Problem 1: Strain gages design

Design a strain gage of height 50 mm, a width of your choice, number of turns of your choice, and a total resistance of 300 Ω. Choose a material, wire length, number of wire turns, and gage diameter. Draw a diagram of your gage and justify your answer. To help with selection, you can search online for a table of materials and their corresponding electrical resistivity and conductivity values.

Problem 2: Strain gage behavior

A metallic wire embedded in a strain gage is 4.2 cm long with a diameter of 0.07 mm. The gage is mounted on the upper surface of a cantilever beam to sense strain. Before strain is applied, the initial resistance of the wire is 64 Ω. Strain is applied to the beam, stretching the wire 0.1 mm, and changing its electrical resistivity by $2 \times 10^{-8} \Omega m$. If Poisson’s ratio for the wire is 0.342, find the change in resistance in the wire due to the strain to the nearest hundredth ohm.

Problem 3: Quarter bridge vs. half bridge

In class we have introduced the single point Wheatstone Bridge (also known as a Quarter Bridge) and the Half Bridge. In the ambient (unloaded) case assume that all resistors and sensors have the same nominal resistance $R_1 = R_2 = R_3 = R_4 = R$.

a) Show that the half bridge has twice the sensitivity (twice the output) of the quarter bridge. The sensors are indicated by the red arrows.

b) What are the tradeoffs with using each setup?
Problem 4: Cantilever load cell

A load cell is constructed by mounting strain gages to the surface of a cantilever beam (Figure 2). Gages 1 and 2 are mounted to the top of the beam, gages 3 and 4 to the bottom. The beam cross section has a moment of inertia $I$ and a Young’s modulus $E$. In the unloaded state, all gages have the same resistance $R_1 = R_2 = R_3 = R_4 = R$, and the gage factor is $G_f$. A load $F$ is applied a distance $L_1$ from gage 2 and 4. For all Wheatstone bridge setups, it can be shown that the relation between bridge output voltage $V_0$ and the input Voltage can be linearized to take the following form:

$$\frac{V_0}{V} = \frac{1}{4} \left[ \frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} - \frac{\Delta R_3}{R_3} + \frac{\Delta R_4}{R_4} \right]$$

(1)

This simplification applies to small changes in resistance, which is almost every problem we will care about in this course.

Using the above information:

a) What is the output voltage $V_0$ in terms of the input voltage $V$ for the below cantilever setup? As we’ve done in class, your expression should contain references to material properties $E$, $I$, $c$ (distance to beam center, sometimes also written as half the beam thickness $t/2$), and $G_f$.

b) If the distance $L_1$ is increased, how does the bridge output change?

c) What is the advantage of this design? Give an example application.

e) (extra credit) show how equation 1 was derived.
Problem 5: Photoresistors and getting the most out of measurement setup

A conductor’s resistance can also be affected by radiation (in this case, more specifically, visible light). Photoresistors (also often called phototransistors or Cadmium sulphide photoconductive photocells) are simple resistors that alter resistance depending on the amount of light they are exposed to. Light is measured in lux, the SI illuminance unit (http://en.wikipedia.org/wiki/Lux). Photoresistors are probably the most common sensor used in measuring light in every day applications, the most affordable (only a dollar a piece), and the easiest of all radiation sensors to implement. A variety of Photoresistors are shown in figure 3.

The use of this sensor is highly dependent upon the application. Using the sensor usually involves anticipating which levels of illuminance the sensor will experience. This often involves exposing the sensor to the brightest environment it will see and then measuring the resistance at this stage - call it $R_b$. This is done for the darkest anticipate illuminance too, by measuring the “dark” resistance $R_d$ (you can measure this by just covering the sensor with your hand). The idea is that future readings of resistance will then fluctuate between $R_d$ and $R_b$ across the lifetime of the sensor deployment.

a) You are using a simple voltage divider circuit (fig 3) to measure the resistance of the sensor. Given a known ballast resistor $R$ in series (see figure), and given that you know $R_d$ (dark resistance) and $R_b$ (bright resistance), derive an expression for $\Delta V_0(R)$, the maximum difference in potential that your voltage divider will output between dark and bright instances.
b) Given that $R_d = 3300 \Omega$, and $R_b = 780 \Omega$, Plot $\Delta V_0(R)$ for a series of values of $R$, starting at $R = 100 \Omega$ and going up to $R = 5000 \Omega$, in 100$\Omega$ increments. What do you notice?

c) Building a voltage divider involves selecting the value of the series resistor $R$. We want to select this resistor to maximize the potential difference $\Delta V_0(R)$. This will reduce the need to amplify our output voltage signal, and will make it easier to discern voltage readings. Derive an expression for the optimal value of $R$ that will maximize this voltage difference.

d) What is the optimal value of $R$ (in Ohms) given the values of $R_d$ and $R_b$ above.

Problem 6: Smart drying machine

You are building a smart clothes dryer. When the clothes lose 50% of their weight they are ready to be taken out. As an engineer, you decided to automate the process by creating a smart drying machine. The machine operates by detecting the weight of the clothes. A standard load of clothes has a weight of 10 kg when “wet” and 5 kg when fully dried. The dryer has a weight of 50 kg.
when empty. The weight sensor is constructed with a strain gauge placed in one of the 4 “feet” of the dryer. In this position, it detects exactly 1/4 of the weight of the machine plus the clothes. The feet are cylindrical, but their weight is negligible. Each foot is essentially a cylinder that has a pre-assembly length of 1 cm, and a stiffness $k$ of 1,000,000 N/m. A metal film strain gauge with a nominal resistance of 1 kΩ and a gage factor $G_f$ of 2.5 is attached to one foot so that the weight of this machine causes compressive strain in the strain gauge that is the same magnitude as the strain in the feet. The assembly is shown in figure 4.

a) If the unloaded length of the foot is 1 cm, what is the length when the entire machine is assembled and mounted upright on the ground?

b) Assume you placed the gage on the foot of the machine, before placing the machine upright. What is the total resistance of the strain gauge when the machine is placed upright?

c) After adding a load of clothes, what is the resistance you expect to see on the strain gage?

d) When the clothes are done drying, what resistance reading would indicate to you that you should cut power off to the dryer?

e) Assuming you chose to use a simple voltage divider to measure the resistance of the strain gage, use the results you obtained in problem 3 to determine the optimal size of the required resistor that you would need to use in the circuit. Now go online (digikey.com, sparkfun.com, or any other site) and find a suitable resistor for this application. Provide the model number and link.

f) The environment can have large temperature variations – leading to resistance variations as large as 4%. Is this a problem for this application? If so, what can be done in the design of a circuit to reduce or eliminate the effect of temperature on the sensor output?

Figure 4: Industrial-scale drying.
Problem 7: Measuring flow

In class we learned about thermistors, which change resistance based on a change in temperature. More specifically, the temperature $T$ at the location of the thermistor can be obtained by

$$T = T_{amb} + G_f \Delta R$$

where $T_{amb}$ is the ambient temperature (starting temperature) at which the resistor has a nominal resistance $R$, and $G_f$ is the thermistor factor. It turns out we can combine two thermistor to measure flow. Figure 5 shows a sensor which is called a thermoanemometer. It is composed of three small tubes immersed into a moving medium. Two tubes contain thermistors $R_0$ and $R_s$. The detectors are thermally coupled to the medium and are thermally isolated from the structural elements and the pipe where the flow is measured. In between the two detectors, a heating element is positioned. Both detectors are connected to electrical wires through tiny conductors to minimize thermal loss through conduction. The sensor operates as follows. The first temperature detector $R_0$ measures the temperature of the flowing medium. A heater warms up the medium in the middle of the pipe and the elevated temperature is measured by the second thermistor $R_s$. When the medium flows, heat dissipation increases due to forced convection. Because the heater is positioned closer to the $R_s$ detector, that detector will register a higher temperature. The higher the rate of flow, the higher the heat dissipation and the lower the temperature that will be registered by the $R_s$ detector. Heat loss can be measured and converted into the flow rate of the medium. A fundamental relationship for this thermometry setup is based on a modified version of King’s law, which states

$$v = \frac{K}{\rho} \left( \frac{dQ}{dt} \frac{1}{T_s - T_0} \right)^{1.87}$$

(2)

where $K$ is a calibration constant, $\rho$ is the density of the liquid or gas flowing through the pipe, $T_s$ is the surface temperature of thermistor $R_s$, $T_0$ is the temperature of sensor $R_0$, and $\frac{dQ}{dt}$ is the thermal loss to the flowing medium over time. The law of conservation of energy demands that the electric power in the heating element be equal to thermal loss to flowing medium. The electric power $W$ across a heater/resistor is given by

$$W = \frac{V^2}{R_h}$$

where $R_h$ is the resistance of the heating element, and $V$ is the voltage across of the heating element.

For this problem, refer to figure 5. $R_0$, $R_s$, $R_1$, and $R_2$ all have the same nominal (ambient) resistance $R$ (e.g. $R_0 = R_s = R_3 = R_4 = R$). $R_1$ and $R_2$ are just part of your measurement circuit and are not sensitive to environmental conditions.
a) Express relation (2) in terms of the voltage $V$ across the heater and its resistance $R_{\text{heater}}$.

b) Derive an expression for the temperature difference $T_s - T_0$ that only depends on the input voltage $V$ the measured voltage $V_0$, the sensor factor $G_f$, and the nominal resistance $R$.

c) Now plug this relation into equation derived in part (a) to obtain a relation for the flow velocity $v$ based on the measured voltage $V_0$ of your circuit.

d) Plot flow velocity $v$ as a function of the output voltage of your circuit $V_0$ (for example, from 0 to 30 mV or whatever looks best to you), for three fluids: water $\rho = 1000\text{kg/m}^3$, propane $\rho = 500\text{kg/m}^3$ and crude oil $\rho = 870\text{kg/m}^3$. Let $K = 45000$, $V = 5\text{Volts}$, $R_h = 10k\Omega$, $G_f = 0.02$, and $R = 1k\Omega$. 

Figure 5: Flow anemometer.