $\xi(\cdot, G, \mathcal{N}, F(p_0))$ defined in (24) below is strictly increasing on $[p_0, \overline{b}]$ and its inverse is differentiable on $[\underline{c}, \overline{c}] \equiv [\xi(p_0, G, \mathcal{N}), \xi(\overline{b}, G, \mathcal{N})]$.

Proof: See Guerre, Perrigne, and Vuong [2000]

$$\xi(\cdot, G, N, F(p_0)) \equiv c = b - \frac{1}{\mathcal{N} - 1} \cdot \frac{1 - G(b) F(p_0)}{g(b) F(p_0)} \tag{24}$$

Theorem 2 is nearly what we need to analyze this market, however in this particular market there is no reserve price. This means we need a different mechanism to explain the observed variation in the numbers of bidders in each auction. The literature provides an abundance of possible settings, each of which results in endogenous entry by firms. The auction settings differ, however, in their implications for firm behavior. Often the analysis of differences between auction settings has to do with when the information discovery step and entry decisions are made. With reserve prices as specified above we would have information revelation occurring before the entry decision, this is because each firm compares their actual cost draw to the reserve price. In some instances it makes sense to use a model of uninformed entry where firms must first decide to participate before they learn their valuation. Settings where valuations are difficult to determine a priori could be analyzed with this type of model. In Kjerstada and Vagstad [2000]'s model of uninformed entry with symmetric equilibrium decisions, the results are driven by firms playing mixed strategy entry decisions in such a way that there is zero expected surplus. The model that most closely captures the institutional details in the school bus route market is examined in Li and Zheng [2009]. In particular they analyze the situation where firms pay an unobserved fixed cost prior to participating in the auction. This cost is incurred after the valuation discovery phase of the auction. One interpretation of the setting used in their model would be that firms must expend costly effort in preparing the bid. This cost is fixed and independent of the firms realized valuation. Following Li and Zheng [2009], we introduce a cutoff cost of c^* where a bidder is indifferent between participating in the auction or not. The firm maximizes expected profit by maximizing

$$E[\pi | c^*] = (b - c^*) \cdot (1 - F(b)) \tag{25}$$

We have assumed that all bidders bid as if they are facing at least one other firm in the auction, otherwise the equilibrium bid would be unbounded for firms with cost c^*. Solving for b^*, the equilibrium bid given c^* costs, yields

$$b^* = c^* + \frac{\int_{c^*}^{\overline{c}} (1 - F(x))\, dx}{1 - F(c^*)} \tag{26}$$

To find the value of c^* we use the zero profit condition in conjunction with (26) above. In equation (27) k is the bid preparation cost. Notice that it is independent of $\mathcal{N}$, yet the resulting cutoff cost is different for each value of $\mathcal{N}$.

$$\underbrace{(b^* - c^*)(1 - F(c^*))^{\mathcal{N}-1} - k}_{\text{zero profit condition}} = \underbrace{\int_{c^*}^{\overline{c}} (1 - F(x))\, dx}_{\text{substituted in } b^*} \cdot (1 - F(c^*))^{\mathcal{N}-2} - k \tag{27}$$

This establishes that for every auction with $\mathcal{N}$ bidders there is an associated cutoff cost above which firms will not participate in the auction. Above this $c^*(\mathcal{N})$ the bid preparation cost k makes the expected profit from participation negative. If we re-interpret $F(p_0)$ as $F(c^*(\mathcal{N}))$ all the same results hold. So rather than needing to estimate the probability of costs being above the reserve price, we need to estimate the probability of costs being below this cutoff cost.

5. Estimation

By combining the results from sections §4.1 − §4.4 we get one unified set of equations (28) and (29) that capture all the various refinements of the base model. We denote the numbers of bidders with $\mathcal{N}_{1it}$ to be the i^{th} firm of type 1 in the marketplace for auction t. Another way of saying this is that auction t has N_{1t} potential bidders of type 1, of which firm i is one.

$$c_{1it} = b_{1it} - \frac{1}{(\mathcal{N}_{1t}-1)\frac{g_1(b_{1it}|z_t,\mathcal{N}_t)\cdot F_1(c^*|\mathcal{N}_t)}{1-G_1(b_{1it}|z_t,\mathcal{N}_t)\cdot F_1(c^*|\mathcal{N}_t)} + \mathcal{N}_{2t}\frac{g_2(b_{1it}|z_t,\mathcal{N}_t)\cdot F_2(c^*|\mathcal{N}_t)}{1-G_2(b_{1it}|z_t,\mathcal{N}_t)\cdot F_2(c^*|\mathcal{N}_t)}} \tag{28}$$

$$c_{2it} = b_{2it} - \frac{1}{\mathcal{N}_{1t}\frac{g_1(b_{2it}|z_t,\mathcal{N}_t)\cdot F_1(c^*|\mathcal{N}_t)}{1-G_1(b_{2it}|z_t,\mathcal{N}_t)\cdot F_1(c^*|\mathcal{N}_t)} + (\mathcal{N}_{2t}-1)\frac{g_2(b_{2it}|z_t,\mathcal{N}_t)\cdot F_2(c^*|\mathcal{N}_t)}{1-G_2(b_{2it}|z_t,\mathcal{N}_t)\cdot F_2(c^*|\mathcal{N}_t)}} \tag{29}$$

Now $G(b, z, \mathcal{N})$ and $g(b, z, \mathcal{N})$ can be nonparametrically estimated from the observed bids, so long as we also observe $\mathcal{N}_t$.[24] For any values of b, z, n we can nonparametrically estimate G_1, G_2, g_1, g_2 and h using equations (30) - (32). The kernels used as well as bandwidth selection are discussed in §6.2.

[24]Situations where the potential bidders are not observed, and therefore must be estimated are discussed in §6.1

$$\hat{G}(b,z,n)=\frac{1}{Th_g}\sum_{t=1}^{T}\frac{1}{\mathcal{N}_t}\sum_{i=1}^{\mathcal{N}_t}\mathbf{1}(\beta_{it}\leq b)\,\kappa\left(\frac{z-z_t}{h_g}.\frac{n-\mathcal{N}_t}{h_{gn}}\right) \tag{30}$$

$$\hat{g}(b,z,n)=\frac{1}{T}\sum_{t=1}^{T}\frac{1}{\mathcal{N}_t}\sum_{i=1}^{\mathcal{N}_t}\frac{1}{h_g^2}\kappa\left(\frac{b-\beta_{it}}{h_g},\frac{z-z_t}{h_g}.\frac{n-\mathcal{N}_t}{h_{gn}}\right) \tag{31}$$

$$\hat{h}(z,n)=\frac{1}{T}\sum_{t=1}^{T}\frac{1}{\mathcal{N}_t}\sum_{i=1}^{\mathcal{N}_t}\frac{1}{h_h}\kappa\left(\frac{z-z_t}{h_h}.\frac{n-\mathcal{N}_t}{h_{hn}}\right) \tag{32}$$

6. Implementation of Estimation Procedure

6.1. Estimating $\mathcal{N}$ & Entry Probability

It is clear from Equations (28) and (29) above that we need an estimate of $F(c^*|\mathcal{N})$, which is simply the probability that the realized costs are less than or equal to the cutoff cost c^* induced by the bid preparation cost k. To estimate this value we fist need an to know $\mathcal{N}$. In some instances the number of potential bidders is known to the econometrician. This could be because all bidders are physically present in the same location or bidders must pre-register or request bid specification documents. (This is often the case with highway contracts, or when the auctioneer wants to pre-screen participants to be sure they meet some criteria.)

When this information is not available, a commonly used estimate of the potential number of bidders is the number of bidders in past auctions in the market. We can approximate $\mathcal{N}$[25] as the maximum number of bidders that has ever been observed in the market.[26]

Estimating N_{bh} with bidding history Our first method for estimating $\mathcal{N}$ is to use our auction data to identify the number of bidders active in a particular district. This is an unbiased estimate if the data we have for that district is a representative sample.

Estimating N_{ph} with payment history We cannot ignore the possibility that some firms only participated in auctions that are not part of our data set. In order to include such firms we formed a second estimate by examining payments to private busing firms over the course of our study. In this method if a firm is receiving funds from a school district for busing services it must be because they won an auction for that route. The drawback is that with this method we include firms that have ceased participating but are still operating

[25] For simplicity we suppress the subscript t in this section, thereby assuming there is only one market and one corresponding "true" $\mathcal{N}$. The estimation procedure doesn't change substantially when we examine markets with different numbers of potential bidders.

[26] This is also suggested by Flambard and Perrigne [2006] as well as Guerre et al. [2000].

contracts won in the past. We combine these two estimates to form our estimate of $\hat{\mathcal{N}} = N_{ph} \cup N_{bh}$

The total number of potential bidders in auction t is given by $\mathcal{N}_t = \mathcal{N}_{1t} + \mathcal{N}_{2t}$. We differentiate this from the observed, or actual, number of bidders by using the notation $\mathcal{N}_t^a = \mathcal{N}_{1t}^a + \mathcal{N}_{2t}^a$ for the latter.

The probability of bidding $\hat{F}(c^*|\mathcal{N}_t)$ can be estimated using our estimate of $\hat{\mathcal{N}}_t$, as well as the actual number of bidders $\mathcal{N}_t^a$. We used the standard Logit model to estimate the probability of bidding. The results of this estimation can be seen in Table 6.

Using the computed values for $F_1(\cdot|\cdot)$,$F_2(\cdot|\cdot)$,$g_1(\cdot|\cdot)$,$g_2(\cdot|\cdot)$,$G_1(\cdot|\cdot)$,$G_2(\cdot|\cdot)$ we have all the elements we need to solve for costs in equations (28) and (29).

Table 6: Bid Probabilities

Near Firms		Far Firms		
Pr(not bid|N)	Pr(bid|N)	Pr(not bid|N)	Pr(bid|N)	N
0.203	0.797	0.3753	0.6247	2
0.2413	0.7587	0.4222	0.5778	3
0.2841	0.7159	0.4705	0.5295	4
0.3313	0.6687	0.5193	0.4807	5
0.3821	0.6179	0.5678	0.4322	6
0.4356	0.5644	0.615	0.385	7
0.4907	0.5093	0.6602	0.3398	8
0.546	0.454	0.7026	0.2974	9
0.6002	0.3998	0.7418	0.2582	10
0.652	0.348	0.7774	0.2226	11

6.2. *Kernels and Bandwidths*

As with anytime nonparametric methods are applied, care must be given to the decision of which kernel(s) and bandwidth(s) to use. Following Racine [2007][27] we use a generalized product kernel, which constructs a multidimensional kernel by multiplying appropriate single dimensional kernels. Since both z_t[28] and bids are continuous we use two Second-Order Gaussian kernels. We treat $\mathcal{N}$[29] in the same manner, which makes our kernel $K(\cdot,\cdot,\cdot) = k(z) \cdot k(z) \cdot k(z)$ where $k(z)$ is the Second Order Gaussian single dimensional kernel:

[27] Also found in Hall et al. [2004]

[28] We define z_t in §6.3

[29] Separate estimates were conducted treating $\mathcal{N}$ as ordered discrete variable. In this case we use the kernel suggested by Wang and Ryzin [1981] for discrete ordered variables:

$$l(xi,x,w)) \begin{cases} 1-w & \text{if } |x_i - x| = 0 \\ (1-w)\cdot w^{|x_i - x|} & \text{if } |x_i - x| \geq 0 \end{cases} \text{ and} w \in (0,1)$$

Therefore $g(b,z,\mathcal{N})$ would be calculated using the product of two Second Order Gaussian Kernels and the kernel suggested above for discrete ordered variables, making the kernel: $K(\cdot,\cdot,\cdot) = k(z)\cdot k(z)\cdot l(x_i,x,w)$. Results were qualitatively similar using this approach.

$$k(z) = \frac{1}{\sqrt{2\pi}} e^{\left(\frac{z^2}{2}\right)} \text{ where } z = {(x_i - 1)}/{h} \text{ and } h > 0 \tag{33}$$

We used the "Rule-of-Thumb" Bandwidth selection procedure to select an appropriate bandwidth for each of the three distributions. This data driven selection method is concisely described in described in R's package 'np' documentation [Hayfield and Racine, 2008] as:

> "... [the] 'rule-of-thumb' bandwidth h_j[is calculated] using the standard formula, $h_j = 1.06 \cdot \sigma_j \cdot n^{\left(-\frac{1}{2P+l}\right)}$ where σ_j is an adaptive measure of spread of the jth continuous variable defined as min(std. dev,$\frac{\text{interquartile range}}{1.349}$), n the number of observations, P the order of the kernel, and l the number of continuous variables."

6.3. Dimensionality reduction

Nonparametric methods offer greater flexibility, and thus avoid the risks associated with mis-specifying a parametric model, however a well know drawback of nonparametric methods is that more data is needed to achieve a given level of accuracy. Nevertheless, the data requirements for nonparametric methods grow quickly as the dimension of the model increases. This is the well known "curse of dimensionality."

One obvious way to address these issues is to collect more data, but when that is not a possibility we must turn to potentially "lossy"[30] data manipulation techniques. There are three variables that we use to characterize a particular auction,

1. # of bus stops on the route (as an estimate for route length)
2. The number of students transported (which captures information about the size of the bus needed to service the route)
3. The year that the auction takes place (captures characteristics that vary over time)

We reduce these three variables down to a one dimensional auction specific variable z_t that is used to condition bids and costs as described in §4.3 . The variable z_t was constructed using Principal Component Analysis (PCA). We also attempted to reduce the dimensionality of our data using a regression to construct a single index. The results were qualitatively similar. With PCA three dimensions are projected down onto one[31], but the axes used for the projection are rotated such that the resulting projection loses the least amount of variation. After de-meaning the variables, z_t is defined as[32]

[30]By lossy we mean that, by necessity, some information contained in the original data is lost during the dimension reduction process.

[31]Projecting down to two dimensions rather than one would preserve more information but add to the data requirements.

[32]See §AppendixD for a more detailed discussion of the dimension reduction process.

$$z_t = \begin{pmatrix} 0.5475 \\ 0.4783 \\ 0.6867 \end{pmatrix}^T \begin{pmatrix} Stops \\ Riders \\ Year \end{pmatrix}$$

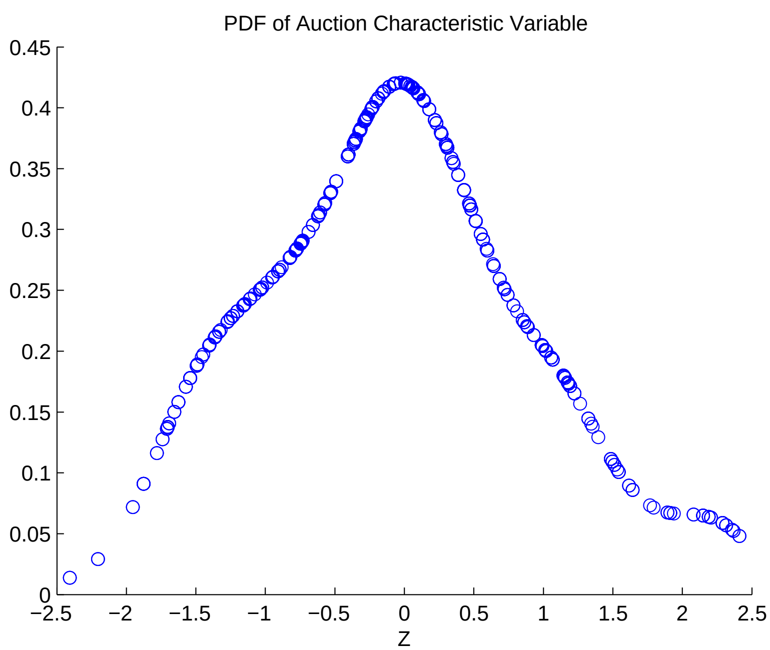
This computation results in z_t values with the following properties:

Table 7: Auction Characteristic Variable

Auction specific characteristic values (z_t)					
Min	Max	Average	Median	Variance	Standard Deviation
-1.95363	2.368767	-0.05775	-0.07228	1.043774	1.021653

Figure 1 plots the points corresponding to the calculated z_ts against the estimated *pdf* of z_t.

Figure 1: Auction Characteristic Variable



6.4. Grouping Decisions

The estimation procedure described in the previous sections is intentionally vague as to what exactly type 1 and type 2 bidders are. This highlights the fact that the procedure can be used for arbitrary groupings of firms. Indeed if the data allows it, we can extend the results to as many distinct groups as we desire. The initial motivation for this analysis led us to first try grouping firms into two qualitative groups, "large" and "small" firms. Differences in the estimated cost distributions could then be interpreted as a measure of the cost (dis)advantages enjoyed by large firms. We also considered two other alternatives: (1) grouping firms into "Laidlaw/First Student" and "Other Firms," with the same type of interpretation being given to the relationship between estimated cost distributions; (2) grouping firms based on the distance of their depots to the schools being served. In the absence of any indisputably correct grouping, we selected the groupings that induced the greatest difference in the estimated cost distributions.[33] This process allowed us to identify the "most important" factor by which firms could be grouped. Based on this criterion, we selected depot-school distance as the best grouping. This result is consistent with the information provided by Jerry Ford that distance is one of the most, if not the most, important cost factor in this market. Distance has also been identified as an important cost factor in other procurement auction settings for instance in Jofre-Bonet and Pesendorfer [2003] as well as Flambard and Perrigne [2006].

It may seem more natural to use the depot-school distance as another continuous variable to condition bids on, much like z_t is used. However without access to a larger set of bid data we hesitate to introduce another dimension into our estimation, the reasons for which are discussed in §(6.3). The following histogram of depot-school distances, suggests two natural candidate distances for our grouping decision, 5 miles and ~10 miles.

We chose to categorize firms with $distance \leq 10$ miles as Type 1, and firms with $distance > 10$ miles as Type 2. The primary reason for this choice is that at the 10 mile mark our data is more evenly split between the two groups, allowing us to estimate costs for each group with a similar number of observations. The corresponding empirical *cdf* is plotted in Figure (3).[34]

6.5. Results

Given the setup of the previous sections, the task of backing out pseudo-costs is relatively straight forward. For each bid we now have a corresponding grouping for the bidder, an auction characteristic variable that describes the particular route being bid on, and a list of firms that competed in the auction (regardless of whether or not firms cost draws were sufficiently low to elicit a bid §6.1). With these values we are able to compute either equation (28) when the bid is from a close firm (Type 1 bidder) or (29) when the bidder is considered a far competitor for that particular route (Type 2 bidder). For the sake of

[33]The greatest difference was determined qualitatively from the resulting cost distribution plots, as well as Mann-Whitney-Wilcoxon test for equal distribution.
[34]Here observations include both bids and non-bids.

Figure 2: Distance from Depot to School

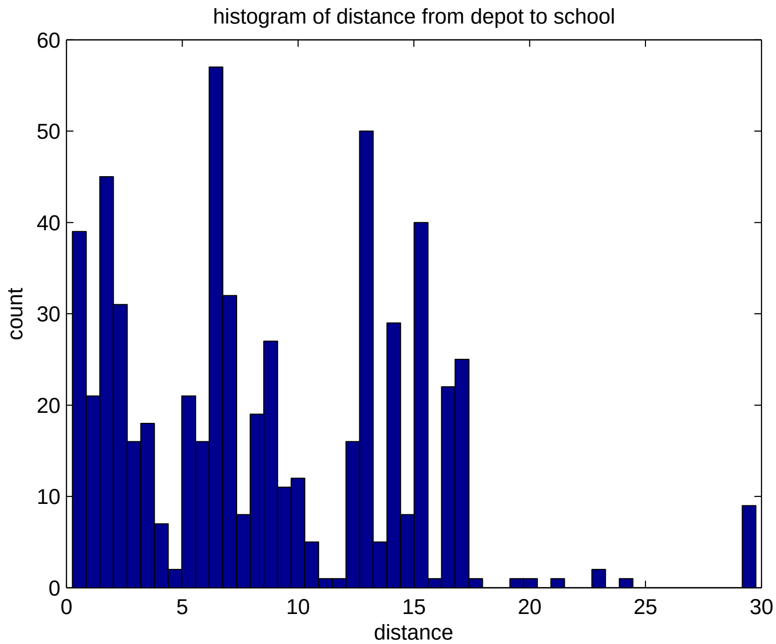
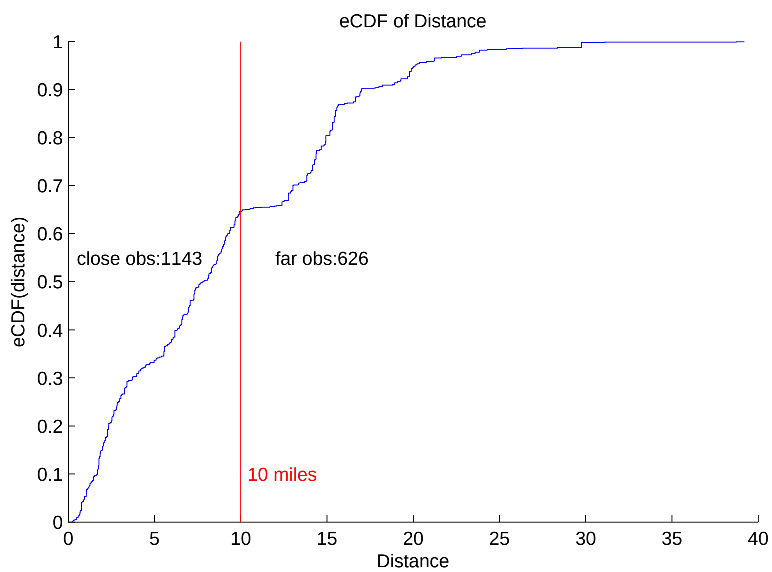


Figure 3: Distance from Depot to School



exposition, consider the case of a close firm.. Then using (28):

$$\overbrace{\hat{c}_{1it}}^{\text{pseudo}-\text{cost}} = \overbrace{b_{1it}}^{\text{observed}} - \frac{1}{\underbrace{\left(\hat{\mathcal{N}}_{1t}-1\right)}_{\mathbb{P}} \cdot \frac{\overbrace{\hat{g}_1\left(b_{1it}|z_t,\hat{\mathcal{N}}_t\right)}^{\mathbb{Q}} \cdot \hat{F}_1\left(c^*|\mathcal{N}_t\right)}{1-\underbrace{\hat{G}_1\left(b_{1it}|z_t,\hat{\mathcal{N}}_t\right)}_{\mathbb{R}} \cdot \underbrace{\hat{F}_1\left(c^*|\hat{\mathcal{N}}_t\right)}_{\mathbb{S}}} + \underbrace{\hat{\mathcal{N}}_{2t}}_{\mathbb{T}} \cdot \frac{\overbrace{\hat{g}_2\left(b_{1it}|z_t,\hat{\mathcal{N}}_t\right)}^{\mathbb{U}} \cdot \hat{F}_2\left(c^*|\hat{\mathcal{N}}_t\right)}{1-\underbrace{\hat{G}_2\left(b_{1it}|z_t,\hat{\mathcal{N}}_t\right)}_{\mathbb{V}} \cdot \underbrace{\hat{F}_2\left(c^*|\hat{\mathcal{N}}_t\right)}_{\mathbb{W}}}}$$

Breaking the equation down into its constituent parts can be informative. The first term in the denominator contains estimated distributions pertaining to close firms (Type 1).

$\mathbb{P}$	$\left(\hat{\mathcal{N}}_{1t}-1\right)$	Estimate of the number of other close competitors. (§6.1,§6.4)
$\mathbb{Q}$	$\hat{g}_1\left(b_{1it}\|z_t,\hat{\mathcal{N}}_t\right)$	Estimated *pdf* of bids (§5, §6.2) for close firms given the total number of competitors $\mathcal{N}_t$(§6.1) and auction characteristic z_t(§6.3)
$\mathbb{R}$	$\hat{G}_1\left(b_{1it}\|z_t,\hat{\mathcal{N}}_t\right)$	Estimated *cdf* of bids (§5, §6.2) for close firms given the total number of competitors $\hat{\mathcal{N}}_t$(§6.1) and auction characteristic z_t(§6.3)
$\mathbb{S}$	$\hat{F}_1\left(c^*\|\hat{\mathcal{N}}_t\right)$	Entry probability (§6.1) of close firms when there are $\hat{\mathcal{N}}_t$ competitors
$\mathbb{T}$	$\hat{\mathcal{N}}_{2t}$	Estimate of the number of far competitors. (§6.1,§6.4)
$\mathbb{U}$	$\hat{g}_2\left(b_{1it}\|z_t,\hat{\mathcal{N}}_t\right)$	Estimated *pdf* of bids for **far** firms evaluated at the **close** firm's bid b_{1it} given the total number of competitors $\hat{\mathcal{N}}_t$ and auction characteristic z_t
$\mathbb{V}$	$\hat{G}_2\left(b_{1it}\|z_t,\hat{\mathcal{N}}_t\right)$	Estimated *cdf* of bids for **far** firms evaluated at the **close** firm's bid b_{1it} given the total number of competitors $\hat{\mathcal{N}}_t$ and auction characteristic z_t
$\mathbb{W}$	$\hat{F}_2\left(c^*\|\hat{\mathcal{N}}_t\right)$	Entry probability of far firms when there are $\hat{\mathcal{N}}_t$ competitors

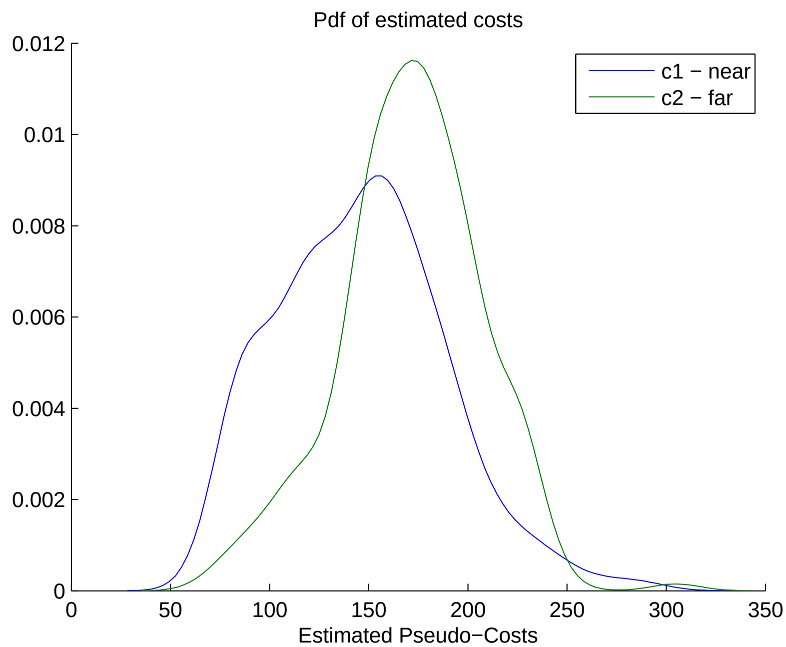
Notice that if $\hat{\mathcal{N}}_{2t} = 0$ then the second term containing $\mathbb{T}, \mathbb{U}, \mathbb{V}$ and $\mathbb{W}$ drops out and we are left with the symmetric case analyzed by Guerre et al. [2000].

Figure 4 shows the smoothed pseudo-cost *pdf*s for each group of firms. Figure 5 shows the empirical *cdf* for the pseudo-costs, as well as the results of a Mann Whitney Wilcox test for equal distributions.[35] We are able to strongly reject that the two groups of firms share a common cost distribution.

With these estimated pseudo-cost curves we are now able to simulate auction outcomes in a variety of settings. In the next section we will apply the results to a set of hypothetical mergers between various firms in the market.

[35] The results of this test, included in Figure 5, strongly rejects the null of equal distributions.

Figure 4: *pdf* of estimated cost distributions



7. Simulations

Rather than study the effects on one particular merger, we take advantage of the newly created pseudo-cost distributions and simulate a large set of possible mergers. Computational concerns suggest not simulating all possible merger pairings. Rather, we identified mergers of interest: those involving firms that we observe bidding against each other. Then the set of mergers to simulate is this list of pairs of firms.[36] The rationale is that a merger of two firms with a history of bidding against each other is more likely to be of interest to antitrust agencies than a merger of two firms with no past history of head-to-head competition. This winnowing process leaves us with a total of 102 mergers of two firms, each of which we simulate separately. These 102 mergers resulted in 276 simulated auctions.[37] For example, if our data set consisted of 2 auctions with bids from 4 firms,

[36]We limit ourselves to the analysis of mergers between two firms at one time.

[37]This number reflects the total number of auctions simulated. Each of these simulations correspond to an auction with a particular type of merger: near firm with another near firm, a far firm with a near firm, or a two far firms.

Figure 5: *cdf* of estimated cost distributions

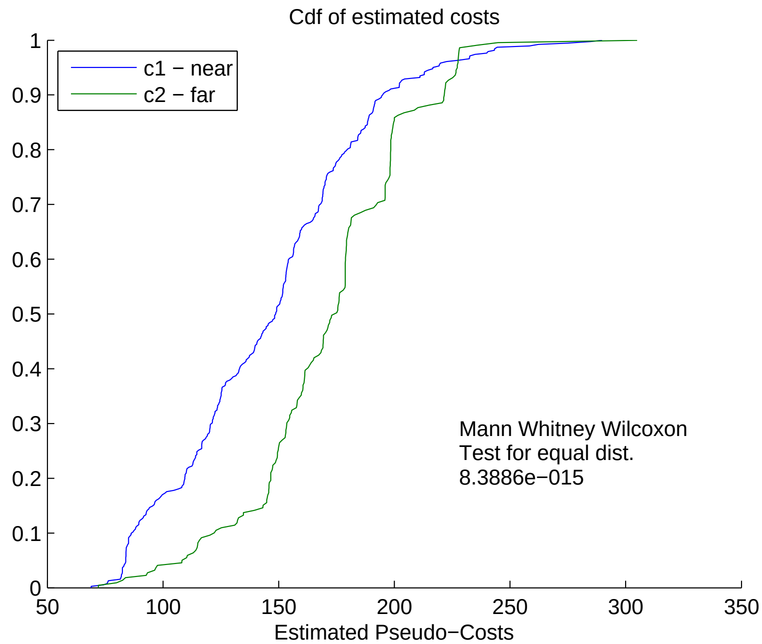


Table 8: Example Auctions

Auction # 1	Observed Bidders	Auction #2	Observed Bidders
	Firms A, B, & D		Firm A, B & C

would result in the simulation of mergers {AB,AD,AC,BD,BC}.[38] Without any limitations on the mergers and auctions to simulate, we would have $\binom{4}{2} = 6 \cdot 2 = 12$ simulations. This can be seen in Table 9, which lists the required simulations under both approaches.

[38]It is important to note here that while we don't simulate the merger of firms D & C that they are both indeed included in the set of potential bidders for each auction. It would be reasonable to simulate auction outcomes for all auctions that the two firms are competitors (regardless of whether or not they ever bid against each other). Since potential bidders (competitors) were determined at a district wide level, doing so would drastically increase the number of mergers and auctions that must be simulated. There is however nothing inherent in our methodology that prevents us from doing so at a future date.

Table 9: Required Simulations

With Heuristics			Without Heuristics	
Merging firms	Simulate auction #1	Simulate auction #2	Simulate auction #1	Simulate auction #2
A,B	Y	Y	Y	Y
A,C	N	Y	Y	Y
A,D	Y	N	Y	Y
B,C	N	Y	Y	Y
B,D	Y	N	Y	Y
C,D	N	N	Y	Y
Total simulated auctions		6	Total simulated auctions	12

Continuing with this example, we take this set of 6 auctions to simulate (AB,1), (AB,2), (AC,2), (AD,1), (BC,2), (BD,1) and simulate the outcomes of each 1000 times. In each of these 1000 repetitions we draw costs for each firm and calculate the bid associated with these costs. Each time auction #1 is simulated we compute bids for each of the 3 firms pre-merger, as well as those for the two post merger firms. Cost draws are held constant pre- and post- merger to ensure that what is measured is only due to the merger, and not due to the randomness of the cost draws. The average over these 1000 outcomes is then taken as the outcome associated in that auction when those two firms merge. Confidence intervals were then constructed based on the bootstrap percentiles.

7.1. Simulation Results

Of the 276 simulated auctions, 162 had post-merger prices that were higher than the simulated pre-merger outcomes, of these 73 are statistically significant at the 95% level. The other 114 had expected procurement costs that were lower post merger. Only 12 of these were significant at the 95% level. Tables 10-12 show the results for each of the simulated auction. "**" indicates significance at the 95% level.

The effects of a particular merger of interest can be calculated by summing the appropriate outcomes in Tables 10-12. For example we can estimate the expected effects of a merger between Laidlaw and First Student as follows. In 14 auctions Laidlaw has a depot that is "near" to the school being serviced and First Student has a depot "far." In 28 other auctions the situation is reversed. The 42 corresponding simulations are reported in Table 11. 15 of the simulated auctions resulted in an increase in the expected procurement costs; 14 of these estimates are significant at the 95% level. Of the 22 auctions with post merger procurement costs lower than the pre-merger estimates, none are significant.

Table 13 shows the auction level results broken down by where the two merging firms are situated relative to each other and the school being served by the route up for bid. When both firms are less than 10 miles from the school, the merger is classified as a "near/near" merger. The other entries are similarly defined.

One might think that the driving factor in our results is the number of potential bidders, pre- and post- merger. Indeed, our analysis indicates that, on average, mergers are associated with modest price increases. However, when we

Table 10: Simulated Auction Outcomes: Near/Near Mergers

ID	Mean Change ($)
1	0.37
8	-0.71
9	-0.43
10	-0.25
19	-0.93**
20	-0.56
21	-0.37
22	-0.67
23	0.00
24	-0.09
32	-0.34
33	-0.13
40	4.82
42	4.89
47	4.17
48	4.20**
53	5.11**
56	-0.99
57	-0.58
62	0.49
63	4.69**
65	4.02
66	3.96**
81	0.22
82	-3.05
84	-0.99
85	-0.67
87	-7.94

ID	Mean Change ($)
94	7.84**
99	1.46
100	0.93
101	-0.52
102	1.60
103	-0.69
104	-0.37
105	-0.81
106	-0.22
107	-0.28
108	-0.23
109	-1.40**
110	-0.25
111	-0.45
112	0.08
113	-0.26
114	0.15
116	0.14
120	-1.30**
121	-1.24**
122	0.18
123	-0.10
124	-0.25
125	-0.14
128	-1.50**
129	-1.63**
130	-1.33**
131	-0.10

ID	Mean Change ($)
132	-0.06
133	-1.36**
134	-0.19
135	-0.40
136	-0.21
137	10.05
139	5.24**
140	4.35**
141	10.95
142	-0.37
146	0.16
156	-0.03
157	0.13
159	-0.02
160	0.16
161	0.18
162	0.05
163	-1.22
164	0.05
166	-0.33
167	-0.71
168	-0.84
169	-0.81
170	0.10
190	-1.38**
191	0.16
192	0.01
195	4.20

ID	Mean Change ($)
196	5.02
198	5.53**
200	3.96
201	4.97
207	0.05
208	0.13
209	-0.10
216	21.85**
221	-0.13
222	-0.44
223	5.07
224	4.12**
225	-7.70
226	2.96
227	3.88**
228	5.21**
229	5.22
230	3.95
240	7.20**
241	7.47**
242	7.28**
243	7.25**
246	7.30**
254	7.82**
259	7.85**
266	7.38**
267	8.02**
268	7.26**

Table 11: Simulated Auction Outcomes: Near/Far Mergers

ID	Mean Change	ID	Mean Change	ID	Mean Change	ID	Mean Change
12	1.83	97	3.16**	200	1.32	253	3.38**
18	3.68	141	8.95	201	2.19	254	3.73**
25	-5.50	146	-1.67	218	4.51**	255	3.00**
34	4.11**	147	-0.32	219	4.41**	256	3.19**
35	3.98**	148	-0.25	220	4.52**	257	3.85**
36	4.23**	149	-0.89	221	-1.32	259	3.80**
37	3.36**	150	-1.62	222	-1.63**	260	3.93**
38	-6.32	151	-1.51	225	-7.80**	262	3.79**
52	-5.69**	152	-0.24	229	2.86	263	3.35**
53	3.57	153	-1.42	230	1.34	265	3.23**
58	7.68	154	-1.90	238	4.08**	266	2.89**
59	2.08	155	-1.64	239	3.28**	267	2.79**
62	-3.33	156	-1.83	240	2.88**	268	3.23**
64	2.02	157	-1.78	241	3.39**	270	3.16**
71	-0.92	158	-1.01	242	3.21**	271	3.45**
86	4.20**	159	-1.74	243	2.98**	272	4.21**
87	-8.08	160	-1.72	244	3.14**	273	2.87**
88	3.32	161	-1.65	245	3.32**	274	3.17**
89	4.00	162	-1.87	246	3.08**	275	3.74
90	3.99**	163	-2.38	247	4.56**	277	3.91**
91	4.07**	164	-1.80	248	2.91**	280	-0.15
92	4.16**	165	-3.76	249	3.96**	283	-1.60
93	4.16	195	1.87	250	3.87**		
94	3.88**	197	6.54	252	4.19		

Table 12: Simulated Auction Outcomes: Far/Far Mergers

ID	Mean Change	ID	Mean Change	ID	Mean Change	ID	Mean Change
17	20.33	146	-0.13	239	1.32	259	1.00
18	6.87**	147	2.08	240	-0.46	260	0.25
25	-3.19	148	1.89	241	-0.81	262	-0.28
34	0.90	155	-0.06	242	-0.95	263	-0.11
35	1.98	156	-0.50	243	-0.48	265	1.59
36	1.61	157	-0.31	244	1.58	266	-0.16
37	6.66**	158	0.95	245	1.59	267	1.51
38	-2.75	159	-0.36	246	-0.54	268	0.31
59	5.49	160	-0.15	247	9.50**	270	0.04
61	18.65**	161	-0.22	248	0.59	271	-0.15
73	0.77	162	-0.58	249	-0.37	272	0.95
74	1.00	163	-0.30	250	0.04	273	0.26
88	3.23	164	-0.40	252	6.83**	274	0.14
89	2.67	165	-0.56	253	-0.37	275	8.10**
90	0.20	175	2.30	254	1.09	277	-0.21
92	0.61	225	-0.68	255	-0.06	283	3.26
94	0.95	229	4.65	256	-0.02		
97	0.44	238	1.60	257	-0.06		

Table 13: Outcomes by Merger Type

Merger Type	Average Change ($)	Count
Near/Near	1.64	70
Near/Far	1.64	94
Far/Far	1.58	112

Table 14: Changes in "Expected N"

Change in expected N	Mean Price Change	Obs
-0.7635	5.79	13
-0.6821	4.71	28
-0.4840	1.45	13
-0.3846	1.78	143
-0.1236	-0.47	14
-0.0116	-0.47	43

examine the distribution of the post-merger price changes a suggestive pattern is revealed. From a theoretical perspective, the relationship between mergers and price changes should depend on the change in the expected number of bidders. We can calculate this value by interacting the number of competitors with the entry probabilities calculated in §6.1. The "change in predicted N" is calculated as follows:

pre-merger expected $N = \mathcal{N}_1 \cdot F_1 \left(c^* | \mathcal{N}_t\right) + \mathcal{N}_2 \cdot F_2 \left(c^* | \mathcal{N}_t\right)$

For mergers of the type near/near, we use

post-merger expected $N = (\mathcal{N}_1 - 1) \cdot F_1 \left(c^* | \mathcal{N}_t^{'}\right) + \mathcal{N}_2 \cdot F_2 \left(c^* | \mathcal{N}_t^{'}\right)$ and $\mathcal{N}_t^{'} = \mathcal{N}_t - 1$ as there is one less firm post merger.

For mergers of the near/far,[39] and far/far types we define the quantity as,

post-merger expected $N = \mathcal{N}_1 \cdot F_1 \left(c^* | \mathcal{N}_t^{'}\right) + (\mathcal{N}_2 - 1) \cdot F_2 \left(c^* | \mathcal{N}_t^{'}\right)$

Table shows the change in expected $\mathcal{N}$ and the change in winning bids. Simply looking at the chart we can see that mergers that result in a decrease in what we are calling "expected N" result in increased post merger pricesWhat is interesting however is that there are some mergers where the interaction between the decrease in total firms in the market and changes in entry probabilities result only a very slight change in the level of competition as measured by expected N.

Table 14 shows the change in expected N and the mean change in winning bids.[40] The larger negative changes in expected N are associated with prices increases; for auctions where the magnitude of the expected change in N is relatively small the average changes in price are small and negative. That is, merger-induced entry can overwhelm the price increases that might be expected with a reduction in the number of potential bidders. We are currently investigating the conditions which favor each of these outcomes, e.g. the type of merger (near/far, etc.), the number of near and far competitors, etc.

[39] In mergers of a near firm with a far firm, we treat the merged firm as a near competitor, meaning that post merger there is one less far firm.

[40] Table 14 does not list entries with 10 or fewer observations. The full table can be found in AppendixE

8. Discussion

A number of extensions and robustness tests are planned. Our current approach to handling cost asymmetries groups firms into near and far categories based on a 10 mile bus depot/school distance threshold; we plan to test the sensitivity of the results to other thresholds, e.g. 5 miles. We may also consider conditioning bids on a continuous measure of this distance. Haile et al. [2003] suggest a possible way to incorporate bidder specific information into the analysis, by assuming an additively separable valuation function.

The typical merger simulation ignores the possibility of post-merger entry. Our framework allows us to compare the no-entry and endogenous entry cases. In the next version of the paper we will include a comparison of the two cases. Since entry in our context is likely to occur in the short run as defined by the US Merger Guidelines (less than 2 years), the results of this comparison are of obvious policy relevance. Indeed, our initial results suggest that the set of potential anti-competitive mergers and the corresponding welfare losses are likely to be smaller when endogenous entry is permitted.

Lastly, an event study using data generated since the Laidlaw/First Student merger should be feasible. Three years of post-merger auction data from New Jersey are now available. We hope to compare actual post-merger auction outcomes with predictions derived from our model.

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AppendixA. District Maps

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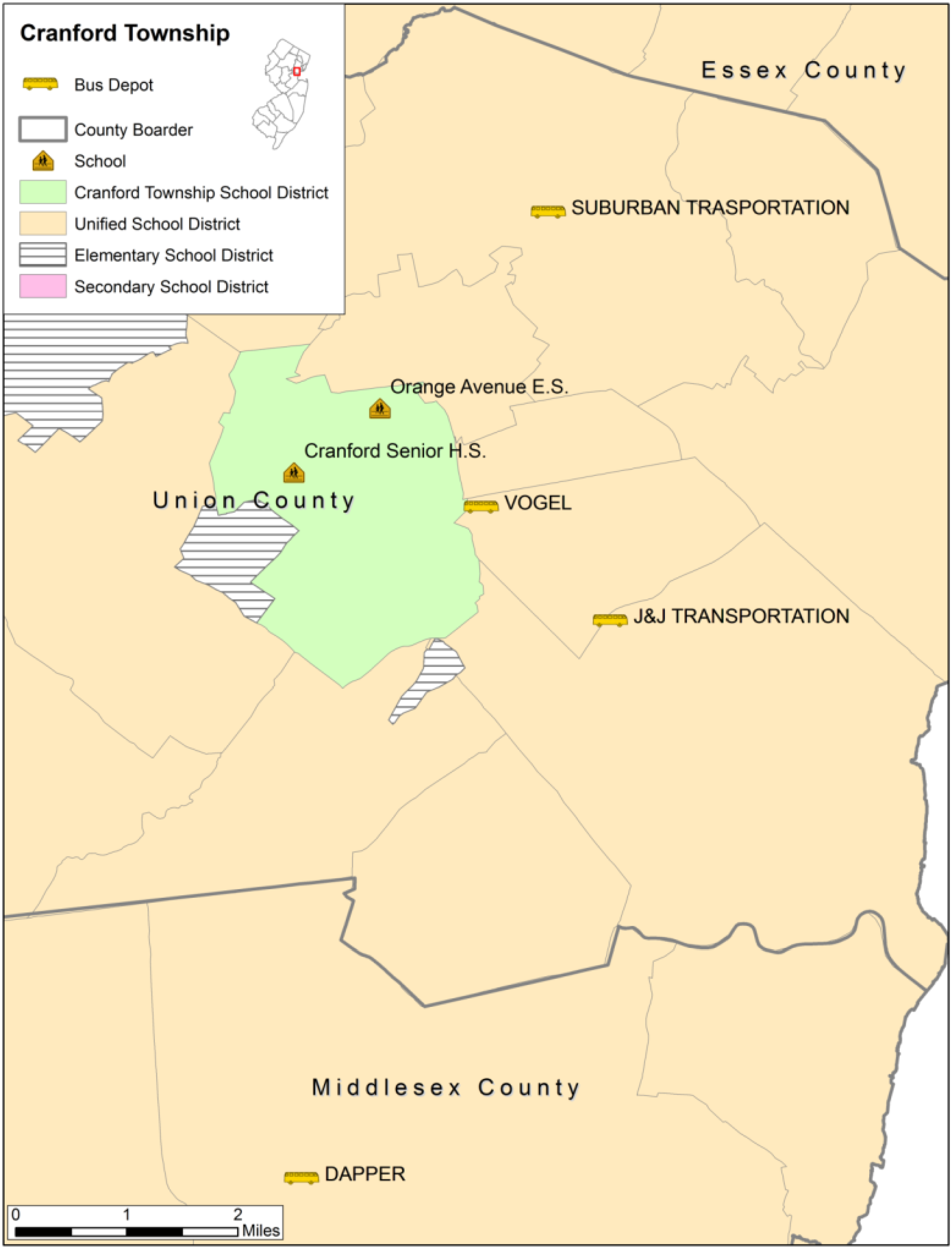
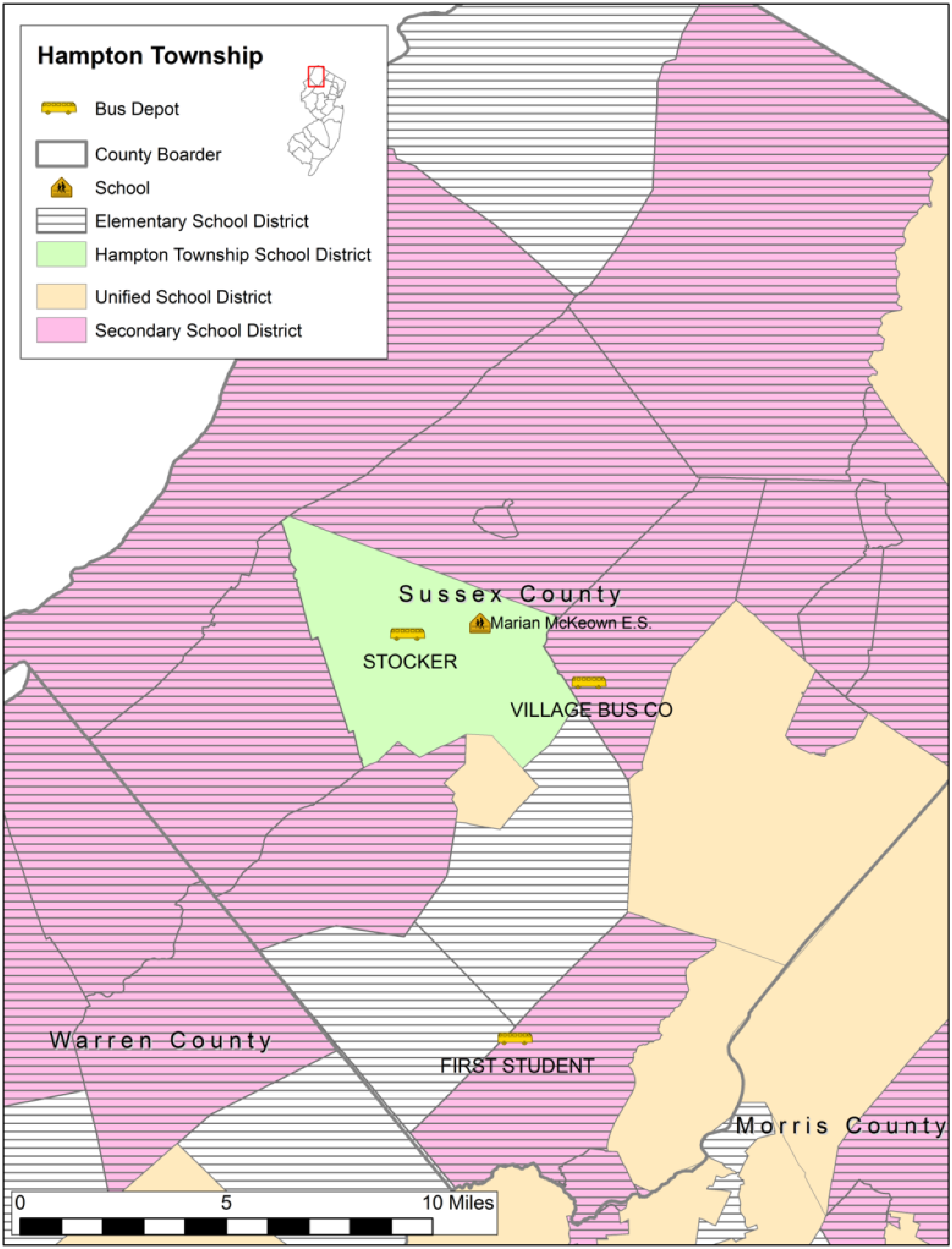


Cranford Township
Bus Depot
County Boarder
School
Cranford Township School District
Unified School District
Elementary School District
Secondary School District
Essex County
SUBURBAN TRASPORTATION
Orange Avenue E.S.
Cranford Senior H.S.
Union County
VOGEL
J&J TRANSPORTATION
Middlesex County
DAPPER
0
1
2
Miles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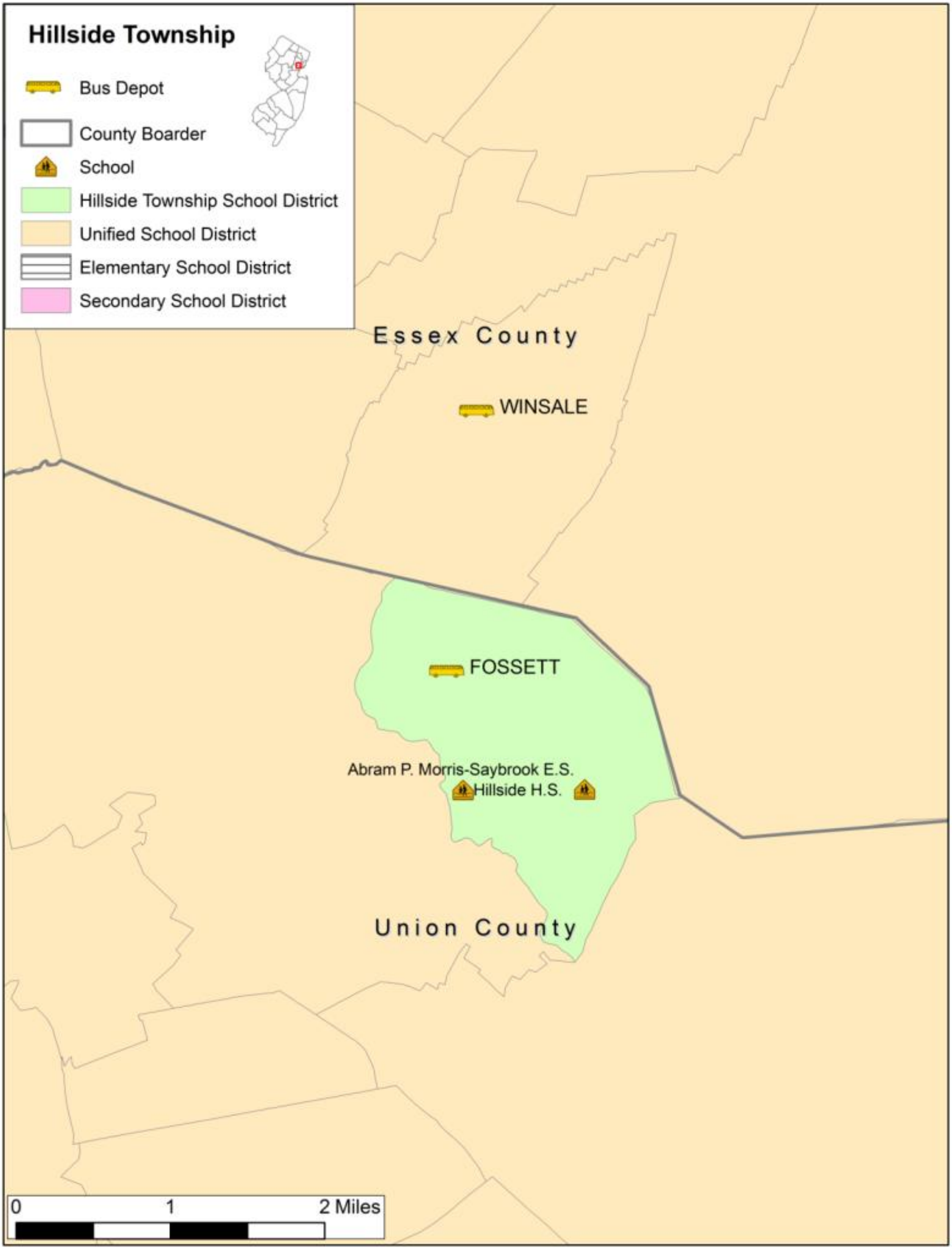
County Boarder
School
Elementary School District
Hampton Township School District
Unified School District
Secondary School District
Sussex County
Marian McKeown E.S.
STOCKER
VILLAGE BUS CO
FIRST STUDENT
Warren County
Morris County
0
5
10 Miles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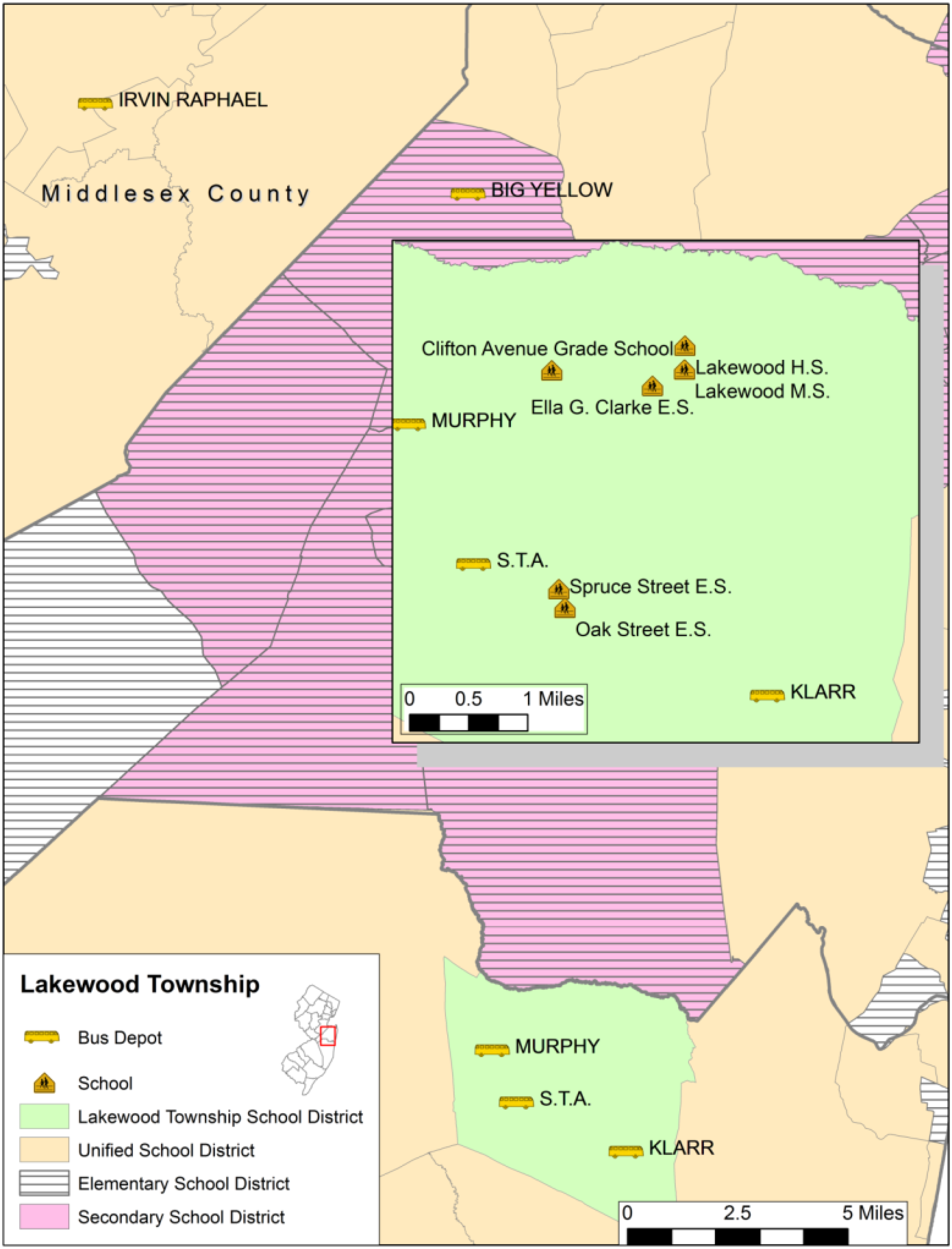
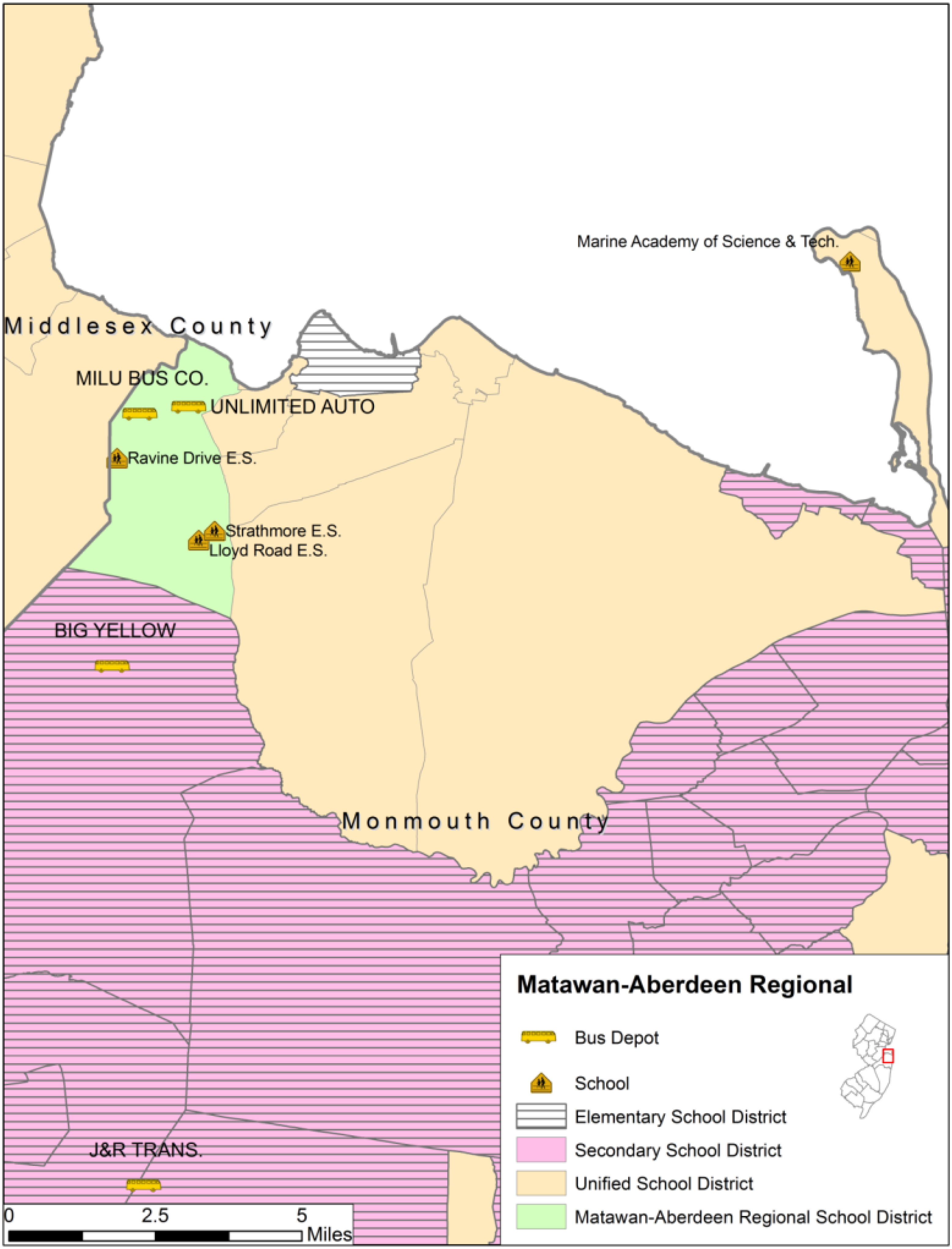
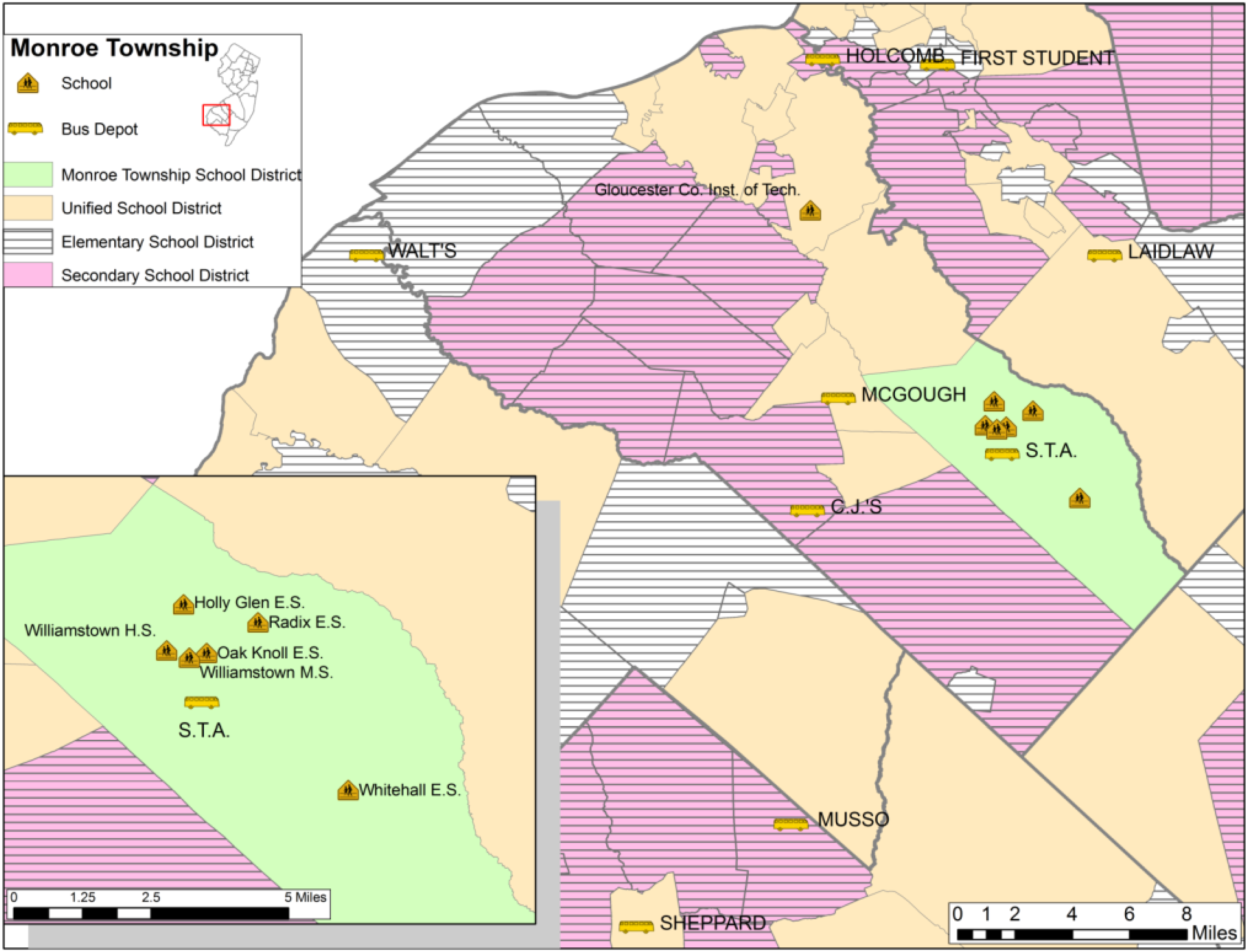
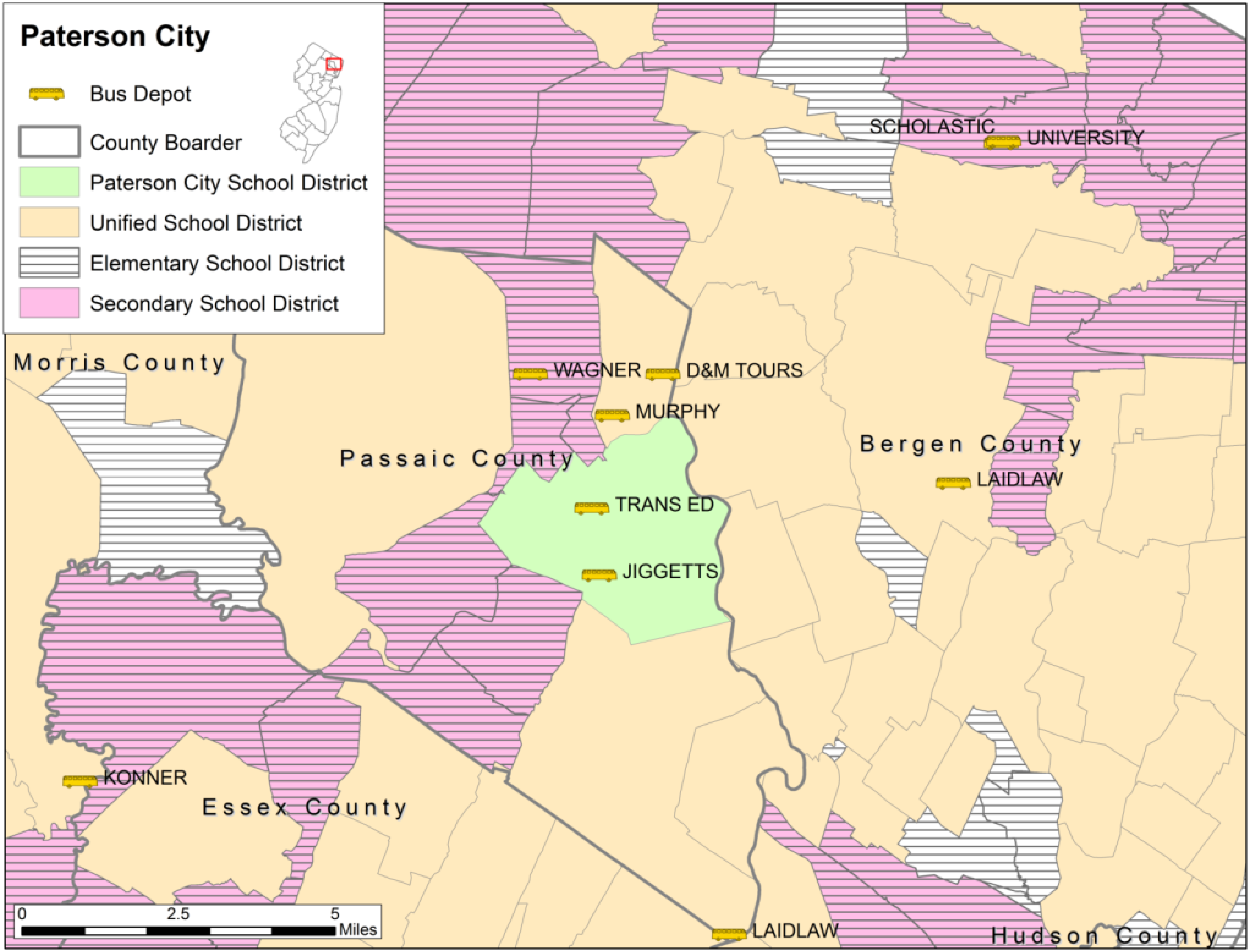
Bus Depot
County Boarder
School
Hillside Township School District
Unified School District
Elementary School District
Secondary School District
Essex County
WINSALE
FOSSETT
Abram P. Morris-Saybrook E.S.
Hillside H.S.
Union County
0
1
2 Miles

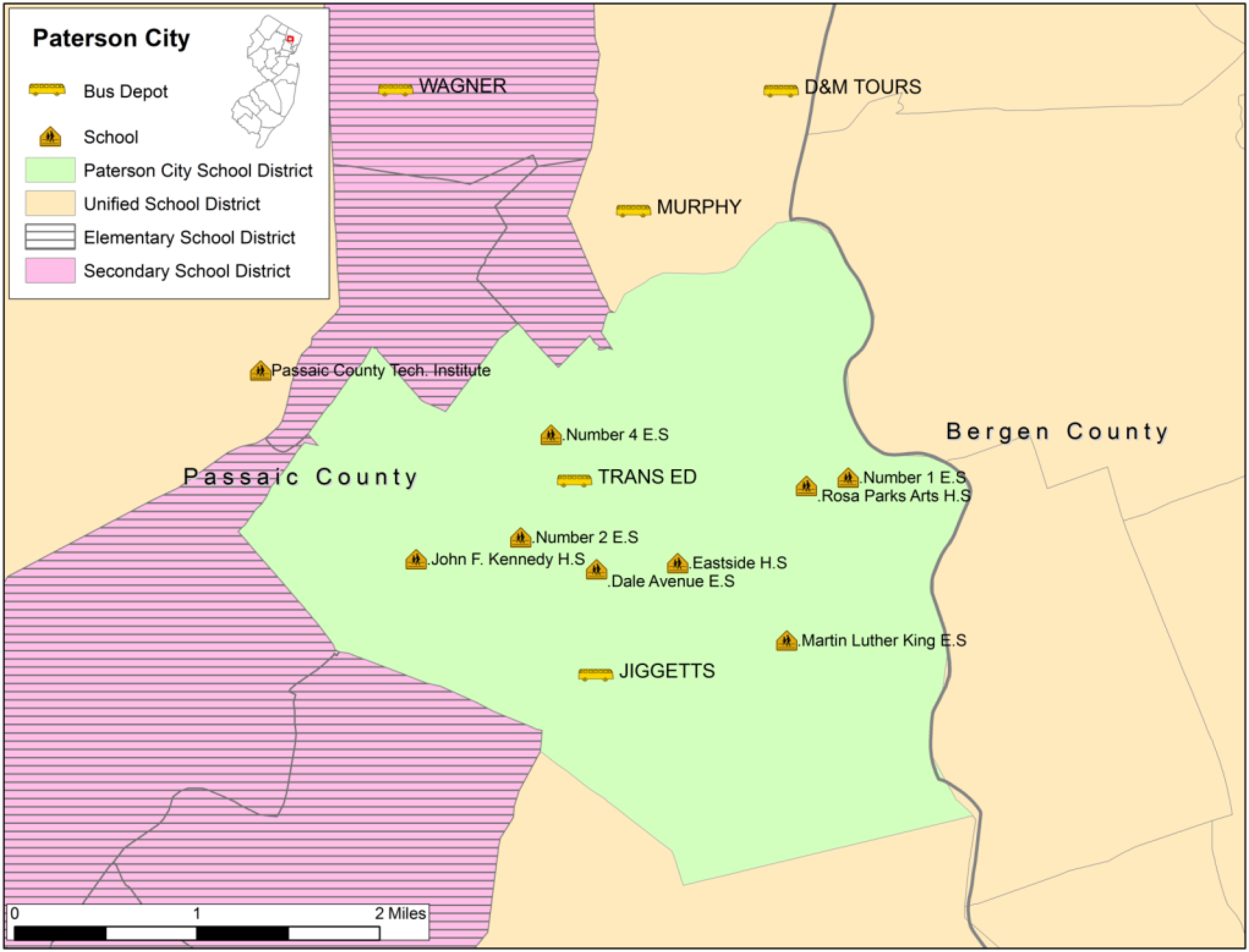

IRVIN RAPHAEL
Middlesex County
BIG YELLOW
Clifton Avenue Grade School
Lakewood H.S.
Lakewood M.S.
Ella G. Clarke E.S.
MURPHY
S.T.A.
Spruce Street E.S.
Oak Street E.S.
KLARR
0 0.5 1 Miles
Lakewood Township
Bus Depot
School
Lakewood Township School District
Unified School District
Elementary School District
Secondary School District
MURPHY
S.T.A.
KLARR
0 2.5 5 Miles

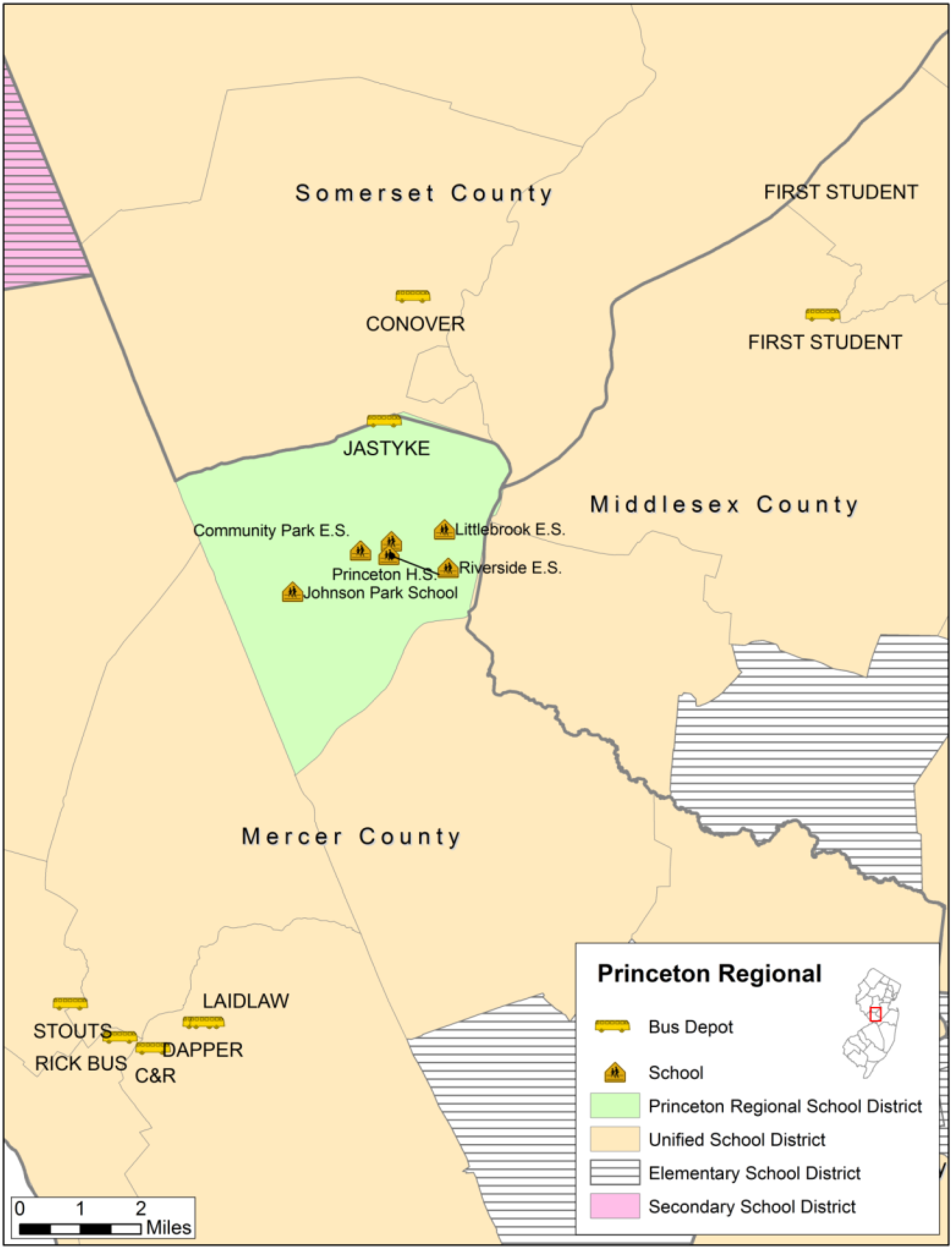

Marine Academy of Science & Tech.
Middlesex County
MILU BUS CO.
UNLIMITED AUTO
Ravine Drive E.S.
Strathmore E.S.
Lloyd Road E.S.
BIG YELLOW
Monmouth County
J&R TRANS.
0
2.5
5
Miles
Matawan-Aberdeen Regional
Bus Depot
School
Elementary School District
Secondary School District
Unified School District
Matawan-Aberdeen Regional School District

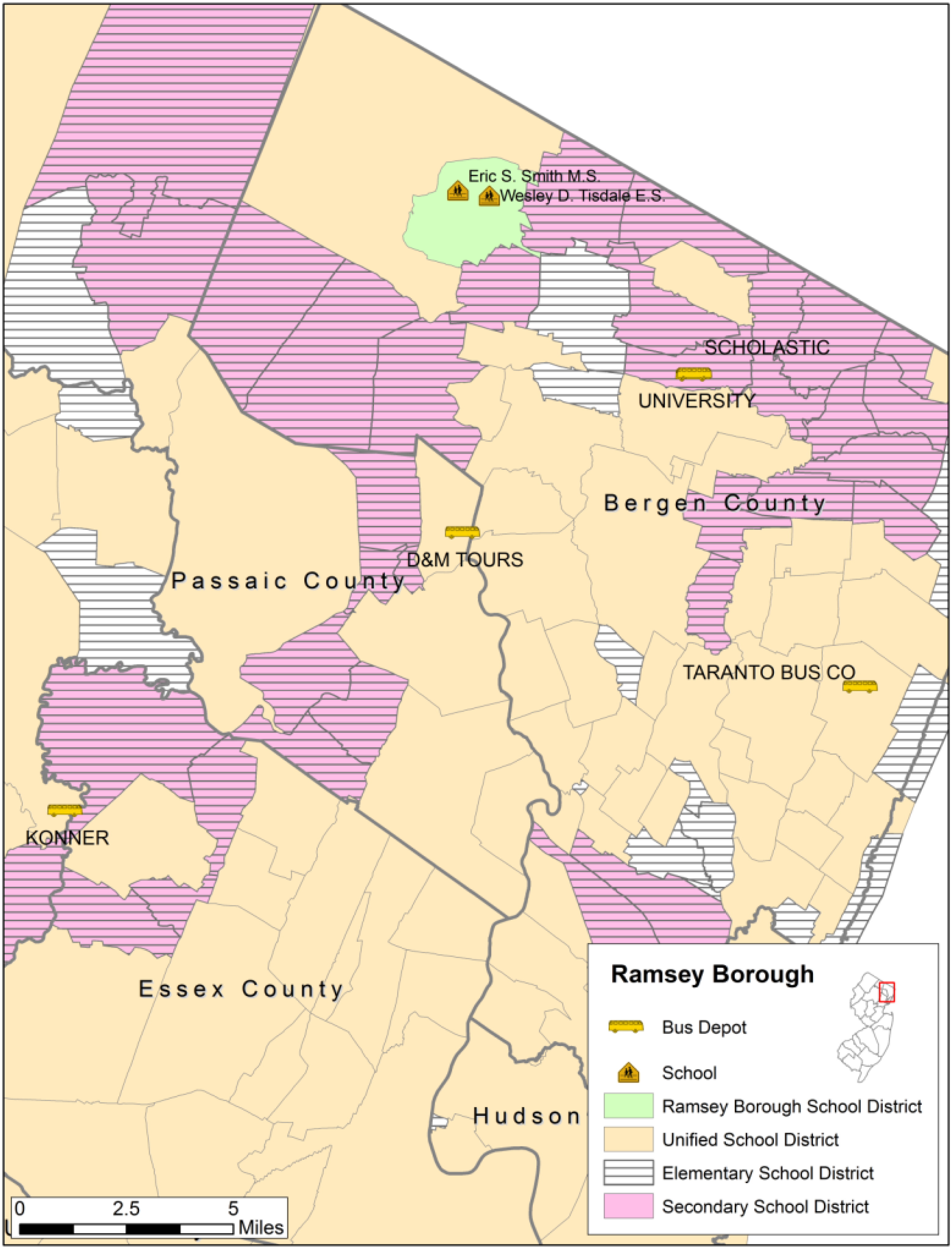

Monroe Township
School
Bus Depot
Monroe Township School District
Unified School District
Elementary School District
Secondary School District
HOLCOMB
FIRST STUDENT
Gloucester Co. Inst. of Tech.
WALT'S
LAIDLAW
MCGOUGH
S.T.A.
C.J.'S
MUSSO
SHEPPARD
0 1 2 4 6 8 Miles
Holly Glen E.S.
Radix E.S.
Williamstown H.S.
Oak Knoll E.S.
Williamstown M.S.
S.T.A.
Whitehall E.S.
0 1.25 2.5 5 Miles



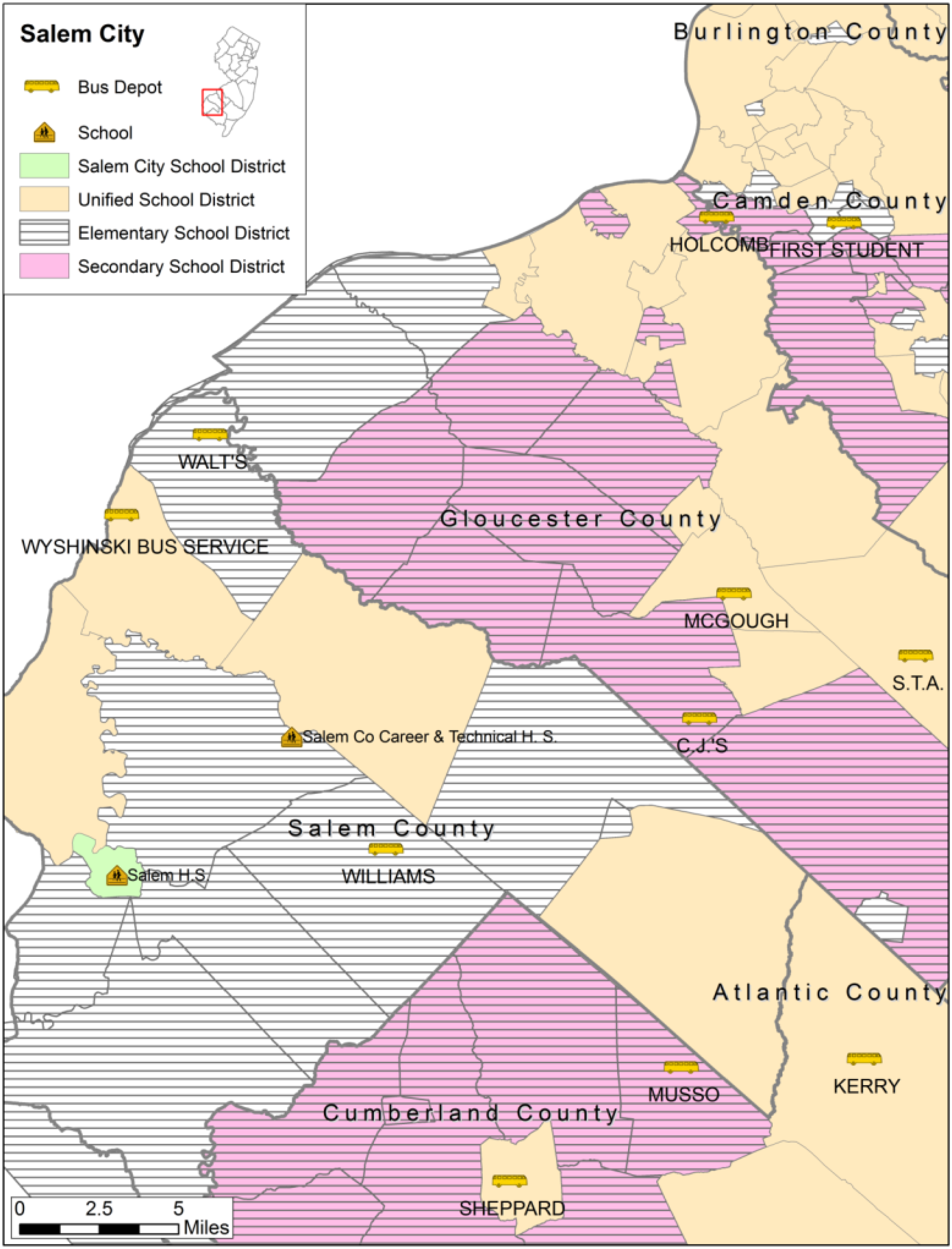
Paterson City
Bus Depot
County Boarder
Paterson City School District
Unified School District
Elementary School District
Secondary School District
Morris County
Passaic County
Bergen County
Essex County
Hudson County
SCHOLASTIC
UNIVERSITY
WAGNER
D&M TOURS
MURPHY
LAIDLAW
TRANS ED
JIGGETTS
KONNER
LAIDLAW
0
2.5
5
Miles


Paterson City
Bus Depot
School
Paterson City School District
Unified School District
Elementary School District
Secondary School District
WAGNER
D&M TOURS
MURPHY
Passaic County Tech. Institute
Number 4 E.S
Bergen County
Passaic County
TRANS ED
Number 1 E.S
Rosa Parks Arts H.S
Number 2 E.S
John F. Kennedy H.S
Eastside H.S
Dale Avenue E.S
Martin Luther King E.S
JIGGETTS
0
1
2 Miles

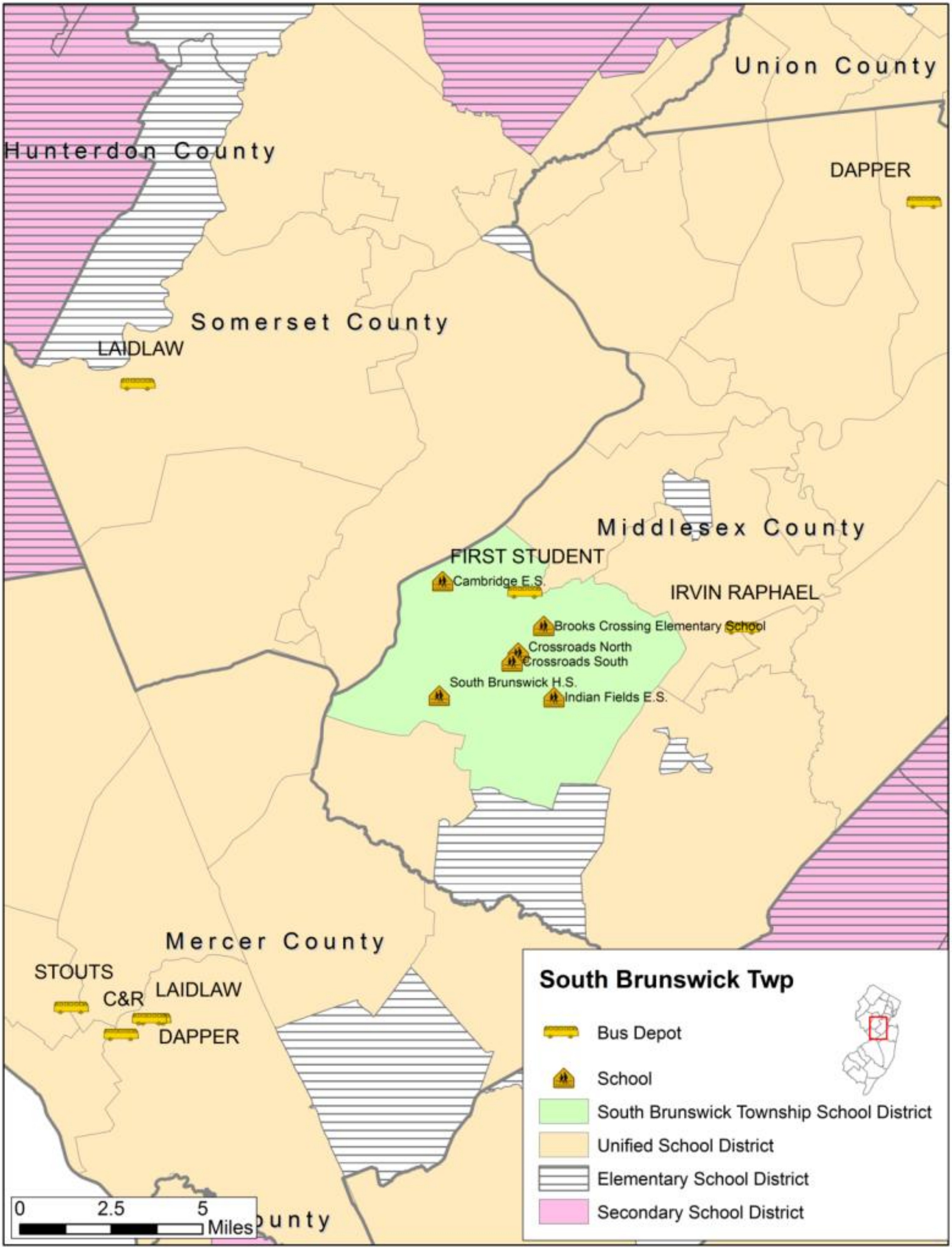

Somerset County
FIRST STUDENT
CONOVER
FIRST STUDENT
JASTYKE
Middlesex County
Community Park E.S.
Littlebrook E.S.
Princeton H.S.
Riverside E.S.
Johnson Park School
Mercer County
LAIDLAW
STOUTS
DAPPER
RICK BUS
C&R
Princeton Regional
Bus Depot
School
Princeton Regional School District
Unified School District
Elementary School District
Secondary School District
0 1 2 Miles



Eric S. Smith M.S.
Wesley D. Tisdale E.S.
SCHOLASTIC
UNIVERSITY
Bergen County
D&M TOURS
Passaic County
TARANTO BUS CO
KONNER
Essex County
Hudson
Ramsey Borough
Bus Depot
School
Ramsey Borough School District
Unified School District
Elementary School District
Secondary School District
0
2.5
5
Miles



Salem City
Bus Depot
School
Salem City School District
Unified School District
Elementary School District
Secondary School District
Burlington County
Camden County
HOLCOMB
FIRST STUDENT
WALT'S
WYSHINSKI BUS SERVICE
Gloucester County
MCGOUGH
S.T.A.
C.J.'S
Salem Co Career & Technical H. S.
Salem County
WILLIAMS
Salem H.S.
Atlantic County
MUSSO
KERRY
Cumberland County
SHEPPARD
0
2.5
5
Miles


Union County
Hunterdon County
DAPPER
Somerset County
LAIDLAW
Middlesex County
FIRST STUDENT
Cambridge E.S.
IRVIN RAPHAEL
Brooks Crossing Elementary School
Crossroads North
Crossroads South
South Brunswick H.S.
Indian Fields E.S.
Mercer County
STOUTS
C&R
LAIDLAW
DAPPER
0
2.5
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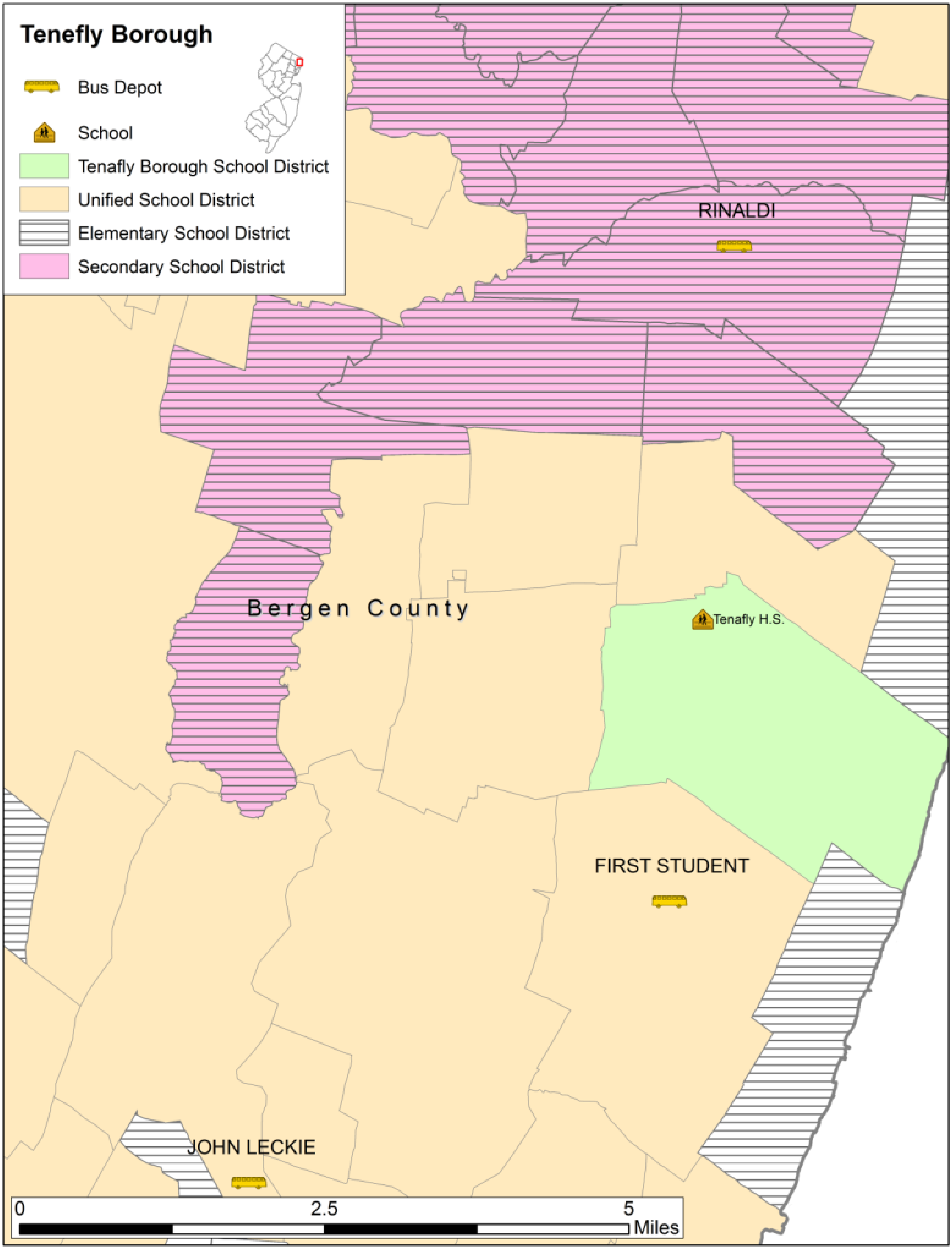
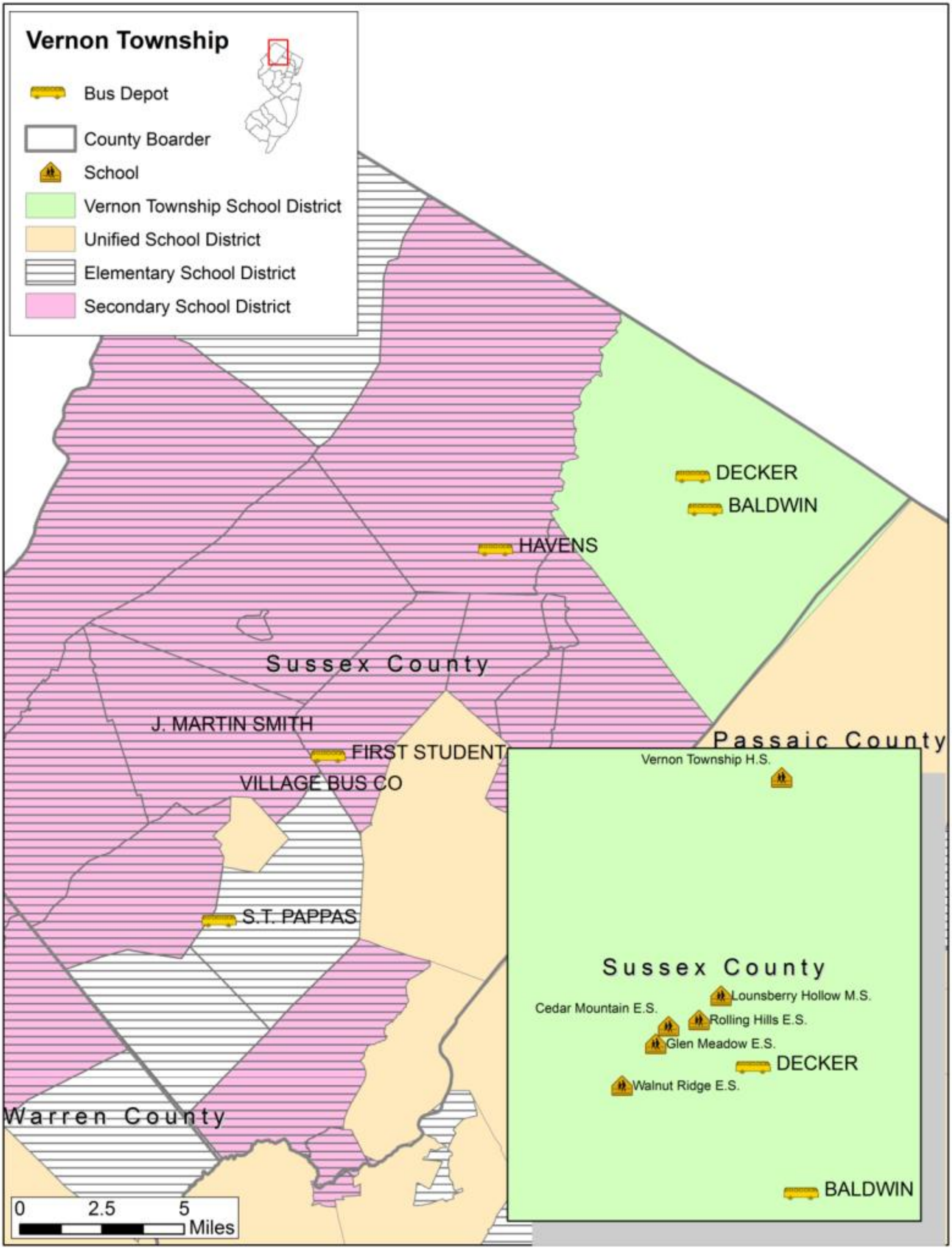
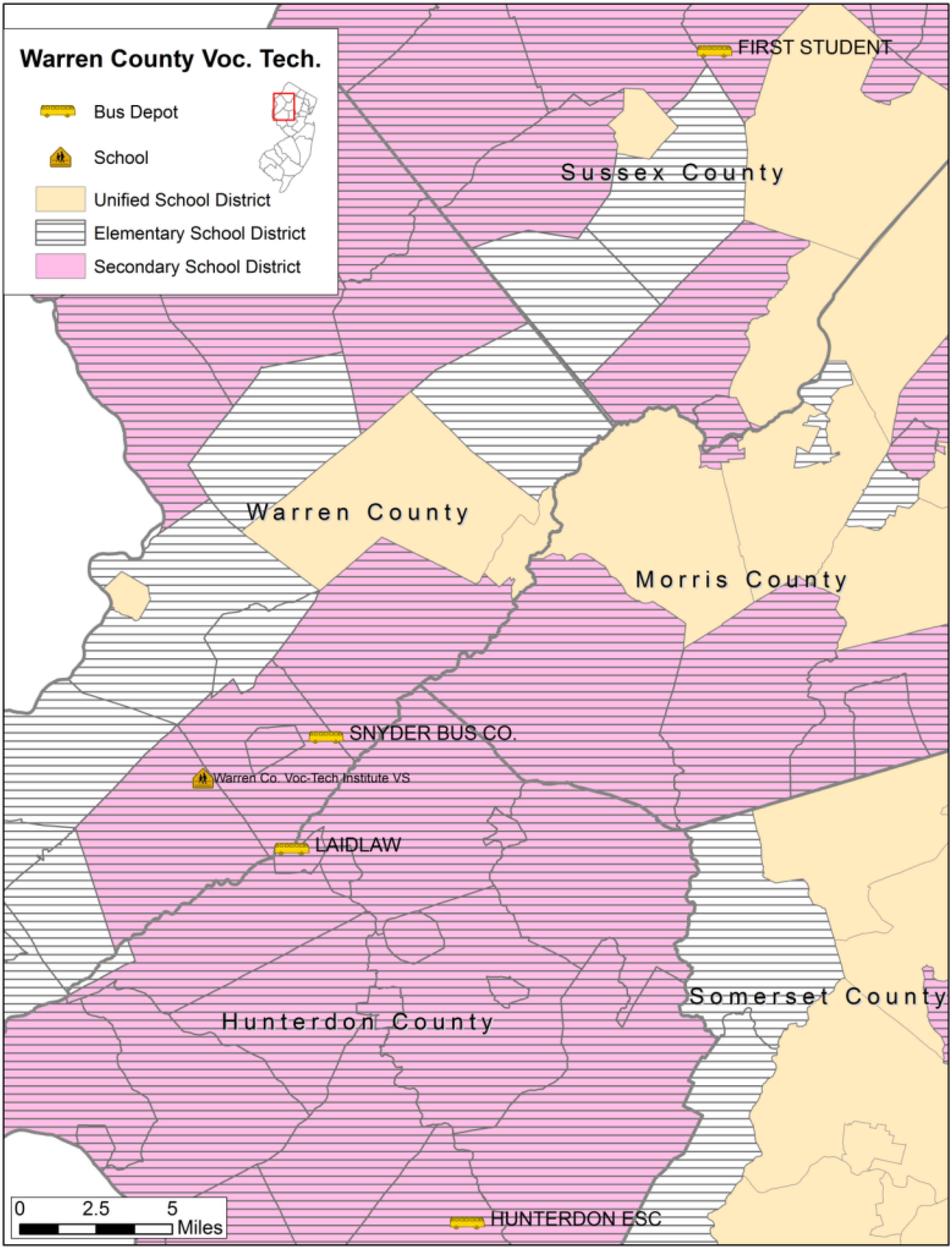
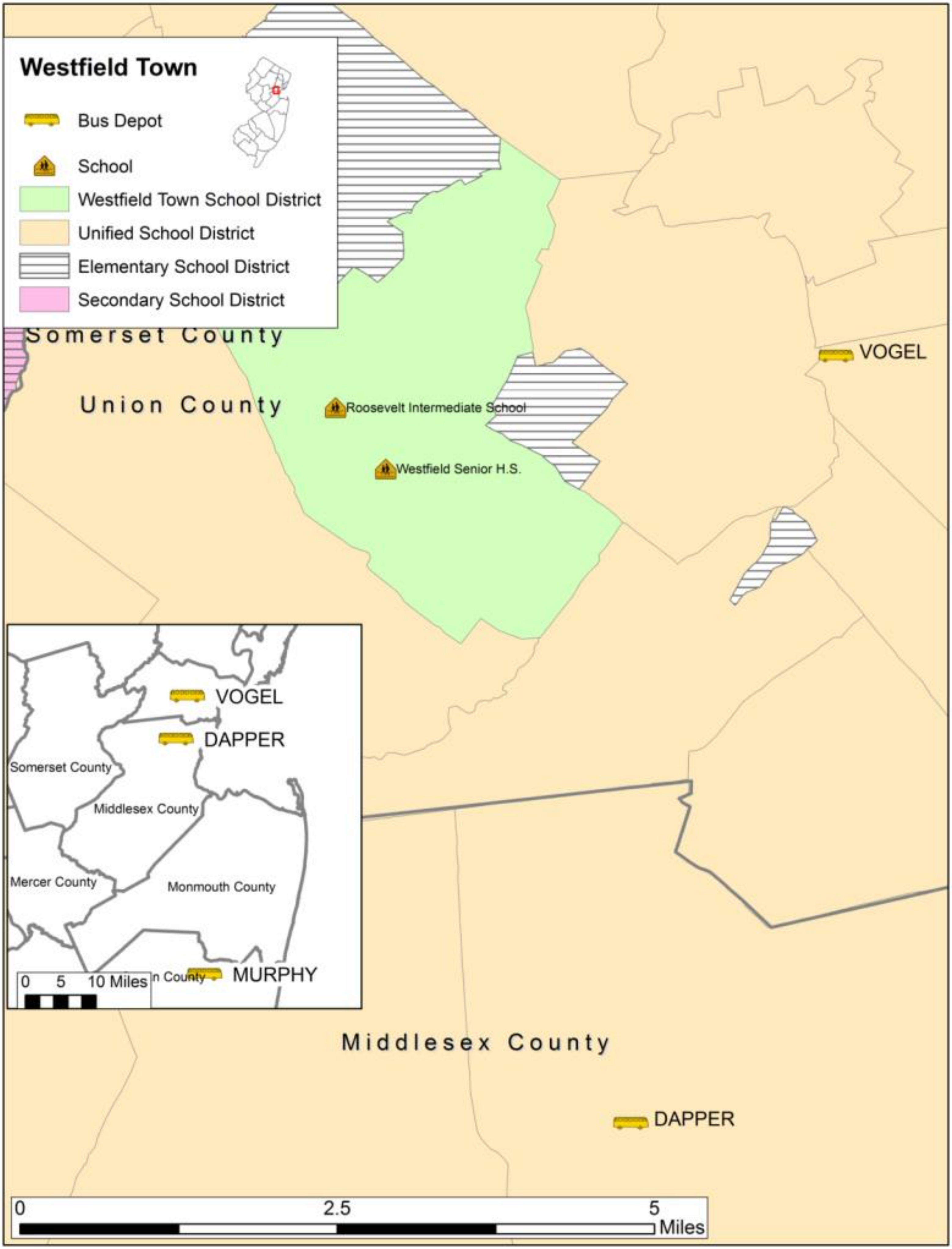
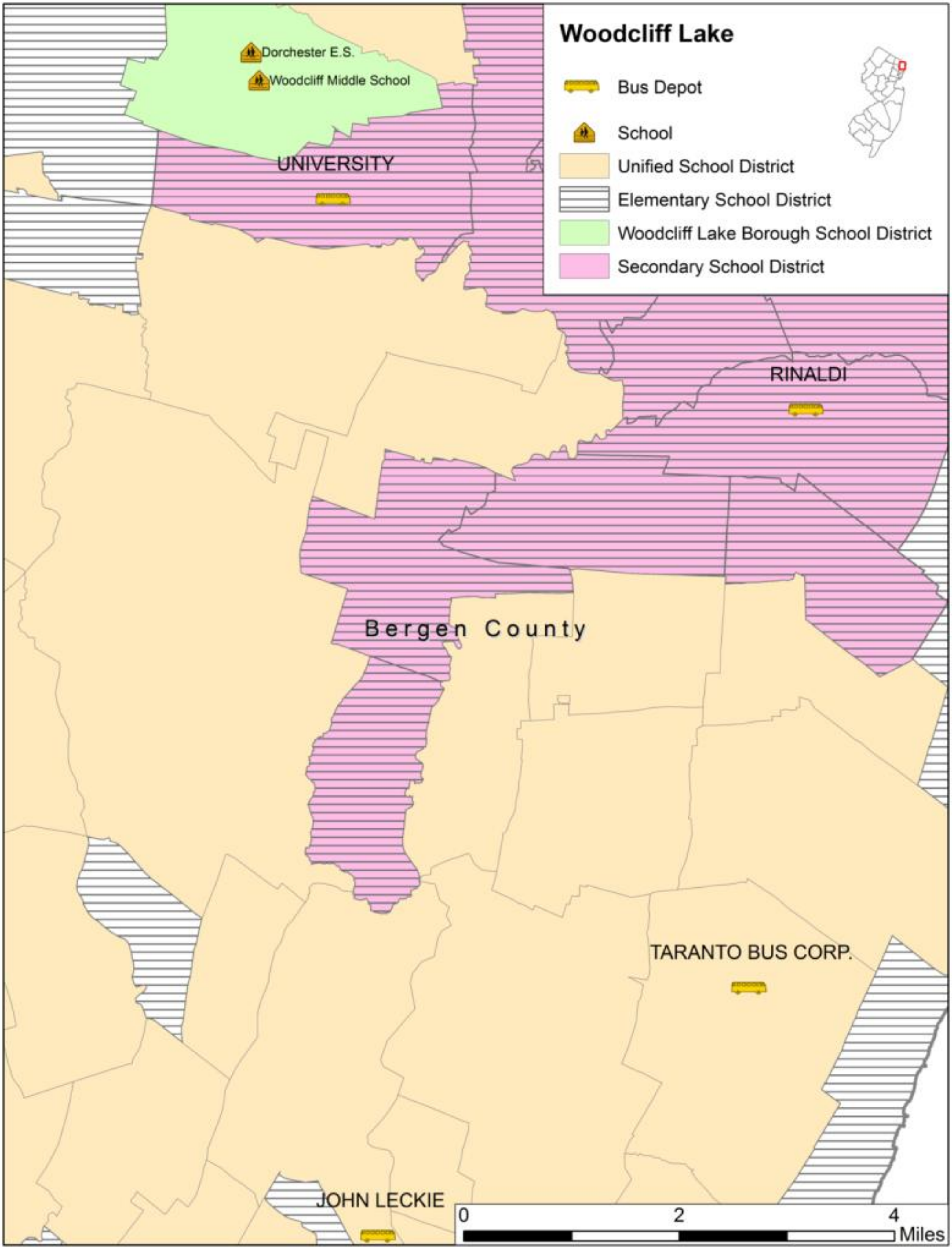
Miles
ounty
South Brunswick Twp
Bus Depot
School
South Brunswick Township School District
Unified School District
Elementary School District
Secondary School District


Tenefly Borough
Bus Depot
School
Tenafly Borough School District
Unified School District
Elementary School District
Secondary School District
RINALDI
Bergen County
Tenafly H.S.
FIRST STUDENT
JOHN LECKIE
0
2.5
5
Miles


Vernon Township
Bus Depot
County Boarder
School
Vernon Township School District
Unified School District
Elementary School District
Secondary School District
DECKER
BALDWIN
HAVENS
Sussex County
J. MARTIN SMITH
FIRST STUDENT
VILLAGE BUS CO
Passaic County
Vernon Township H.S.
S.T. PAPPAS
Sussex County
Lounsberry Hollow M.S.
Cedar Mountain E.S.
Rolling Hills E.S.
Glen Meadow E.S.
DECKER
Walnut Ridge E.S.
Warren County
BALDWIN
0
2.5
5
Miles


Warren County Voc. Tech.
Bus Depot
School
Unified School District
Elementary School District
Secondary School District
FIRST STUDENT
Sussex County
Warren County
Morris County
SNYDER BUS CO.
Warren Co. Voc-Tech Institute VS
LAIDLAW
Somerset County
Hunterdon County
HUNTERDON ESC
0
2.5
5
Miles


Westfield Town
Bus Depot
School
Westfield Town School District
Unified School District
Elementary School District
Secondary School District
Somerset County
Union County
Roosevelt Intermediate School
Westfield Senior H.S.
VOGEL
VOGEL
DAPPER
Somerset County
Middlesex County
Mercer County
Monmouth County
MURPHY
0 5 10 Miles
Middlesex County
DAPPER
0
2.5
5
Miles


Woodcliff Lake
Bus Depot
School
Unified School District
Elementary School District
Woodcliff Lake Borough School District
Secondary School District
Dorchester E.S.
Woodcliff Middle School
UNIVERSITY
RINALDI
Bergen County
TARANTO BUS CORP.
JOHN LECKIE
0
2
4
Miles

AppendixB. DRTRS - Students Database

Field Name	Use	Other Information
CO	County code	Undocumented: A unique 2 digit county code where the student resides. Each county is assigned two county codes. Even codes are used when the student is transported to a private school, an odd numbered code is used otherwise
CONAME	County name	County in which the student resides.
DIST	School district code	4 digit code for the district in which the student resides.
DISTNAME	School district name	School district in which the student resides.
CITY	Student resident city	
GRADE	Student current grade	
	PK-12=Regular ed	
	S1=PK-8 special ed	
	S2=9-12 special ed	
OTHERLOC	Is the student transported to another location? (Y-N)	Some students are transported between different locations during a single day. This is more common with Special Education students
SCHTYPE1	Type of school attending	The type of the FIRST school attended.
	1=Public	
	2=Non-public	
	3=Private school for the handicapped	
	4=Charter school	
	5=Early childhood community provider	
ATCO1	Attending school county code	2 digit county code
ATDIST1	Attending school district code	4 digit district code
ATSCH1	Attending school code	3 digit school code
MILES1	Miles from home to school one way	Distance from residence to FIRST school attended
DIFFBUS1	Does student ride a different bus from/to school? (Y-N)	
RTIDTO1	Local route number to school	The route number for the trip taken TO school
LICTO1	Vehicle license plate to school	
RTIDFR1	Local route number from school	(optional) Some students may ride a different bus home than they do to school
LICFR1	Vehicle license plate from school	(optional)
CURTESY1	Subscription busing participant (Y-N)	
SPNEEDIA	Special needs code	(optional) Information regarding any special needs for the student
	1=Wheelchair	
	2=Aide/nurse	
	3=Extended year program	
SPEENDIB	4=Other	
IEP1	Does IEP require transportation (Y-N)	A student's Individualized Education Program may require that a student be transported even when they might not otherwise.
SCHTYPE2	Second attending school	(optional)
...	...	All fields starting with school type are duplicated to allow for situations when a student attends multiple schools in one day.
DOB	Date of birth	

AppendixC. DRTRS - Routes Database

Field Name	Use
CO	Reporting party county code
DIST	Reporting party school district code
CONAME	Reporting party county name
DISTNAME	Reporting party school district name
ROUTENO	Route number
LICENSE	Vehicle license plate
VEHCAP	Vehicle student capacity
OPCAT	Operator code (party being paid for the service) 1=District Owned 2=Contracted 3=Host district 4=Host educational services commission 5=Parent contract
OPCO	Operator county location
OPDIST	DRTRS documentation: Operator school district location Note this appears to be the same as the firm's OPCODE Undocumented: This 4 digit code is not the school district code, rather it is a unique code assigned to each depot. Also called OPCODE (operator code) in other documentation.
OPNAME	Operator name
ROUTCOST	District input cost of route

AppendixD. Dimensionality Reduction – PCA

Principal components/correlation	Number of obs = 2157	Number of comp. = 1
	Trace = 3	
Rotation: (unrotated = principal)	Rho = 0.3668	

Component	Eigenvalue	Difference	Proportion	Cumulative
Comp1	1.09365	0.0919747	0.3646	0.3646
Comp2	1.00168	0.0970066	0.3339	0.6984
Comp3	. 904671		0.3016.	1

Principal components (eigenvectors)	Comp1	Unexplained
STOPS	0.3787	0.8432
RIDERS	-0.5935	0.6148
YEAR	0.7102	0.4484

Kaiser-Meyer-Olkin measure of sampling adequacy	
Variable	kmo
STOPS	0.4941
RIDERS	0.4975
YEAR	0.4982
Overall	0.4973

AppendixE. Changes in Expected $\mathcal{N}$

Table E.15: Changes in “Expected N”

Change in expected N	Mean Price Change	Obs
-0.7635	5.79	13
-0.6821	4.71	28
-0.6693	-0.37	3
-0.5875	-0.02	10
-0.4840	1.45	13
-0.3846	1.78	143
-0.2855	-4.24	2
-0.2434	0.87	4
-0.1236	-0.47	14
-0.0234	-0.15	1
-0.0116	-0.47	43
0.0946	0.83	2