"Bach's extraordinary temperament: Our Rosetta Stone"

Appendix: comparison of Bach's method with other temperaments

Name	PC=Pythagorean Comma, SC=Syntonic Comma; circ=Circulating temperament (all keys usable)																							
1/12 PC: Equal Temperament circ	**Ab**	-60	**Eb**	-60	**Bb**	-60	**F**	-60	**C**	-60	**G**	-60	**D**	-60	**A**	-60	**E**	-60	**B**	-60	**F#**	-60	**C#**	-60	**G#**
1/4 SC regular meantone	**Ab**	1095	**Eb**	-165	**Bb**	-165	**F**	-165	**C**	-165	**G**	-165	**D**	-165	**A**	-165	**E**	-165	**B**	-165	**F#**	-165	**C#**	-165	**G#**
1/5 PC regular meantone	**Ab**	864	**Eb**	-144	**Bb**	-144	**F**	-144	**C**	-144	**G**	-144	**D**	-144	**A**	-144	**E**	-144	**B**	-144	**F#**	-144	**C#**	-144	**G#**
1/5 SC regular meantone	**Ab**	732	**Eb**	-132	**Bb**	-132	**F**	-132	**C**	-132	**G**	-132	**D**	-132	**A**	-132	**E**	-132	**B**	-132	**F#**	-132	**C#**	-132	**G#**
1/6 PC regular meantone	**Ab**	600	**Eb**	-120	**Bb**	-120	**F**	-120	**C**	-120	**G**	-120	**D**	-120	**A**	-120	**E**	-120	**B**	-120	**F#**	-120	**C#**	-120	**G#**
1/6 SC regular meantone	**Ab**	490	**Eb**	-110	**Bb**	-110	**F**	-110	**C**	-110	**G**	-110	**D**	-110	**A**	-110	**E**	-110	**B**	-110	**F#**	-110	**C#**	-110	**G#**
Bach 1/12 PC circ	**Ab**	-60	**Eb**	-60	**Bb**	60	**F**	-120	**C**	-120	**G**	-120	**D**	-120	**A**	-120	**E**	0	**B**	0	**F#**	0	**C#**	-60	**G#**
Bach 1/12 PC circ in D	**Ab**	0	**Eb**	-60	**Bb**	-60	**F**	-60	**C**	60	**G**	-120	**D**	-120	**A**	-120	**E**	-120	**B**	-120	**F#**	0	**C#**	0	**G#**
Bach 1/12 SC (1/13 PC) circ	**Ab**	-60	**Eb**	-55	**Bb**	0	**F**	-110	**C**	-110	**G**	-110	**D**	-110	**A**	-110	**E**	0	**B**	0	**F#**	0	**C#**	-55	**G#**
Bach 1/12 SC (1/13 PC) circ in D	**Ab**	0	**Eb**	-55	**Bb**	-60	**F**	-55	**C**	0	**G**	-110	**D**	-110	**A**	-110	**E**	-110	**B**	-110	**F#**	0	**C#**	0	**G#**
Barnes "Bach" 1/6 PC circ	**Ab**	0	**Eb**	0	**Bb**	0	**F**	-120	**C**	-120	**G**	-120	**D**	-120	**A**	-120	**E**	0	**B**	-120	**F#**	0	**C#**	0	**G#**
Billeter "Bach" 1/12 SC circ (1979)	**Ab**	0	**Eb**	0	**Bb**	0	**F**	-60	**C**	-55	**G**	-55	**D**	-220	**A**	-220	**E**	-55	**B**	-55	**F#**	0	**C#**	0	**G#**
Vallotti 1/6 SC circ	**Ab**	-10	**Eb**	-10	**Bb**	-10	**F**	-110	**C**	-110	**G**	-110	**D**	-110	**A**	-110	**E**	-110	**B**	-10	**F#**	-10	**C#**	-10	**G#**
Circ 1/5 PC: C-G-D-A-E-B	**Ab**	0	**Eb**	0	**Bb**	0	**F**	0	**C**	-144	**G**	-144	**D**	-144	**A**	-144	**E**	-144	**B**	0	**F#**	0	**C#**	0	**G#**
Circ 1/5 PC: F-C-G-D-A-E	**Ab**	0	**Eb**	0	**Bb**	0	**F**	-144	**C**	-144	**G**	-144	**D**	-144	**A**	-144	**E**	0	**B**	0	**F#**	0	**C#**	0	**G#**
Circ 1/6 PC: Vallotti F-C-G-D-A-E-B	**Ab**	0	**Eb**	0	**Bb**	0	**F**	-120	**C**	-120	**G**	-120	**D**	-120	**A**	-120	**E**	-120	**B**	0	**F#**	0	**C#**	0	**G#**
Circ 1/6 PC: Young C-G-D-A-E-B-F#	**Ab**	0	**Eb**	0	**Bb**	0	**F**	0	**C**	-120	**G**	-120	**D**	-120	**A**	-120	**E**	-120	**B**	-120	**F#**	0	**C#**	0	**G#**
Jorgensen "Handel" 1/25 PC circ	**Ab**	0	**Eb**	-28	**Bb**	-28	**F**	-28	**C**	-144	**G**	-116	**D**	-58	**A**	-58	**E**	-86	**B**	-116	**F#**	-58	**C#**	0	**G#**
Kellner (Werck/"Bach") 1/5 PC circ	**Ab**	0	**Eb**	0	**Bb**	0	**F**	0	**C**	-144	**G**	-144	**D**	-144	**A**	-144	**E**	0	**B**	-144	**F#**	0	**C#**	0	**G#**
Kirnberger 2: 1/2 SC circ *	**Ab**	0	**Eb**	0	**Bb**	0	**F**	0	**C**	0	**G**	0	**D**	-330	**A**	-330	**E**	0	**B**	0	**F#**	-60	**C#**	0	**G#**
Kirnberger 3: 1/4 SC circ *	**Ab**	0	**Eb**	0	**Bb**	0	**F**	0	**C**	-165	**G**	-165	**D**	-165	**A**	-165	**E**	0	**B**	0	**F#**	-60	**C#**	0	**G#**
Lindley Grosvenor 1/12 PC circ	**Ab**	0	**Eb**	60	**Bb**	60	**F**	-120	**C**	-120	**G**	-120	**D**	-120	**A**	-120	**E**	-120	**B**	-60	**F#**	-60	**C#**	0	**G#**
Neidhardt #1: 1/12 PC circ	**Ab**	0	**Eb**	0	**Bb**	-60	**F**	-60	**C**	-120	**G**	-120	**D**	-120	**A**	-60	**E**	0	**B**	-60	**F#**	-60	**C#**	-60	**G#**
Neidhardt #2: 1/12 PC circ	**Ab**	0	**Eb**	-60	**Bb**	-60	**F**	0	**C**	-180	**G**	-60	**D**	-120	**A**	-60	**E**	0	**B**	-60	**F#**	-60	**C#**	-60	**G#**
Neidhardt #3: 1/12 PC circ	**Ab**	0	**Eb**	-60	**Bb**	0	**F**	0	**C**	-120	**G**	-120	**D**	-120	**A**	-120	**E**	-60	**B**	-60	**F#**	0	**C#**	-60	**G#**
Sorge 1744: 1/12 PC circ	**Ab**	0	**Eb**	-60	**Bb**	-60	**F**	0	**C**	-120	**G**	-120	**D**	-120	**A**	0	**E**	-120	**B**	-60	**F#**	-60	**C#**	0	**G#**
Sorge 1758: 1/12 PC circ	**Ab**	-60	**Eb**	-60	**Bb**	-60	**F**	0	**C**	-120	**G**	-120	**D**	-120	**A**	-60	**E**	0	**B**	-60	**F#**	-60	**C#**	0	**G#**
Siracusa 1/4 SC almost circ	**Ab**	105	**Eb**	0	**Bb**	0	**F**	0	**C**	-165	**G**	-165	**D**	-165	**A**	-165	**E**	-165	**B**	0	**F#**	0	**C#**	0	**G#**
Stanford: Lindley sugg #2 1/12 PC circ	**Ab**	-60	**Eb**	60	**Bb**	60	**F**	-120	**C**	-120	**G**	-120	**D**	-120	**A**	-120	**E**	-120	**B**	0	**F#**	0	**C#**	-60	**G#**
Stanford: Vogel 1/5 PC circ	**Ab**	0	**Eb**	0	**Bb**	144	**F**	-144	**C**	-144	**G**	-144	**D**	-144	**A**	-144	**E**	-144	**B**	0	**F#**	144	**C#**	-144	**G#**
Stanford: Vogel 1/5 PC & 1/4 SC mt	**Ab**	969	**Eb**	-165	**Bb**	-165	**F**	-144	**C**	-144	**G**	-144	**D**	-144	**A**	-144	**E**	-144	**B**	-165	**F#**	-165	**C#**	-165	**G#**
Vogel/Ahrend Norden 1/5 PC circ	**Ab**	144	**Eb**	144	**Bb**	0	**F**	-144	**C**	-144	**G**	-144	**D**	-144	**A**	-144	**E**	-144	**B**	-144	**F#**	0	**C#**	0	**G#**
Werckmeister III: 1/4 PC circ	**Ab**	0	**Eb**	0	**Bb**	0	**F**	0	**C**	-180	**G**	-180	**D**	-180	**A**	0	**E**	0	**B**	-180	**F#**	0	**C#**	0	**G#**
Werckmeister IV: 1/3 PC circ	**Ab**	240	**Eb**	240	**Bb**	-240	**F**	0	**C**	-240	**G**	0	**D**	-240	**A**	0	**E**	-240	**B**	0	**F#**	-240	**C#**	0	**G#**
Werckmeister V: 1/4 PC circ	**Ab**	180	**Eb**	0	**Bb**	0	**F**	-180	**C**	0	**G**	0	**D**	-180	**A**	-180	**E**	0	**B**	0	**F#**	-180	**C#**	-180	**G#**
Bendeler 1: 1/3 PC circ	**Ab**	0	**Eb**	0	**Bb**	0	**F**	0	**C**	-240	**G**	-240	**D**	0	**A**	0	**E**	0	**B**	-240	**F#**	0	**C#**	0	**G#**
Bendeler 2: 1/3 PC circ	**Ab**	0	**Eb**	0	**Bb**	0	**F**	0	**C**	-240	**G**	0	**D**	-240	**A**	0	**E**	0	**B**	0	**F#**	-240	**C#**	0	**G#**
Bendeler 3: 1/4 PC circ	**Ab**	-180	**Eb**	0	**Bb**	0	**F**	0	**C**	-180	**G**	-180	**D**	0	**A**	0	**E**	-180	**B**	0	**F#**	0	**C#**	0	**G#**
Rameau 1/4 SC circ	**Ab**	218	**Eb**	217	**Bb**	-165	**F**	-165	**C**	-165	**G**	-165	**D**	-165	**A**	-165	**E**	-165	**B**	0	**F#**	0	**C#**	0	**G#**
Lambert/Chaumont/Legros 1/4 SC circ	**Ab**	501	**Eb**	132	**Bb**	132	**F**	-165	**C**	-165	**G**	-165	**D**	-165	**A**	-165	**E**	-165	**B**	-165	**F#**	-165	**C#**	-165	**G#**
Mercadier 1/12 SC circ	**Ab**	0	**Eb**	0	**Bb**	0	**F**	-60	**C**	-110	**G**	-110	**D**	-110	**A**	-110	**E**	-55	**B**	-55	**F#**	-55	**C#**	-55	**G#**
Neidhardt/Marpurg 1/6 PC fake ET	**Ab**	0	**Eb**	-120	**Bb**	0	**F**	-120	**C**	0	**G**	-120	**D**	0	**A**	-120	**E**	0	**B**	-120	**F#**	0	**C#**	-120	**G#**
Sparschuhe/Zapf	**Ab**	-44	**Eb**	-60	**Bb**	-40	**F**	-95	**C**	-142	**G**	-95	**D**	-127	**A**	0	**E**	0	**B**	0	**F#**	-50	**C#**	-67	**G#**

Fig. 1: Comparison of Bach's temperament with other representative temperaments. The focus of the present article is on the temperament here labeled "Bach 1/12 PC circ".

Figures 1 and 2 show analyses of Bach's temperament next to its competition today. All the major thirds are sharp in all of these, except the several that have fifths tempered by ¼ syntonic comma (165 units, here) yielding some pure major thirds.

Name	**55edo Notes**	Worst Maj3	**Shape**	Regular Naturals	**SC+# Maj3s**	Maj3 Sizes	**Semitone Sizes**
1/12 PC: Equal Temperament circ	**2**	All/None	**0**	7	**0**	1	**1**
1/4 SC regular meantone	**2**	B/F#/Db/Ab	**V**	7	**4**	2	**2**
1/5 PC regular meantone	**3**	B/F#/Db/Ab	**V**	7	**4**	2	**2**
1/5 SC regular meantone	**5**	B/F#/Db/Ab	**V**	7	**4**	2	**2**
1/6 PC regular meantone	**12**	B/F#/Db/Ab	**V**	7	**4**	2	**2**
1/6 SC regular meantone	**9**	B/F#/Db/Ab	**V**	7	**4**	2	**2**
Bach 1/12 PC circ	**6**	E	**B**	6	**0**	8	**8**
Bach 1/12 PC circ in D	**6**	F#	**V**	5	**0**	8	**8**
Bach 1/12 SC (1/13 PC) circ	**6**	E	**B**	6	**0**	8	**8**
Bach 1/12 SC (1/13 PC) circ in D	**6**	F#	**V**	5	**0**	8	**8**
Barnes "Bach" 1/6 PC circ	**6**	F#/Db	**V**	6	**2**	5	**6**
Billeter "Bach" 1/12 SC circ (1979)	**3**	F#/C#	**V**	3	**2**	7	**8**
Vallotti 1/6 SC circ	**7**	B/F#/C#	**V**	7	**0**	5	**0**
Circ 1/5 PC: C-G-D-A-E-B	**4**	B/F#/Db/Ab	**V**	6	**4**	5	**6**
Circ 1/5 PC: F-C-G-D-A-E	**4**	E/B/F#/Db	**V**	6	**4**	5	**6**
Circ 1/6 PC: Vallotti F-C-G-D-A-E-B	**7**	B/F#/Db	**V**	7	**3**	5	**6**
Circ 1/6 PC: Young C-G-D-A-E-B-F#	**7**	F#/Db/Ab	**V**	6	**3**	5	**6**
Jorgensen "Handel" 1/25 PC circ	**4**	Db	**V**	3	**0**	8	**10**
Kellner (Werck/"Bach") 1/5 PC circ	**4**	F#/Db/Ab	**V**	5	**3**	5	**5**
Kirnberger 2: 1/2 SC circ *	**2**	Db/Ab/Eb/Bb	**V**	5	**4**	5	**6**
Kirnberger 3: 1/4 SC circ *	**3**	Db/Ab	**V**	5	**2**	7	**8**
Lindley Grosvenor 1/12 PC circ	**7**	Db	**V**	7	**3**	8	**10**
Neidhardt #1: 1/12 PC circ	**4**	F#/Db/Ab	**V**	4	**0**	6	**6**
Neidhardt #2: 1/12 PC circ	**3**	Ab	**V**	2	**0**	6	**5**
Neidhardt #3: 1/12 PC circ	**6**	Ab	**V**	5	**0**	7	**6**
Sorge 1744: 1/12 PC circ	**4**	B/F#/Db/Ab	**V**	4	**0**	4	**6**
Sorge 1758: 1/12 PC circ	**4**	E	**B**	4	**0**	6	**5**
Siracusa 1/4 SC almost circ	**3**	B/F#/Db/Ab	**V**	6	**4**	5	**6**
Stanford: Lindley sugg #2 1/12 PC circ	**7**	Db	**V**	7	**1**	8	**10**
Stanford: Vogel 1/5 PC circ	**4**	B/F#/Db/Ab	**V**	7	**4**	5	**7**
Stanford: Vogel 1/5 PC & 1/4 SC mt	**3**	B/F#/Db/Ab	**V**	7	**4**	4	**5**
Vogel/Ahrend Norden 1/5 PC circ	**3**	F#/Db	**V**	7	**4**	7	**8**
Werckmeister III: 1/4 PC circ	**4**	F#/Db/Ab	**V**	4	**3**	4	**4**
Werckmeister IV: 1/3 PC circ	**5**	F#/Db/Ab	**V**	2	**4**	4	**5**
Werckmeister V: 1/4 PC circ	**4**	Db/Ab	**V**	3	**2**	3	**4**
Bendeler 1: 1/3 PC circ	**3**	F#/Db/Ab	**V**	4	**3**	3	**3**
Bendeler 2: 1/3 PC circ	**3**	Db/Ab	**V**	3	**2**	3	**3**
Bendeler 3: 1/4 PC circ	**2**	D...Eb	**V**	3	**0**	2	**3**
Rameau 1/4 SC circ	**2**	F#	**V**	7	**4**	10	**11**
Lambert/Chaumont/Legros 1/4 SC circ	**2**	Db/Ab	**V**	7	**4**	6	**6**
Mercadier 1/12 SC circ	**5**	Db/Ab	**V**	5	**0**	8	**9**
Neidhardt/Marpurg 1/6 PC fake ET	**2**	All/None	**0**	2	**0**	2	**1**
Sparschuher/Zapf	**5**	A	**S**	2	**0**	11	**11**

Fig. 2: Comparison of Bach's temperament with other representative temperaments.

For this comparison, the unit of measurement is the "Temperament Units" system devised by organ-builder John Brombaugh. It is a logarithmic scale assigning 720 microscopic portions to the Pythagorean comma. The syntonic comma then works out to 660 TU, the schisma (difference between them) 60. All of these numbers are easy to work with on paper, divisible by 3, 4, 5, 6, and 12: the fractions used in most descriptions of theoretical temperaments. The TU system therefore allows a theorist or player to work almost exclusively with integers in sorting out the concepts of keyboard tuning.

Each row of the chart sums to –720: the distribution of the PC around the circle. To find the quality of the major thirds, sum the four intervening fifths and add 660, then divide the result by 660. For example, in Barnes's temperament the third Eb-G is 420/660 or 63.6% sharp of pure: the same amount that all the major thirds of equal temperament are (and therefore also the average of all twelve thirds, in every temperament).

The column "55edo Notes" shows the count of notes that coincide with the "55-division" of Sauveur et al, i.e. a double spiral of regular 1/6 Pythagorean comma fifths including all the enharmonic notes from Bbb to Gx.

The column "Worst Maj3rd" shows which major third(s) in the given temperament are the widest, i.e. the farthest from a pure 5/4 interval. The accompanying column "Shape" summarizes that. If the worst major third is one or more of the traditional meantone diminished fourths, I have labeled it "W" (for "workmaster"). If that peak is at E, I use "B" (for "brook")…simply to distinguish it from "W" and show how uncommon it is in its ability to handle enharmonic equivalences. Equal and fake-equal temperaments have "0" shape since all the thirds are the same as one another; these also handle enharmonic swaps smoothly. Sparschuhe/Zapf, with its peak at A, is labeled "S". The broader point here is: most of the tuning literature presents *only* the "W" options as the viable alternatives to equal temperament, as if the properties of those temperaments are automatically true of all unequal temperaments.

The column "Regular Naturals" shows the count of consecutive identical fifths among the notes F-C-G-D-A-E-B: where singers and other instrumentalists expect regularity of intonation, for ease of matching pitches.

The column "SC+# Maj3s" shows the count of major thirds that are 100% syntonic comma sharp (i.e. Pythagorean thirds), or higher. The quality of the major third determines most of the harshness or smoothness of the triads.

The columns "Maj3 Sizes" and "Semitone Sizes" count the different intonations of these intervals available in the temperament, as expressive resources for the composer and performers.

Explanation of the included temperaments

A syntonic comma interpretation of Bach's diagram is as follows, for completeness. If one uses the SC, the closing of the circle results in a pure fifth at Bb-F (at the open ends of Bach's diagram). This is equivalent in practice to a 1/13 PC reading of the diagram: since there are 13 *Spuren*, we simply split the PC into 13 pieces and map it as he shows. By mathematical coincidence this aligns almost perfectly with a 1/12 division of the SC. One of the "1/12" SC fifths is actually the schisma, giving a total of 13 of these 12ths, and I have assigned it to the usual meantone position of "leftovers": between Eb and G#. This reading of the temperament has the same basic harmonic shape, with E major still the brightest, but slightly faster-beating major thirds F-A and C-E.

The PC and SC versions of Bach's temperament also appear here in the "D" transposition as described in the section "Ensemble music". Their purpose is to simulate the performance situation of an organist playing in *Chorton* while the orchestra and singers are reading in *Cammerton*, a tone lower. That is, they facilitate the use of modern performing materials for Bach's Leipzig vocal works: having the keyboard part transposed down to match the orchestra and transposing the temperament itself back up a step to offset that.

Owen Jorgensen's "Handel" temperament is his theoretical interpretation of a mathematically vague tuning method printed c1780 in London, and attributed to George Friderich Handel (who had died in 1759). Full description is in pages 158-172 of Jorgensen's book *Tuning* (1991).

The details of Harald Vogel's "Norden" temperament (c1984 for a restoration by Jürgen Ahrend) are from John Brombaugh, with whom I have been in conversation during this project.

Vogel's two temperaments devised for the Stanford University organ are described in the booklet of his CD "Buxtehude und seine Zeit", Organa 3208: allowing the organ to played in either this circulating temperament or hybrid near-meantone layout by moving a lever. Lindley discussed this further in "A Suggested Improvement for the Fisk Organ at Stanford" (1988), yielding his own recommendations (one of which is shown here).

Lindley's "Grosvenor" temperament is for the restoration of the 18th century organ Grosvenor Chapel, Mayfair, London: in collaboration with Joost de Boer, and William Drake. A description is at http://www.btinternet.com/~william.drake/GrosvenorTemperament.htm

The early 17th century "Siracusa" temperament is a discovery by Andrea Grisendi in the archives of the cathedral of Siracusa. It is described in the booklet of the CD Tactus 601901: Bernardo Storace's keyboard works played by Francesco Cera. This is an Italian example of "ordinary" temperament with tastefully raised sharps and lowered flats. Contemporary French methods were in use as well: see Lindley's articles "Temperament" and "Well-Tempered Clavier" in *New Grove* (2001).

In Johann Philipp Kirnberger's temperaments (from *Die Kunst des reines Satzes…* (1771/79)), I have corrected his divisions to use true square roots where he had used approximations (e.g., the division of 81/80 into the two parts 162/161 x 161/160). Kirnberger was a former student of Bach. His theoretical writings explored the possibilities of deriving temperaments from just-intonation values, working as far as possible with simple fractions: an elegant approach having nothing to do with Bach. As an easy practical scheme converting regular ¼ syntonic comma to a smooth circulating temperament, simply by straightening out as many fifths as possible outside the C-G-D-A-E core, his methods could have been discovered independently by anyone 150-200 years before he did. (Indeed, the "Siracusa" is nearly the same as "Kirnberger 3" working with a tuning hammer in hand.) "Kirnberger 3" is also known variously as another of Sorge's 1744 temperaments (but in a Pythagorean comma version), D'Alembert 1752, and Prinz/Farey 1808. Moroney has used this temperament in his recording of William Byrd's music, taking it back hypothetically another 150 years before Sorge, as a "modified Pythagorean" Elizabethan tuning.

Barnes' temperament is technically derivative from Kellner's, and Kellner's in turn from Werckmeister's, each "smoothing" the predecessor's sound. Respectively these methods split 1/6, 1/5, and 1/4 Pythagorean comma. Their common shape is to take pure fifths all the way around the flat side ([C-]F-Bb-Eb-Ab-Db-Gb), put a single tempered fifth at B-Gb, and put all the other tempered fifths in the naturals from C (where they would be from regular 17th century layouts). Werckmeister's has tempered fifths C-G-D-A; Kellner's C-G-D-A-E; and Barnes' F-C-G-D-A-E. Kellner claimed that Werckmeister probably secretly used the 1/5 comma version (i.e. Kellner's own modification) while publishing only the 1/4 comma version.

The three Neidhardt temperaments shown here are as reported in Lindley's *New Grove* article "Well-tempered clavier". "#1" is for a small city in 1724, a large city in 1732. "#2" is for a large city in 1724. "#3" is for a village in 1724, a small city in 1732. Further Neidhardt temperaments are available in Barbour's book.

The other temperaments shown here are from sources cited in the notes of the present article: most are also readily available in Lindley's articles, Barbour's book, Devie's book, and Paul Poletti's booklet *Temperaments For Dummies*. I have omitted the 18th century quasi-equal temperaments by Marpurg, Sorge, and Neidhardt (as contrasted with his earlier ones shown here) as their intended *effect* for listeners is that of true equal temperament, while being easier to tune in practice. These are covered both by Barbour and Lindley, and elsewhere in the tuning literature.

The final chart, Figure 3, gives another comparative view of the temperaments that distribute 1/12 portions of the PC, reoriented to the same sequence as in Bach's diagram. The note C is taken in common, for reference. The notes that match their positions in Bach's temperament are in boldface. It can be seen how some of these temperaments may be converted easily from one to another, simply by nicking several of the notes up or down slightly by 1/12 or 2/12 PC. (For example, "Barnes" and "Vallotti" differ by only the single note B.) The Barnes temperament has 8 of Bach's 12 notes correctly, but the notes F#, C#, and G# are tuned too flat, and the A# too sharp.

The Bach temperament transposed into D is remarkably close to equal temperament, from the orchestra's perspective in the ease of matching pitches. Five of the pitches are exactly the same as in equal temperament, and the other seven are nearby. Most notably it has lower-pitched B, F#, C#, and G#, being more euphonious in their tonal functions as thirds of major triads.

	F	C	G	D	A	E	B	F#	C#	G#	D#	A#	*(E#)*
1/12 PC: Equal Temperament circ	1	**0**	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	*-11*
1/6 PC regular meantone	**2**	**0**	**-2**	**-4**	**-6**	**-8**	-10	-12	-14	-16	-6	-8	*-10*
Bach 1/12 PC circ	**2**	**0**	**-2**	**-4**	**-6**	**-8**	**-8**	**-8**	**-8**	**-9**	**-10**	**-11**	*-10*
Bach 1/12 PC circ in D	1	**0**	1	-1	-3	-5	-7	-9	-9	**-9**	-9	-10	*-11*
Barnes "Bach" 1/6 PC circ	**2**	**0**	**-2**	**-4**	**-6**	**-8**	**-8**	-10	-10	-10	**-10**	-10	*-10*
Circ 1/6 PC: Vallotti F-C-G-D-A-E-B	**2**	**0**	**-2**	**-4**	**-6**	**-8**	-10	-10	-10	-10	**-10**	-10	*-10*
Circ 1/6 PC: Young C-G-D-A-E-B-F#	0	**0**	**-2**	**-4**	**-6**	**-8**	-10	-12	-12	-12	-12	-12	*-12*
Lindley Grosvenor 1/12 PC circ	**2**	**0**	**-2**	**-4**	**-6**	**-8**	-10	-11	-12	-12	-12	**-11**	*-10*
Neidhardt #1: 1/12 PC circ	1	**0**	**-2**	**-4**	**-6**	-7	-7	**-8**	-9	-10	**-10**	-10	*-11*
Neidhardt #2: 1/12 PC circ	0	**0**	-3	**-4**	**-6**	-7	-7	**-8**	-9	-10	**-10**	**-11**	*-12*
Neidhardt #3: 1/12 PC circ	0	**0**	**-2**	**-4**	**-6**	**-8**	-9	-10	-10	-11	-11	-12	*-12*
Sorge 1744: 1/12 PC circ	0	**0**	**-2**	**-4**	**-6**	-6	**-8**	-9	-10	-10	**-10**	**-11**	*-12*
Sorge 1758: 1/12 PC circ	0	**0**	**-2**	**-4**	**-6**	-7	-7	**-8**	-9	**-9**	**-10**	**-11**	*-12*
Stanford: Lindley sugg #2 1/12 PC circ	**2**	**0**	**-2**	**-4**	**-6**	**-8**	-10	-10	-10	-11	-12	**-11**	*-10*
Werckmeister III: 1/4 PC circ	0	**0**	-3	-6	-9	-9	-9	-12	-12	-12	-12	-12	*-12*
Werckmeister IV: 1/3 PC circ	0	**0**	-4	**-4**	-8	**-8**	-12	-12	-16	-16	-8	-12	*-12*
Werckmeister V: 1/4 PC circ	3	**0**	0	0	-3	-6	-6	-6	-9	-12	-9	-9	*-9*
Bendeler 1: 1/3 PC circ	0	**0**	-4	-8	-8	**-8**	**-8**	-12	-12	-12	-12	-12	*-12*
Bendeler 2: 1/3 PC circ	0	**0**	-4	**-4**	-8	**-8**	**-8**	**-8**	-12	-12	-12	-12	*-12*
Bendeler 3: 1/4 PC circ	0	**0**	-3	-6	**-6**	-6	-9	-9	-9	**-9**	-12	-12	*-12*
Neidhardt/Marpurg 1/6 PC fake ET	**2**	**0**	0	-2	-2	-4	-4	-6	-6	-8	-8	-10	*-10*

Fig. 3: Placement of the notes with the unit of 1/12 PC. The most extraordinary features of Bach's temperament, from this perspective, are the unusually high C# and G# allowing them to function so smoothly as Db and Ab. Bach's and the 1758 Sorge are the only two here that have a gentler B major triad than the E major triad.