The Haptic Probe: Mechanized Haptic Exploration and Automated Modeling

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Abstract

It seems likely that humans build internal models of objects that they explore haptically, and that the complexity of an internal model is not necessarily associated with complex exploratory procedures or the existence of visual input. Can a robot build a model of an object it touches or presses, and if so, can that model be used to render the object through a haptic interface? Or even simpler, can a robot estimate parameter values for a model already provided using data produced only by haptic exploration? This paper addresses the problem of parameter identification of hybrid dynamical systems, a class which contains systems of objects which make and break contact with one another. We propose the use of a hybrid sensitivity model as a basis for the construction of parameter identification algorithms. We distinguish between two components of a hybrid sensitivity model: sensitivity of states and sensitivity of switching instants to parameter variation, and we present two algorithms, one essentially based on each type of sensitivity. Simulation results demonstrating each algorithm are also presented.

1 Introduction

The ultimate test of fidelity for a haptically rendered virtual object is a side-by-side comparison of the rendered object and the real thing, where comparison involves haptic exploration by a human user. Given a sufficiently faithful rendering, the user could presumably not tell the difference between the rendered and real object. Internal models of the rendered and real objects, constructed in the mind of the user by observing responses to applied mechanical excitation, would match. Or would they? Perhaps a rendering that is sufficiently faithful for one user would nevertheless admit detectable differences for another user.

Imagine instead a mechanized haptic explorer, a machine that can haptically probe a real or rendered object and make quantitative comparisons. We wish to acknowledge the work of MacLean [9], who called this envisioned device the *haptic camera*. Although our aims are virtually the same, we call our device the haptic probe to emphasize its reliance on mechanical excitation and to limit its association with the visual sense, which is typically less [inter]active than haptic exploration.

If a haptic probe were to actually exist, it could be used not only to compare rendered to real objects (verify), but also to help realize the rendered object in the first place (automated modeling). We are in fact primarily interested in the use of the haptic probe for automatic generation of the virtual environment. The model that the haptic probe constructs through mechanized haptic exploration of an object could be used as a kind of executable specification: it could be rendered through real-time simulation to re-create that object through a haptic interface. Mathematically, the mechanical impedance of an object could be characterized using a haptic probe, then that impedance model could be used as the essential algorithm during haptic rendering. When the impedance of the object in question is linear (perhaps because of limitations imposed on the types of haptic exploration available) then the problem is relatively simple. When, however, the impedance is nonlinear, as will be the case if the object in question is in fact a system of objects that may undergo changing contact conditions, then the matter of constructing the model from observed behavior under known excitation is a bit more complicated. Given the ubiquity of changing contact conditions in the physical world as regularly manipulated and explored by humans, the inclusion of nonlinear impedance models within the abilities of a haptic probe is highly desirable.

The haptic probe idea is by no means new. MacLean [8] [9] developed an apparatus and method that measured force-displacement data from a toggle switch, created a model, and then rendered that model for a user to feel. A piece-wise linear fit to the recorded force/displacement curve allowed parameters of a piece-wise continuous impedance to be estimated. Howe et.al. [6] and Schulteis et.al. [16] present various algorithms that process sensory information collected during a human-directed telemanipulation task to automatically identify certain properties of the remote environment. Data processing involves segmentation, automatic identification of subtasks and states, and identification of object properties. Okamura [14] has also addressed the construction of geometric and surface property models of the environment from interaction data, but by placing the production of that data (haptic exploration) under automatic control. In another work, Miller and Colgate [12] adapted a nonlinear system identification technique, called a wavelet network, to provide a tool that is capable of identifying environments with static 1-D nonlinear features.

We are interested in characterizing the mechanical impedance of objects that are manipulated strictly by pushing or turning. That is, we restrict ourselves to objects whose feel can be rendered on a single axis haptic interface. Our prototypical problem is the characterization of the feel of single keys on keyboards, including typewriter, computer, piano, and synthesizer keyboards. These are all objects that are generally just pressed or pushed, but nevertheless posses behaviors that strongly influence human-machine interaction. The mechanical response generated by a key-press on a typewriter or computer keyboard has been implicated as an influence on forces used by typists which in turn influences the occurrence of repetitive stress injury [11]. Also, the mechanical response of a musical keyboard determines the availability of certain types of musical expression [3]. Figure 1 shows a haptic probe in use to characterize the feel of a piano key.

We do allow, however, changing contact conditions, either between human finger and key (haptic interface end-effector) or between elements internal to a system of objects. Changing contact conditions among elements of the piano action and strong nonlinearities arising from buckling membranes in computer keyboards are examples of the kinds of phenomena we are interested in capturing. These are the phenomena that generate the interesting (and often most easily perceived) details that influence human-machine interaction. These phenomena support the development and the multiplicity of technique on the part of the human user. Thus the models that we use whose parameters are to be identified are hybrid dynamical models: models incorporating both discrete and continuous variables.

The literature in hybrid dynamical systems is rich and varied. While parameter identification has been addressed in some detail for certain classes of systems [1] [5], a method for the identification of hybrid models representing simple interacting rigid bodies that make and break contact with one another is not yet available. In this paper, we present a method for identifying the parameters of a hybrid dynamical model using data collected with a haptic probe (force and motion data alone, where the force and

Figure 1. On the right a motorized instrumented probe presses on a piano key. In the foreground a prototype force reflecting synthesizer keyboard is visible.

motion are power-conjugates or their ratio expresses a driving point impedance). Our canonical identification problem is the bouncing ball: we will attempt to characterize the elastic, dissipative, and inertial properties of a ball bouncing on a paddle under the control of the haptic probe from force and motion sensors resident on the paddle itself. Access to the ball position is not available: only the paddle position and interaction force are given. From these data, the mass, stiffness, and damping parameter values are to be extracted. The hybrid dynamical model comprises contact and flight phases. We are particularly interested in developing a parameter identification technique which is based on adaptive feedback control. We presume that a feedback technique will be more extensible to models for which analytical solutions do not exist.

The bouncing ball is actually inspired by the piano action. A grossly simplified piano action model is analogous to the bouncing ball, with the hammer resting or bouncing on a lever-like paddle. A key-press accelerates the hammer toward the string and the appropriately placed keybed causes a launch (release, or transition into free-flight) of the hammer before hammer/string impact begins. After hammer/string interaction, the hammer lands back on the key. The effects of release and re-capture of the hammer on the key are haptically perceptible to the player, such that a consistent loudness can be produced (corresponding to a stable juggle), and this without any view of the hammer.

We have investigated two approaches to the problem of parameter identification for hybrid systems.Both of these approaches make use of sensitivities obtained from a sensitivity model. A sensitivity model may be derived from the original system model by taking partial derivatives with respect to the parameters. When a model is hybrid, a full hybrid sensitivity model comprises two types of sensitivities: the sensitivity of the state values, and the sensitivity of the switching instants to parameter variations, as further reviewed below. One of the two approaches we have developed borrows from a paper by Hiskens [4], and essentially makes use of sensitivities of the states to parameters with an extension utilizing sensitivities of switching instants to improve the results. Incidentally, we demonstrate this approach below not using a parameter identification problem, but rather using a reconstruction problem. The second approach to parameter identification presented below concentrates on the underlying discrete dynamics of the hybrid system and uses only the sensitivities at the switching instants. The two parameter identification approaches are presented in turn in the body of this paper. We also briefly describe an envisioned method, a true hybrid approach, that is based on a combination of these two basic approaches.

The paper is organized as follows. In section 2 we review important concepts in sensitivities of hybrid systems. We describe the first approach to parameter identification using a reconstruction problem in section 3. Section 4 concentrates on the second approach, namely estimation in discrete time. For discrete time estimation, Newton observers are used which are briefly reviewed in subsection 4.1. In section 5, preliminary simulation results are presented. Finally, section 6 concludes the paper and discusses extensions of this work.

2 Sensitivities

Parametric sensitivity of a dynamic system, which follows from the Taylor series expansion of its flow, provides a first order approximation of the change in its nominal trajectory due to parameter changes. Parametric sensitivity analysis defines a way of quantifying the changes in the flow of a model due to (small) changes in parameters and initial conditions.

Problems of parametric sensitivity of a dynamic system can be classified into two groups: direct problems that aim to calculate changes in the nominal trajectory given parameter variations and inverse problems that aim to estimate change in parameter values given differences between nominal and perturbed trajectories. The gradient-type information contained in the parametric sensitivity trajectories can be used to guide the search in the inverse problems. Parameter estimation is an inverse problem and trajectory sensitivities are very useful since they can be directly used in many gradient based estimation methods.

Hybrid systems exhibit both discrete state and continuous state dynamics. In these systems, continuous and discrete dynamics not only coexist, but *interact*. That is, changes occur both in response to discrete, instantaneous events and in response to dynamics as described by differential or difference equations in time.

In this paper, we'll adapt a model for hybrid systems that is suitable for development of trajectory sensitivity analysis. We'll describe our hybrid system with a model of the form

$$\begin{aligned} \dot{x}_i &= F_i(x_i, t, \alpha) && (1) \\ x_i^+ &= \phi_i(x_i^-, t_i, \alpha) && (2) \\ 0 &= f_i(x_i^-, t_i, \alpha) && (3) \\ t_0 &= t_0(\alpha) && (4) \\ x^+(t_0) &= x^+(t_0, \alpha) && (5) \end{aligned}$$

In this model, i represents discrete states (modes) and F_i governs continuous dynamics in each mode. Switchings from one mode to another (events) occur when the switching conditions f_i are satisfied. Whenever an event occurs, states in the new mode are updated using the reset conditions ϕ_i. Finally, x represents continuous states, t_i represents switching instants, α represents model parameters. x_0 is the initial conditions and t_0 is the initial time. The notations $()^+$, $()^-$ signify just before and just after the events, respectively.

Analysis of hybrid systems is relatively difficult. The significant feature that makes their analysis complex is their nonlinear non-smooth time-dependent dynamics. For this reason although parametric sensitivity analysis for continuous systems has been extensively studied, sensitivity analysis for hybrid systems has only recently been established [2]. However, the first work in the area dates back to 1967 when Rozenvasser [15] applied sensitivity analysis to dynamic models containing discontinuities.

Since hybrid system models are characterized by the interaction between continuous and discrete event dynamics, in general, sensitivity models extracted from these systems also have a hybrid nature. Changes in parameters of a hybrid system affect not only the continuous dynamics, but also the discrete event dynamics. As pointed out in [4], while calculating sensitivity trajectories of a hybrid system, evaluating sensitivity of switching instants with respect to parameters is as important as correctly detecting and locating discrete events in the hybrid model itself. Hybrid sensitivity models have discrete event dynamics similar to the system model and they include jumps in sensitivity values when event transitions occur. Jumps occur according to reset conditions which are highly influenced by the difference between instants when the nominal and perturbed trajectories reach the event triggering hypersurface, i.e. sensitivity of switching instants with respect to parameters.

Defining sensitivity of continuous states with respect to parameters as λ and sensitivity of switching instants with respect to parameters as λ_t, the sensitivity trajectories for a hybrid system can be calculated by the hybrid sensitivity model

$$\lambda \triangleq \frac{\partial x}{\partial \alpha} \tag{6}$$

$$\lambda_t \triangleq \frac{dt_i}{d\alpha} \tag{7}$$

$$\dot{\lambda} = \frac{\partial F_i}{\partial x}\lambda + \frac{\partial F_i}{\partial \alpha} \tag{8}$$

$$\lambda^+ = \lambda^- + \Delta\lambda_i \tag{9}$$

$$0 = f_i(x_i^-, t_i, \alpha) \tag{10}$$

$$\lambda_0 = \frac{dx_0}{d\alpha} - \dot{x}_0 \frac{dt_0}{d\alpha} \tag{11}$$

where

$$\Delta\lambda_i = [-\Delta F_i + (\frac{\partial \phi_i^-}{\partial x} - I)F_i^- + \frac{\partial \phi_i^-}{\partial t}]\lambda_t + (\frac{\partial \phi_i^-}{\partial x} - I)\lambda_i^- + \frac{\partial \phi_i^-}{\partial \alpha} \tag{12}$$

$$\Delta F_i = F_i^+ - F_i^- \tag{13}$$

$$\lambda_t = -\frac{\frac{\partial f_i^-}{\partial x}\lambda_i^- + \frac{\partial f_i^-}{\partial \alpha}}{\frac{\partial f_i^-}{\partial x}F_i^- + \frac{\partial f_i^-}{\partial t}} \tag{14}$$

In this hybrid sensitivity model the reset conditions at events ensure that the sensitivities accurately reflect trajectory perturbations at and beyond the perturbed event time. Over the intervening time interval when nominal and perturbed models are in different modes, the sensitivities cannot directly represent perturbations.

3 Reconstruction using Sensitivities and Continuous Time Estimation

In this section, we explore the first approach to parameter estimation problem by making use of a direct problem, namely the reconstruction of the trajectory of the same system with a different parameter value using sensitivities. In principle, the parameter estimation problem is the inverse of the reconstruction problem: Given two trajectories (nominal and perturbed) the aim is to calculate the amount of perturbation. Therefore one can get valuable insight from the reconstruction problem to do parameter estimation.

Figure 2 demonstrates a simple hybrid system that we have used to pose test problems. Physically, this system resembles the model of the temperature in a thermostat controlled room. The mathematical model for this hybrid system is given in Table 1. Table 2 contains the parameters used for the simulations in this section.

In Figure 2, the dashed line represents the nominal trajectory of the system and the solid line represents a perturbed trajectory. The dotted line is the reconstruction of perturbed trajectory from the nominal trajectory using sensitivity of the output to the perturbed parameter. From the figure it can be clearly observed that the approximated trajectory is in agreement with the perturbed trajectory when

Table 1. Model - First Order Switching System

System Model		
IC	$x_0 = 0$	
Flows	$F_1 \triangleq \dot{x} = -\tau x + u_1$	$F_2 \triangleq \dot{x} = -\tau x + u_2$
Switchings	$f_1 \triangleq x - s_1 = 0$	$f_2 \triangleq x - s_2 = 0$
Resets	$\phi_1 \triangleq x^+ = x^-$	$\phi_2 \triangleq x^+ = x^-$

Table 2. Parameter Set

τ	$\hat{\tau}_0$	u_1	u_2	s_1	s_2
0.5	0.6	1.5	-0.5	2	0

both nominal and perturbed systems are in the same mode. This agrement is ensured by the reset condition in the sensitivity model. However, over the intervening time interval, sensitivities fail to give a good approximation, since nominal and perturbed systems are in different modes. We can also observe that it is meaningless to try to approximate after nominal and perturbed systems completely lose synchronization.

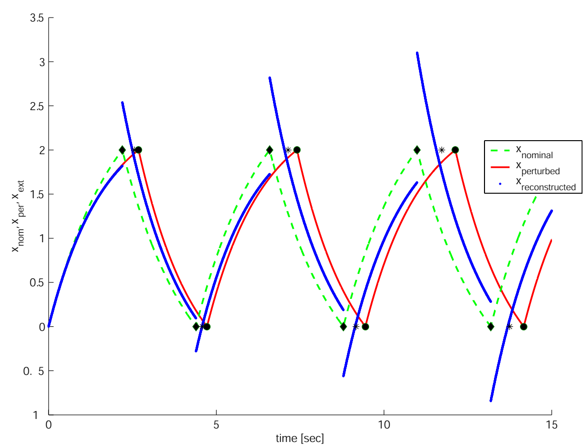


Figure 2. Reconstruction of perturbed trajectory using λ only

In Figure 3 the same hybrid system is studied as in Figure 2. However, this time sensitivity of switching instants with respect to the perturbed parameter is taken into account during reconstruction. In particular, we make use of a refinement to the reconstruction formula proposed by Hiskens et. al. [4] and simulate both the nominal system and its sensitivity model without changing the mode for an extended amount of time, until the estimated switching time for the perturbed system is reached. The results for such an approach are given by the dotted lines in Figure 3. It is clear that use of this extra information allows a better approximation for the perturbed trajectory.

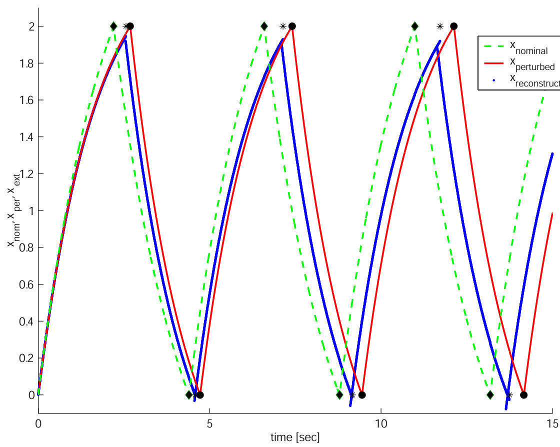


Figure 3. Reconstruction of perturbed trajectory using both λ and λ_t - Extended simulation

It is worth mentioning that, in this section, we have observed that it is possible to gather information about parameter perturbation directly using sensitivities when the nominal and perturbed trajectories are both in the same mode; therefore, we can conclude that as long as the modes of the nominal and the perturbed systems agree one can use gradient based continuous-time adaptation methods to perform parameter updates.

4 Discrete-Time Estimation

In the previous section we have considered the parameter estimation problem using only continuous-time approaches. However, hybrid systems have interacting continuous and discrete dynamics. Therefore, basing the adaptation algorithm only on continuous dynamics and neglecting the information offered by the discrete dynamics is not necessarily the best approach to the parameter estimation problem of a hybrid system.

The hybrid system model defined in section 2 can be characterized by a finite number of nonlinear dynamical models together with a set of rules for switching among these models. Therefore, this model description naturally causes a partitioning of the state space into cells. The boundaries of each cell are in effect switches between different nonlinear systems. In order analyze this type of hybrid system, it is natural to investigate the behavior of the system as it flows from one switching surface to the next switching surface since one of the most interesting phenomena in the study of hybrid systems is the behavior of the discontinuities in the dynamics.

A useful notion that will be used in this paper is that of an *impact map*. The notion of an impact map can be considered a generalization of a Poincaré map. An impact map is simply a map from one switching surface to the next switching surface. The impact map of a hybrid dynamical system defines a discrete-time dynamical system which in general has very complex dynamics. In many cases, it is not even possible to have explicit expressions for the impact maps. However, these maps can be obtained by advancing the hybrid model through time and sampling at the impact instants. Similarly, the sensitivities of an impact map can be obtained by advancing the hybrid sensitivity model in time.

Note that, in general, while constructing the impact map, the states of the hybrid system are augmented with time so that switching instants are also included in the impact map.

In this section, we will concentrate on the discrete-time system defined by the impact maps of a hybrid system and as our second approach we will study the parameter estimation problem using sensitivities of the impact maps only.

State estimation and parameter estimation are similar problems. State estimators can be used for simultaneous parameter estimation by augmenting the state space with parameters α having (no) dynamics $\dot{\alpha} = 0$. Moreover, if one uses an augmented observer for parameter estimation, then the identifiability conditions are simply given by the observability of the parameter augmented system.

The next subsection briefly reviews a very general discrete-time state estimator, namely a *Newton Observer*, developed by Moraal and Grizzle [13]. Newton observers have proven useful for us to do parameter estimation in the discrete-time domain using impact map sensitivities.

4.1 Newton Observers

The basic idea behind Newton observers is to formulate the state estimation problem as one of solving a sequence of nonlinear inversion problems and to use Newton's algorithm to result in an asymptotic observer. Under observability and smoothness assumptions, Newton's algorithm can be shown to have a uniform rate of convergence over the entire sequence of inversion problems, defining a quasi-local, exponential observer for the discrete-time nonlinear system

$$x_{k+1} = F(u_k, u_k) \tag{15}$$

$$y_k = h(x_k, u_k) \tag{16}$$

where $x \in R^n, u \in R^m$ and $y \in R^p$.

While defining Newton observers, first, the concept of a N-lifted system should be introduced. Lifting is performed to ensure that enough measurements are considered to satisfy the observability rank condition. In particular, the dynamics of the N-lifted system is the dynamics of the original discrete-time system iterated N-times. Similarly, the state of the lifted system is the state of the original system sampled in windows of length N.

For notational simplicity, let $F^u(x) \triangleq F(x,u)$ and $h^u(x) \triangleq h(x,u)$. Also let " $\circ$ " denote composition. Then the N-lifted system is defined as

$$x_{N(j+1)} = \Phi(x_{Nj}, U_{[N(j-1)+1,Nj]}) \quad (17)$$
$$Y_{[N(j-1)+1,Nj]} = H(x_{Nj}, U_{[N(j-1)+1,Nj]}) \quad (18)$$

where, the set of N consecutive measurements and controls are given by

$$Y_{[k-N+1,k]} \triangleq \begin{pmatrix} y_{k-N+1} \\ \vdots \\ y_k \end{pmatrix} \; U_{[k-N+1,k]} \triangleq \begin{pmatrix} u_{k-N+1} \\ \vdots \\ u_k \end{pmatrix} \quad (19)$$

Furthermore, the evolution of states from window to window Φ and the state to measurement map H read as

$$\Phi(x_{Nj}, U_{[k-N+1,k]}) \triangleq F^{u^N_{[k-N+1,k]}} \circ \ldots \circ F^{u^1_{[k-N+1,k]}}(x_{Nj}) \quad (20)$$

$$H(x_{Nj}, U_{[k-N+1,k]}) \triangleq \begin{pmatrix} h^{u^1_{[k-N+1,k]}}(x_{Nj}) \\ \vdots \\ h^{u^N_{[k-N+1,k]}} \circ \ldots \circ F^{u^1_{[k-N+1,k]}}(x_{Nj}) \end{pmatrix} \quad (21)$$

In [13], Moraal and Grizzle present a theorem which interprets Newton's method as a quasi-local exponential observer for discrete time systems.

In particular, suppose that N has been fixed, and for notational convenience, let $Y_k = Y_{[k-N+1,k]}$ and $U_k = U_{[k-N+1,k]}$. Define

$$\Theta^{Y_k,U_k}(\zeta) = \zeta + \left[\frac{\partial H}{\partial x}(\zeta, U_k)\right]^{-1} (Y_k - H(\zeta, U_k)) \quad (22)$$

and let $(\Theta^{Y_k,U_k})^d(\xi)$ represent $\Theta^{Y_k,U_k}(\xi)$ composed with itself d-times. Then the theorem states that

$$z_{k+1} = (\Theta^{Y_k,U_k})^d(F(z_k, u_{k-N})) \quad (23)$$
$$\hat{x}_k = F^{u_{k-1}} \circ F^{u_{k-2}} \circ \ldots \circ F^{u_{k-N}}(z_k) \quad (24)$$

is a quasi-local, exponential observer for the system (15)-(16).

Note that when the state to measurement map H is not square but the observability rank condition is satisfied, then the set of equations is over-determined. In this case, one can use a method for solving a nonlinear least squares problem, in particular, one may utilize a pseudo-inverse in the Newton observer to result in an exponential observer [13].

5 Simulation Results

As outlined in section 1, our canonical problem is a ball bouncing on a moving paddle. We treat the bouncing ball using two paddle/ball contact models: a rigid body model, and a compliant model. In the rigid body model, the contact takes place in infinitesimal time and the relative velocity after collision is related to the relative velocity before collision by the simple factor ϵ, called the coefficient of restitution. In the compliant contact model, interaction occurs in a period of finite duration and is governed by dynamical equations in which the ball and paddle motions are coupled. Typically a spring/damper coupler is placed between the ball and paddle during contact. Various compliant contact models are commonly used in engineering applications. Among these are the linear spring damper model with contact force

$$F_c = k\rho + b\dot{\rho} \quad (25)$$

and the general nonlinear compliant contact model with contact force

$$F_c = k|\rho|^{n-1}\rho + b|\rho|^n\dot{\rho} \quad (26)$$

introduced by Hunt [7]. In both of these models ρ denotes the relative position of the contacting bodies, while k and b are spring and damping coefficients, respectively. Parameter n is a shape coefficient that depends on the contact geometry.

In our analysis, we treat the rigid body model and the general nonlinear compliant contact model. However, we do not treat the linear compliant contact model since this model yields physically inconsistent interaction forces. Simple simulations using the linear contact model of a ball bouncing on fixed ground, for example, will produce tensile (sticky) and discontinuous contact forces which cannot be physically justified [10].

Figures 4 and 5 demonstrate a sample run of the estimation procedure applied to the rigid body contact model.

A Newton observer is used to generate the parameter updates while the bouncing ball is driven open-loop by a sinusoidally moving paddle. In Figure 4 time trajectories of the reference ball and the adjusting model are given. Figure 5 shows the estimated model parameter as a function of time. From these two plots we can observe that, as expected, ball/paddle strike instants synchronize and bounce heights regularize after a few bounces as the parameter is updated on the model bouncing ball. The parameter is correctly estimated.

Similarly, Figures 6 and 7 demonstrate a sample run of the estimation procedure for the nonlinear complaint bouncing ball problem. In particular, Figure 6 gives time trajectories of the system to be identified and the adjusting model. Trajectories initially behave differently but asymptotically converge as the parameter is correctly identified.

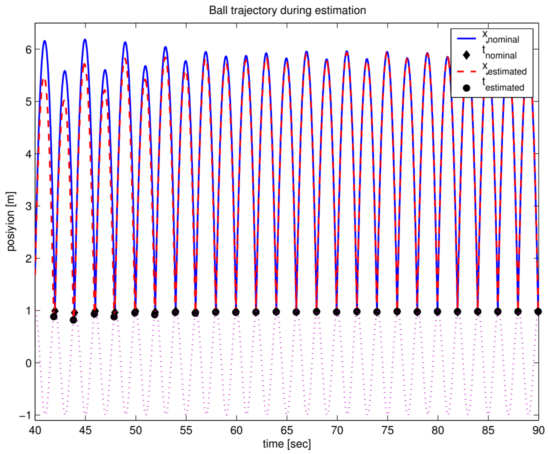


Figure 4. Trajectories of the system to be identified and the adjusting model

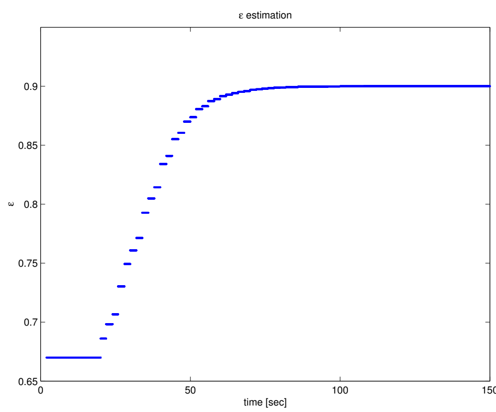


Figure 5. Parameter convergence in ϵ estimation

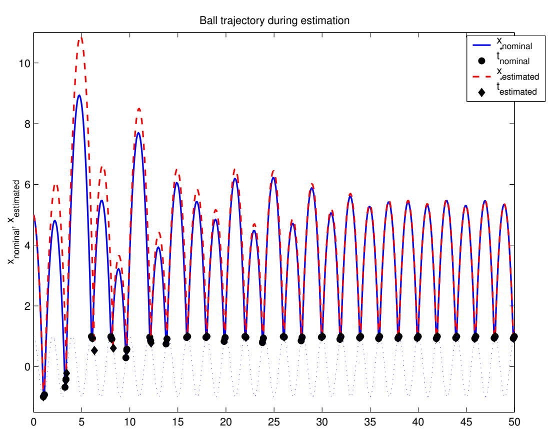
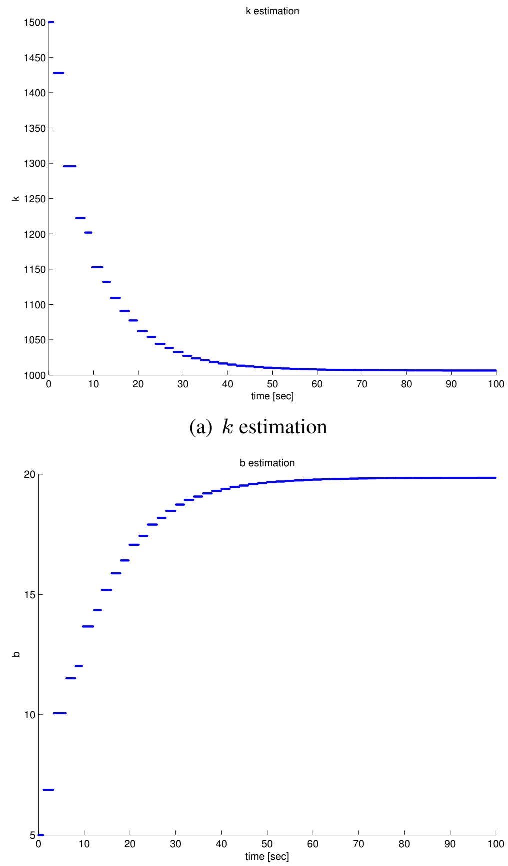


Figure 6. Trajectories of the system to be identified and the adjusting model

Figure 7 shows the estimated model parameters $\hat{k}$ and $\hat{b}$ as a function of time. The parameters are correctly estimated.



(a) k estimation

(b) b estimation

Figure 7. Parameter convergence for the nonlinear contact model

6 Conclusions and Future Work

This paper has presented two approaches to the parameter identification of certain hybrid dynamical systems. Both approaches are based on a feedback control architecture, and should be extensible to more complex hybrid systems than the bouncing ball example presented herein. Both approaches are basically gradient-descent algorithms that use sensitivity models to generate the gradients. One method uses sensitivities of the states to parameter variation in continuous time while the other method uses sensitivities at the switching instants.

Our future work includes the combination of the two approaches into a single feedback-control architecture. In particular, we envision a procedure that makes use of these two methods to result in an asymptotically stable hybrid estimator. A true hybrid estimator can be achieved by a proper combination of the continuous time and discrete-time adaptation methods. The combination is not trivial and as seen in the reconstruction problem, special care should be taken concerning the intervening time intervals when nominal and perturbed systems are in different modes.

We are also interested in investigating questions of identifiability, with plans to delineate which parameters can be identified using which kinds of sensitivities. Other open topics include excitation persistence and associated plans for closing the loop on the excitation of the hybrid system: the excitation could be placed under the direction of the parameter identification scheme rather than simply running open-loop as in the present work.

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