Recovering Bach’s tuning from the *Well-Tempered Clavier*

*[Colloquium presentation, University of Colorado: October 11, 2010]*

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Why use unequal temperaments on harpsichords and organs? (part 1)

• Better resonance and projection of the instrument
• It compensates for the inability to play dynamic contrasts from note to note
• The melodic and harmonic tensions inspire sensitive players to listen closely, and to shape the musical delivery with appropriate expression
• Differences among the keys help to sustain the listener’s interest during a composition, and to provide contrasts during a concert or church service
• It makes the instrument seem “alive”, not merely like a box full of levers
Why use unequal temperaments on harpsichords and organs? (part 2)

• The keys and scales used most frequently can be better in tune than equal temperament allows
• Good unequal temperaments are easier and much faster to set up than equal temperament is
• The blend with other instruments and voices can be more harmonious
• Historical accuracy: play older compositions in the temperament style for which the music was originally conceived, if this information is reasonably knowable
  [Presumably, the composers knew what they were doing, and wanted their music to sound good....]

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Which notes do we need to install into our keyboard instruments?

12 most common note names

[Spiral of 5ths]

[The harpsichord for this demonstration is set up with regular 1/6 comma tempering on the upper manual, and Bach/Lehman on the lower manual.]
A harpsichord sounds “out of tune” whenever we play notes that are misspelled for the way it was tuned.

..., Fb, Cb, Gb, Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#, D#, A#, E#, B#, ...

Ebb, Bbb, Fb, Cb, Gb, Db, Ab are typically too low in pitch, because they were tuned as D, A, E, B, F#, C#, and G#.

D#, A#, E#, B#, Fx, Cx, Gx are typically too high, because they were tuned as Eb, Bb, F, C, G, D, and A.

[On harpsichord with regular 1/6 comma: play B-Eb, F#-Bb, G#-C, C#-F which are really diminished 4ths; compare against the sound of correctly-spelled major 3rds. Also play melodically: B-C#-Eb, F-G-F#, F#-G#-Bb, and some correctly-spelled scales.]
Before we can tune an instrument, we have to study how scales are put together.

Where do we get all our notes: A, B, C, D, E, F, G, and the sharps and flats?
Notes are all related to one another by 5ths. This is the way scales are generated.

A series of 5ths makes the simplest sequence to tune keyboard instruments by ear. Starting from C, tune G by playing both notes together and adjusting G.

From G, tune D. From D, tune A. From A, tune E; etc. We can copy pure octaves to keep the notes within a manageable range. We can also tune by 5ths flatward: C-F-Bb-Eb-etc.

NOTE: after we have tuned four 5ths, such as C-G-D-A-E, we can test the sound of the resulting major 3rd: C-E.
Each major scale uses seven consecutive notes from the spiral of 5ths: **Fa, Ut, Sol, Re, La, Mi, Ti.**

“Ut” is an older name for the note “Do”, the starting note of a major scale. (“Ut, a deer, a female deer...”)

All seven diatonic notes are generated by 5ths: Fa-Ut-Sol-Re-La-Mi-Ti.

Rearrange them as the major scale: Ut-Re-Mi-Fa-Sol-La-Ti.

[The extra notes outside Ut-Re-Mi-Fa-Sol-La-Ti are used only if you change key temporarily during the composition....]
We have each letter name once and only once: an “A something”, a “B something”, a “C something”, etc.

To modulate from F major to C major, stop using Bb and start using B.

Fa, Ut, Sol, Re, La, Mi, Ti.
Modulation: the old Ut becomes the new Fa, or vice versa. The whole Fa-Ut-Sol-Re-La-Mi-Ti sequence simply rotates to a new position.

To modulate from C major to G major, stop using F and start using F#.

C major → G major

Chromatic change: C# replaces C, but they are both “C something”.

D major

[Play D minor and D major triads in alternation, showing the change of “color” in that “chromatic” difference]
There is always a break where the “circle of 5ths” does not connect.

Working by 5ths, all keyboard temperaments must have a break somewhere: a diminished 6th where the name changes from sharps to flats.

[Again demonstrate the good major 3rds against the diminished 4ths: the triangles in this drawing.]
Some examples from music published in 1722

Christoph Graupner, *Monatliche Clavir Früchte* (“Monthly Keyboard Fruit”), suites named for all 12 months

...in C, G, g, c, F, f, D, d, A, a, E, e.
...tonic on F-C-G-D-A-E, with compositions in both major and minor.

Notes needed:

January, C: 12 notes: Bb, F, C, G, D, A, E, B, F#, C#, G#, D#.
March, g: 12 notes: Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#.
April, c: 13 notes: Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#.
June, f: 12 notes: Gb, Db, Ab, Eb, Bb, F, C, G, D, A, E, B.
August, d: 13 notes: Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#, D#.
October, a: 12 notes: Bb, F, C, G, D, A, E, B, F#, C#, G#, D#.
Some examples from music published in 1722

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March, g: 12 notes: Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#.
April, c: 13 notes: Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#.
June, f: 12 notes: Gb, Db, Ab, Eb, Bb, F, C, G, D, A, E, B.
August, d: 13 notes: Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#, D#.
October, a: 12 notes: Bb, F, C, G, D, A, E, B, F#, C#, G#, D#.

[If a note is needed for two different functions, you can leave it set as one or the other, or you can try to make a compromise in pitch so it will work passably as either one.

If the composition is in major, look for overlap points at the minor 3\(^{rd}\) and augmented 2\(^{nd}\) : needing Ab and G# in F major, or Bb and A# in G major.

If the composition is in minor, look for the minor 2\(^{nd}\) and the augmented unison : needing F natural and E# in E minor, or Eb and D# in D minor.]
Some examples from music c1720-22

[Play some music on the regular 1/6 comma manual to demonstrate how it sounds if any misspelled notes come in.]

Graupner, Menuet from “Januarius” (January), C major.
11 notes: Bb, F, C, G, D, A, E, B, F#, C#, G#. [“Pure” effect, no wrong notes]

Graupner, Menuet and Praeludium from “Junius” (June), F minor.
11 notes: Db, Ab, Eb, Bb, F, C, G, D, A, E, B.

Graupner, Allemande from “Aprilis” (April), C minor.
12 notes: Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#

Graupner, Menuet from “November”, E major.
10 notes: A, E, B, F#, C#, G#, D#, A#, E#, B#

Bach, Allemande from “English” suite in A major. [Later in the lecture]

Bach, Sarabande from “English” suite in G minor. [Later]
14 notes: Cb, _, Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#.
What kind of practical tuning challenges does the *Well-Tempered Clavier* (the book) present?

• Bach provided exemplary preludes and fugues in all 24 major and minor scales.

• The book as a whole calls for 27 differently-named notes!

• Most of the preludes and fugues use 13, 14, or more notes...not only a convenient 12.

• Therefore, tasteful compromises MUST be made: for example, in C major, choosing a pitch for the G# and Ab keyboard lever that can serve passably as either one, in all its harmonic and melodic contexts.
A roster of all the named notes in the *Well-Tempered Clavier (book 1, 1722)*:

Only one prelude and fugue (F major) can be played in old-fashioned meantone temperaments. Five others are confined to using only 12 notes, but they would require retuning. All the rest of the preludes and fugues go beyond 12 notes.

C major: 13 notes: Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#
C minor: 12 notes: Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#
C# major: 15 notes: A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx, Cx, Gx, Dx, Ax
C# minor: 14 notes: G, D, A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx, Cx
D major: 14 notes: Bb, F, C, G, D, A, E, B, F#, C#, G#, D#, A#, E#
D minor: 13 notes: Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#, D#
Eb major: 14 notes: Cb, Gb, Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#.
Eb minor/D# minor: 25 notes: Bbb, Fb, Cb, Gb, Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx, Cx, Gx
E major: 13 notes: C, G, D, A, E, B, F#, C#, G#, D#, A#, E#, B#
F major: 12 notes: Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#
F# major: 14 notes: D, A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx, Cx, Gx
F# minor: 14 notes: G, D, A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx
G major: 14 notes: Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#, D#, A#
G minor: 12 notes: Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#
Ab major: 12 notes: Gb, Db, Ab, Eb, Bb, F, C, G, D, A, E, B
G# minor: 14 notes: D, A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx, Cx, Gx
A major: 12 notes: G, D, A, E, B, F#, C#, G#, D#, A#, E#, B#
A minor: 14 notes: Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#, D#, A#
Bb major: 14 notes: Gb, Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#
Bb minor: 13 notes: Fb, Cb, Gb, Db, Ab, Eb, Bb, F, C, G, D, A, E
B major: 12 notes: D, A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx
B minor: 17 notes: Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx

(including 13 different notes already in the statement of the fugue subject!)

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A roster of all the named notes in the *Well-Tempered Clavier (book 1, 1722)*:

Overlap to 13, 14, or more notes in a composition....

C# minor: 14 notes: G, D, A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx, Cx

D major: 14 notes: Bb, F, C, G, D, A, E, B, F#, C#, G#, D#, A#, E#

F# major: 14 notes: D, A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx, Cx, Gx

G major: 14 notes: Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#, D#, A#

A minor: 14 notes: Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#, D#, A#

Bb major: 14 notes: Gb, Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#

[When the composition needs 14 different notes and is in a major key, the overlap points are most often: the minor 3\(^{rd}\) and augmented 2\(^{nd}\) above the tonic, and the major 3\(^{rd}\) below the tonic. For example, in D major, we need the F and the E#, along with the Bb a major 3\(^{rd}\) below D.

When the composition needs 14 different notes and is in a minor key, the overlap points are most often: the minor 2\(^{nd}\) and the augmented unison, plus both notes at the tritone. For example, in A minor, we need the Bb and the A#, plus both the Eb and D# at the tritone.

There are exceptions, but these are the most crucial spots where there must be some compromise.

Furthermore, if we want to play more than one piece, it’s useful to find a temperament that we can leave in one position for a whole day....or for years, on a clavichord.]
A roster of all the named notes in the *Well-Tempered Clavier (book 2)*:

All of book 2 (compiled in the 1740s) goes beyond 12 notes per piece.

C major: 14 notes: Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#
C minor: 13 notes: Gb, Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#
C# major: 13 notes: A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx, Cx, Gx
C# minor: 14 notes: C, __, D, A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx, Cx
D major: 14 notes: Bb, F, C, G, D, A, E, B, F#, C#, G#, D#, A#, E#
D minor: 13 notes: Db, __, Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#
Eb major: 16 notes: Fb, Cb, Gb, Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#
D# minor: 14 notes: A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx, Cx, Gx, Dx
E major: 14 notes: G, D, A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx, Cx
F major: 15 notes: Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#, D#
F minor: 13 notes: Cb, Gb, Db, Ab, Eb, Bb, F, C, G, D, A, E, B
F# major: 15 notes: D, A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx, Cx, Gx, Dx
F# minor: 14 notes: C, G, D, A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx
G major: 13 notes: Bb, F, C, G, D, A, E, B, F#, C#, G#, D#, A#
G minor: 15 notes: Cb, __, Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#
Ab major: 16 notes: Ebb, Bbb, Fb, Cb, Gb, Db, Ab, Eb, Bb, F, C, G, D, A, E, B
G# minor: 17 notes: C, G, D, A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx, Cx, Gx, Dx
A major: 13 notes: C, G, D, A, E, B, F#, C#, G#, D#, A#, E#, B#
A minor: 15 notes: Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#, G#, D#, A#
Bb major: 15 notes: Cb, Gb, Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#
Bb minor: 16 notes: Ebb, Bbb, Fb, Cb, Gb, Db, Ab, Eb, Bb, F, C, G, D, A, E, B
B major: 14 notes: D, A, E, B, F#, C#, G#, D#, A#, E#, B#, Fx, Cx, Gx
So, that is the basic problem to solve: How can we play in all these notes that keep getting renamed throughout the book?
A closer look at the top section:
Isolate Bach’s drawing at the top of the page. Why isn’t it symmetrical?

Why does it have three different types of spirally loops in it?
Within the need to play in all 24 different major and minor scales, using 27 different notes smoothly: (Premises, part 1)

• Tune the harpsichord by ear as quickly as possible.
• Make the first half of the notes as normal as possible, coming from everyday practice.
• Install the naturals first, because they are the center of melody and harmony.
• Tuning is done by 5ths and 4ths, checking major 3rds as they become available. 5ths are made deliberately narrow, or 4ths wide, so the major 3rds will turn out nicely.
Within the need to play in all 24 different major and minor scales, using 27 different notes smoothly: (Premises, part 2)

• The first six notes are the hexachord in the home key, the naturals, C major: C-D-E-F-G-A; that’s F-C-G-D-A-E when working by 5ths/4ths.

• Fit the remaining notes (B and the sharps) into place carefully, compromising them sharpward so they can also serve well as flats.

• That is: they are tempered less tightly by 5ths/4ths than the starting 5ths/4ths were. They are more nearly pure, or perhaps some are pure.

• Late 17th century practice of “modified meantone”: stretching the sharps higher and the flats lower until they meet.

• What might such a process look like on paper?
My interpretation of Bach’s drawing (part 1)

• It is a practical diagram for tuning.
• The adjustments are all practical motions of the wrist, not a pretext for calculations. The tuner works hands-on at the harpsichord, and doesn’t need to know any mathematics.
• Bach was demonstrating with this book a better tuning method than equal, and better than any of the other expert or amateur systems around him.
• Bach drew the diagram at the bottom of the page, with the page rotated 180 degrees. It is now at the top of the page, upside-down.
My interpretation of Bach’s drawing (part 2)

• The loops of the drawing indicate how much to temper each 5th in turn.
• The beginning notes are the naturals, and are done the same way as normally: F-C-G-D-A-E by 5ths, narrowed by the usual amount each.
• The remaining notes are all placed slightly higher than they would have been in that system. This allows them to function well as either sharps or flats.
• E-B-F#-C# are pure, because the loops of the drawing are empty.
• The final notes C#-G#-D#-A# are fit into position using average 5ths, only very slightly tempered.
• From the point where the 5th would be sounding as pure, the tuner nudges the upper note flatward (turning the tuning pin counterclockwise) by 2, 0, or 1 nudge of the wrist. “About that much.”
My interpretation of Bach’s drawing (part 3)

• The point is to make all 24 major and minor scales (and music in them) playable and beautiful, with a healthy variety of character and with no obviously “out-of-tune” problems.

• The same temperament can be left in place to play the entire book, with all 27 of the different notes it asks for.

• The temperament is not tied to any specific frequency or pitch. The starting C, F, or A can be any convenient tension for the given instrument, and all the rest is derived from there by ear, listening for relationships.
Adjusting the regular 1/6 comma temperament to become “Bach/Lehman”
(Bach’s own practice?)

• Leave most of the naturals where they were: F, C, G, D, A, and E.
• Raise the B, F#, C#, and G# (the sharps all get sharper).
• Lower the Bb and Eb (the flats get flatter) to become a better A# and D#.
• The spiral of 5ths meets itself nearly enough to sound like a circle of 5ths on a standard keyboard with 12 key levers per octave.

[Demonstrate how the naturals are in unison with the other manual on this harpsichord, but the remaining notes are adjusted higher or lower for their dual functions.]
The Bach/Lehman layout
Some musical outcomes of this temperament: (part 1)

• Music in sharp keys gets a bright and brilliant sound.
• Music in flat keys is warmer, gentler, and mellower.
• Music in major keys tends to sound smooth, genial, poised.
• Music in minor keys tends to have much stronger contrasts and forceful gestures; then, it relaxes when it moves to major.
• Tonal music has firm arrival points.
• Music has tension and relaxation as it goes along. (By contrast, when the same music is played in equal temperament it has “nowhere to go”, because equal temperament offers only a one-dimensional surface....)
• Chromatic music sounds especially intense.
Some musical outcomes of this temperament: (part 2)

• Performance is easier: it is not necessary to work so hard to make the musical effects project well. (Bach: “Just hit all the right notes at the right times, and the instrument plays itself.”)

• This temperament works not only for harpsichords and clavichords, but also for organs and pianos.

• If we can assume that a Leipzig organ was in or near this temperament, at least for a few accompanying stops: Bach’s vocal music for his job there emerges with strongly dramatic contrasts. The modulations and the choices of key fit the moods closely. That is to say: the expressive character of this temperament may have inspired some of Bach’s compositional ideas, both instrumental and vocal.
Some musical outcomes of this temperament: (part 3)

• This temperament also fits (and inspired?) the most volatile and intense music by others in Bach’s family: especially the two oldest sons, Wilhelm Friedemann Bach and Carl Philipp Emanuel Bach.
• CPE Bach described something similar, and possibly this specific temperament, in his own book (Essay on the True Art of Playing the Keyboard, 1753): in the correct modern temperament, one “takes away from most of the 5ths a scarcely noticeable bit from their absolute purity....”
Some examples from music c1720-22

[Play now on the manual with Bach/Lehman....]

Graupner, Menuet from “Januarius” (January), C major.
  11 notes: Bb, F, C, G, D, A, E, B, F#, C#, G#.

Graupner, Menuet and Praeludium from “Junius” (June), F minor.
  11 notes: Db, Ab, Eb, Bb, F, C, G, D, A, E, B.

Graupner, Allemande from “Aprilis” (April), C minor.
  12 notes: Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#

Graupner, Menuet from “November”, E major.
  10 notes: A, E, B, F#, C#, G#, D#, A#, E#, B#

Bach, Allemande from “English” suite in A major.

Bach, Sarabande from “English” suite in G minor.
  14 notes: Cb, __, Db, Ab, Eb, Bb, F, C, G, D, A, E, B, F#, C#.
The Bach/Lehman layout

Resources

• Lehman, Bradley. “Bach’s extraordinary temperament: our Rosetta Stone” (Early Music, 2005)
• Lehman. Several other articles in Clavichord International, Diapason, BBC Music, and CD booklet notes.
• Lehman. Web site www.larips.com with free copies of the articles, analysis, musical examples, CDs for purchase, YouTube video demonstrations, and more.
• At least a dozen pipe organs in North America and Europe with this temperament in their permanent installation.