Problem 1 - Piezoelectric accelerometer

You are interested in measuring vibrations that are induced by an earthquake. You decide to accomplish this by building your own piezoelectric accelerometer. This is done by sandwiching a ceramic piezo disk between a seismic mass $m$. The sensors base is fastened to a surface you are measuring, so that when an acceleration is applied the seismic mass will move up and down, causing the piezo disk to generate a charge (think of the capacitor model we derived in class). For this simplified analysis we will assume that the piezo disk is only generating charge if a force is applied in the vertical direction (due to acceleration), proportional to the piezo charge coefficient $d_{yy}$. The circular piezoelectric disk is a 0.5 cm in diameter and 1 mm thick (denote thickness by the variable $l$). The seismic mass is a steel cylinder, 1 cm diameter, 1 cm long. Figure shows that the proposed setup can be modeled as a spring mass system, where the piezo disk acts as the spring. You also know the following physical properties:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>71GPa</td>
</tr>
<tr>
<td>$d_{yy}$</td>
<td>0.559 Coulombs/meter</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>450</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>$8.86\times10^{-12}$</td>
</tr>
</tbody>
</table>

a) What is the mechanical spring constant $k$ of the piezo disk if we treat the sensor as the spring mass model.

b) What is the capacitance of the piezo disk when undeformed.

c) Derive a symbolic expression for the acceleration of the mass $m$ that is only a function of the known material properties and dimensions, as well as the output voltage $V$ of the piezoelectric accelerometer. To help you simplify your expression you may assume the terms that contain references to $y^2$ (square of small changes of the displacement) cancel out.

Problem 2 - Loop detectors

Loop detectors are used to detect cars on the road. When no car is present, the inductance of the loop is a steady, known value. When a car passes over
the loop, the inductance changes. The circuit in figure 2 is a common tool for measuring the change of an unknown inductor. Based on known principles of RC and LC impedances, we can obtain a value for $L_3$ (the loop detector of unknown inductance). To obtain this value, a variable capacitor $C_2$ is digitally adjusted until the bridge circuit is balanced ($V_0 = 0$).

a) For the this balanced condition, derive an expression for $L_3$ that only contains the references to known values of $R_1$, $R_4$, and $C_2$.

b) extra credit) This is an exploratory exercise. Assume that you can not adjust $C_2$, but it is instead constant. Chose some nominal values for $R_1$, $C_2$, and $R_4$. Chose a variable sinusoidal input voltage (you pick the frequency and amplitude). Using MATLAB, or your favorite software, simulate what happens to the measured output $V_0$ as $L_3$ varies. How could you integrate your observations into an automated system to detect a change in inductance (e.g., detect a car)?
Problem 3 - Amplifying signals

In lecture we derived the relation of the non-inverting amplifier where \( V_o = (1 + \frac{R_2}{R_1}) \cdot F \). Find the output for the amplification circuit in figure 3. Speculate on the benefit of this configuration compared to the non-inverting amplifier.

![Amplification circuit](image)

Figure 3: Amplification circuit.

Problem 4 - Mathematical operations on analog signals

For the three circuits in figure 4 derive an expression for the output voltage \( V_0 \) given the input voltage signals \( V_a \) and \( V_b \). Which mathematical operation does each circuit represent? Assume each op-amp has a nominal gain \( A \), and every resistor has a nominal resistance \( R \). Assuming that \( A \) is very large may help you reduce to simpler relations (as we’ve done in class).

![Two measuring circuits](image)

Figure 4: Two measuring circuits.

Problem 5 - Digital to analog conversion

An N-bit digital to analog converter (DAC) can be used to provide \( 2^N \) analog voltage outputs for a number of applications. In the case of our SAR ADC, the
digital to analog converter was used to source a voltage comparator, helping
us isolate the input voltage coming into the ADC. Digital to analog converter
can also be used to drive a number of external peripherals and actuators (more
on this later). Needless to say, they are super handy. As this problem will
show, they are also fairly straightforward to construct using some resistors and
an a non-inverting summation op-amp. Figure 5 shows a 4-bit DAC. A digital
controller is used to supply a voltage \( V_{ref} \) to four switches \((D_0 - D_3)\). When a
switch is flipped into the on position, it outputs voltage \( V_{ref} \) into a a network
of resistors. For example, setting \( D_0 \) to on (logic 1) causes a voltage \( V_{ref} \) to
be output through the bottom resistors. Setting \( D_0 \) to off (logic 0), causes the
switch \( D_0 \) to be grounded. Different output voltages \( V_0 \) can be created through
different settings of the switches.

a) Derive an expression for \( V_0 \), the output of the DAC, which only depends
on \( V_{ref}, \), \( R \), and \( R_f \), and \( D_0 \) through \( D_3 \). The values for \( D_0 - D_3 \) should be
binary (0 or 1).

b) Use your solution above to derive a general expression for \( V_0 \) of an N-bit
DAC.

\[
V_0 = \sum_{i=0}^{3} D_i \cdot V_{ref}
\]

\[
V_0 = \sum_{i=0}^{N-1} D_i \cdot V_{ref}
\]

Figure 5: Digital to analog conversion.

**Problem 6 - ADC selection**

a) An analog voltage channel is used to transmit a signal to an analog to dig-
tital converter. The input voltage can vary over the range of \( \pm 2V \), and signal
noise level corresponds to 1.25 mV. How many different code symbols would
be required to record all possible signal values without being affected by noise?
What ADC accuracy (in bits) does this correspond to? Texas Instruments is
one of many manufacturers that makes ADCs. Go to their website (click here) and select an ADC chip that is suitable for this application. Justify your answer and attach the first few pages of the ADC data sheet.

b) What is the minimum number of binary bits required for an ADC to digitize the output of a pressure sensor whose range is 800 to 1100 kPa, with an accuracy of 0.05 kPa?

**Problem 7 - SAR ADC approximation algorithm**

In class we saw how a successive approximation (SAR) ADC uses a digital-to-analog converter to compare an input voltage $V_{in}$ to a value $V_{DAC}$ produced by the $N$-bit DAC. As we know, an $N$-bit DAC produces $2^N$ possible output voltages. This number can be quite substantial for large $N$. A simple algorithm aiming to find $V_{in}$ could start by setting the DAC to 0 volts, and then incrementing it step by step until our comparator matches it to $V_{in}$. As you can imagine this may take some time, especially when we have high input voltages. In such cases, the ADC would have to make almost the full $2^N$ comparisons before isolating the value of the input. Such a method is called digital ramp approximation. For this problem, come up with a better approximation approach that will make no more than $N$ comparisons before finding $V_{in}$.

**Problem 8 - Fiber optical strain measurements**

Fiberoptic strain gage sensors work by transmitting light across a Bragg grating. This Bragg grating forms the sensing component. As we mentioned in class, the light that is reflected by the Bragg grating is an indicator of strain (see figure 6). The frequency of the reflected light is given by

$$\lambda_B = 2n_{eff}\Lambda$$

where $n_{eff}$ is the refraction index (ratio of the speed of light in vacuum divided by the speed of light in a material), and $\Lambda$ is the bragg spacing (distance between etch marks, see figure) inside the sensor. It can be shown that $\frac{\Delta \lambda_B}{\lambda_B} = G_f \frac{\Delta \Lambda}{\Lambda}$, where $G_f$ is the gage factor and $\frac{\Delta \Lambda}{\Lambda}$ is the strain being measured by the fiberoptic sensor. In class it was asked what a realistic value for $G_f$ would be. This problem will help you answer that question. $n_{eff}$ and $\Lambda$ are independent of each other, but both are sensitive to strain fluctuations. The change in the refraction index $n_{eff}$ is given by $\frac{\Delta n_{eff}}{n_{eff}} = p_e \frac{\Delta \Lambda}{\Lambda}$, where

$$p_e = -\frac{n_{eff}^2}{2} \left[p_{12} - v(p_{11} - p_{12})\right]$$

For a fiber optic fiber made from germanium silicate, laboratory experiments show that $p_{11} = 0.113$, $p_{12} = 0.252$, $v = 0.16$ and $n_{eff} = 1.482$. 


Figure 6: Fiberoptic Bragg sensor, used to measure strain.

a) Use this information to derive the expression for $G_f$ and calculate its value.
b) For your value of $G_f$ plot the measured wavelength of reflected light $\lambda_B$ as a function of strain ranging across ±8000 µm.

**Problem 9 - Solar radiation measurements**

You are interested in measuring terrestrial solar radiation for an experiment near the Great Lakes. The experiment is part of a study by really famous climate scientists who want to know how much energy reaches the surface of the earth from the sun. They also want to know which fraction of this energy is absorbed or reflected by the ground. Their models take solar energy (Watts/m²) as an input. While they would like to obtain the best and most accurate measurements, cost will always be a factor. As the engineer, they want you to design the experiment and give them realistic measurements they can feed into their models. This is a realistic scenario, so there is no “perfect answer.”

a) For the region of interest, do some research and find realistic solar radiation values. What is the range of these values? How sensitive do your measurements need to be to pick up daily fluctuations? How frequently should you sample?

b) Do some research on commercially available solar radiation sensors and pick one that you think would be suitable for the experiment. What is the range of this sensor? What’s the precision of your sensor? Cost (if possible)?

c) What is your sensor output? Will you need to amplify your signal? If so, what amplification factor do you want to apply before inputting the reading.
into the ADC. Choose an ADC and justify your answer in terms of resolution, sampling speed and architecture. If your sensor has a built-in ADC, explain why the manufacturer chose it. How do you convert this reading to Watts/m²?

e) If you were to pick an A/D converter to convert the readings of the sensor, how many bits would you need to get the resolution you require.

f) How would you measure the fraction of light/energy reflected by the ground.