On the technique of extrapolation to obtain wall correction factors for ion chambers irradiated by photon beams

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Wall correction factors, which correct ion chamber response for photon attenuation and scatter, can differ by as much as 1.0% for spherical chambers depending on whether they are obtained experimentally by extrapolation measurements or by Monte Carlo simulation. This difference is not explained by experimental or calculational statistics which lie in the range 0.05%–0.2%. In this paper it is demonstrated that linear extrapolation of experimental data for spherical chambers is inappropriate, owing to the curvature of the chamber walls. A simple nonlinear theory is constructed that resolves the difference. The Monte Carlo calculations and the nonlinear theory are compared with extrapolation measurements for the NIST (formerly NBS) spherical chambers. It is concluded that wall correction factors should be obtained by Monte Carlo calculation for spherical chambers and that linear-extrapolation techniques should be regarded with suspicion for all chambers.

Key words: extrapolation measurements, wall correction, wall perturbation, ion chambers, Monte Carlo

I. INTRODUCTION

When a thick-walled ionization chamber is irradiated by a photon beam, the walls of the chamber perturb the incident beam by attenuating it and generating scattered photons. Historically, Standards' laboratories have corrected for this perturbation by performing extrapolation measurements whereby the walls of the chamber are varied, keeping the walls thick enough so that full buildup and charged particle quasi-equilibrium are maintained, fitting the data linearly, and then extrapolating over the nonequilibrium region to zero wall thickness.[1,2] An example of an extrapolation measurement from the National Research Council of Canada (NRCC)[2] is presented in Fig. 1. This example depicts extrapolation measurement data for the sidewall and end cap of a cylindrical chamber similar to NRCC's primary exposure standard. The points in the equilibrium region, greater than 3 mm or so, are fitted to a straight line. The ratio of the intercept at zero wall thickness to the ordinate at the standard-chamber thickness provides the experimentally determined wall correction factor for the side wall. Note that the points in the nonequilibrium region fall sharply away from the fitted line since buildup there is incomplete. The stated accuracy of wall correction factors obtained by extrapolation experiments is typically about 0.2%.

Recently, Monte Carlo simulations have been applied to the calculations of wall correction factors.[3–7] The estimated error on these calculations is in the vicinity of 0.1%. Differences between calculated and experimentally determined wall correction factors have been published[6] with differences as high as 0.7% although the different values of the electron-drift factor K_{cep}, employed as a further correction to the experimental data, obfuscated a clear conclusion.

A basic assumption about the analysis of the experimental data, which so far has remained unchallenged, is that the extrapolation to zero wall thickness is linear, irrespective of the shape of the chamber. In this paper it is found that the extrapolation is nonlinear for spherical chambers owing to the nonlinear scaling of the effective wall thickness when it is changed. These nonlinearities result in differences of the order of 1% for spherical chambers.

To see how nonlinear effects may be important for curved-walled chambers, consider Fig. 2 depicting the passage of a photon through the curved wall of a spherical or cylindrical chamber. An incident photon passes through the curved

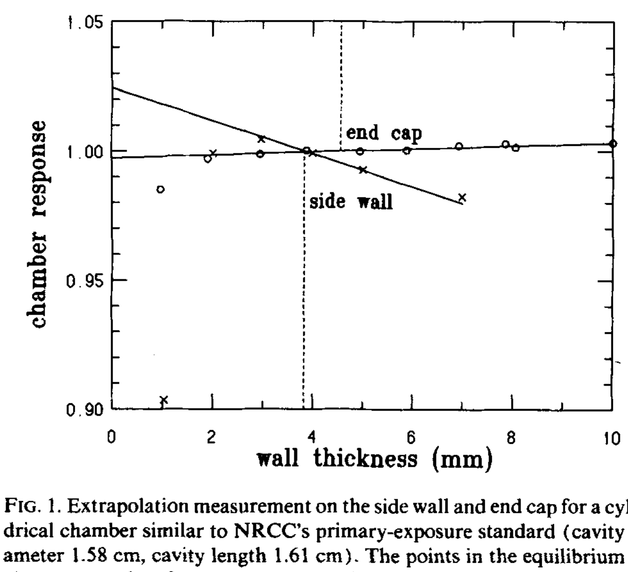

FIG. 1. Extrapolation measurement on the side wall and end cap for a cylindrical chamber similar to NRCC's primary-exposure standard (cavity diameter 1.58 cm, cavity length 1.61 cm). The points in the equilibrium region, greater than 3 mm or so, are fitted to a straight line. The ratio of the intercept at zero wall thickness to that of the thickness of standard chamber (dotted line) provides the experimentally determined wall-correction factor for the side wall or end cap. (Taken from Shortt and Ross,[2] by permission.)

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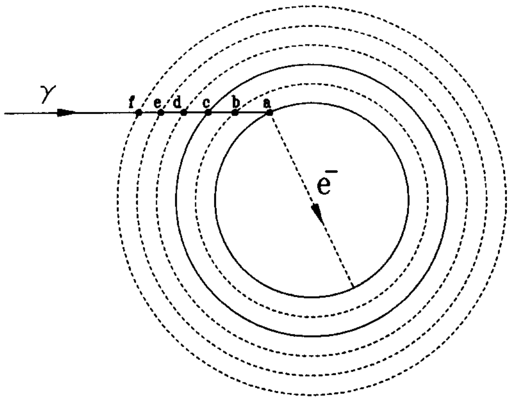

FIG. 2. An incident photon passes through the curved wall of a spherical or cylindrical chamber interacting near the surface a of the cavity generating an electron that produces ionization in the cavity. Full buildup thickness is at c. The thicknesses d, e and f are other wall thicknesses used to obtain extrapolation data. Despite equal increments in wall thickness, the photon pathlength increments are not the same leading to nonlinear extrapolation effects.

wall interacting near the surface **a** of the cavity generating an electron that produces ionization in the cavity. Full buildup thickness is at **c**. The thicknesses **d**, **e** and **f** are other wall thicknesses used to obtain extrapolation data. Each increment in wall thickness is the same. Note that despite equal increments in wall thickness, the photon pathlength increments are not the same. That is, **fe** < **ed** < **dc** < **cb** < **ba**. The nonlinearity increases as the exterior radius decreases suggesting that the extrapolation is nonlinear for curved walls. A simple nonlinear theory is developed in the next section to explain this effect.

II. THEORY

Analytic theories of ion chamber response have been used to predict, with good success, point-source nonuniformity correction factors.[8,9] Using the correction-factor formalism of Bielajew[10,11] [in these references a different notation is used—the connection is $A_{\text{wall}} = (K_W K_{\text{cep}})^{-1}$, where K_{cep} accounts for electron drift] with the same approximations as Kondo and Randolph,[8] the correction factor K_W may be expressed as

$$K_W^{-1} = 1 - \frac{\mu_{\text{eff}}}{4\pi V_{\text{cav}}} \int dA \, |\mathbf{R} - \mathbf{r}_s| \int dV \frac{\cos \eta}{u^2}, \qquad (1)$$

where V_{cav} is the volume of the cavity, μ_{eff} is the effective attenuation coefficient, $|\mathbf{R} - \mathbf{r}_s|$ is the magnitude of the distance between the point at which the photon hits the exterior of the chamber $\mathbf{R}$ and the point on the cavity surface $\mathbf{r}_s$, dA is a surface integration element of the cavity, dV is a volume integration element of the cavity, u is the distance between dA and dV, and $\cos \eta$ is the cosine the u-line makes with the inward normal from dA. The effective attenuation constant μ_{eff} accounts for both attenuation of the primary beam and photon scatter. For this work μ_{eff} is treated as a parameter that is obtained by fitting the experimental or Monte Carlo data. Note that the $4\pi V_{\text{cav}}$ normalization of Eq. (1) is derived by Kondo and Randolph[8] who proved the relationship $4\pi V_{\text{cav}} = \int dA \int dV \cos \eta / u^2$. Hence, Eq. (1) may be rewritten $K_W^{-1} = 1 - \mu_{\text{eff}} \overline{|\mathbf{R} - \mathbf{r}_s|}$, where the effective wall thickness is interpreted as the average value of $|\mathbf{R} - \mathbf{r}_s|$.

The essential assumptions made in deriving Eq. (1) were (i) the photon beam is uniform and parallel; (ii) only one order of scatter is considered and the theory is only developed to the first order in the number of photon mean-free paths through the ion chamber (e.g., the factor, $\mu_{\text{eff}} |\mathbf{R} - \mathbf{r}_s|$ is obtained from a first-order expansion of the photon-attenuation factor, $\exp[(-\mu_{\text{eff}} |\mathbf{R} - \mathbf{r}_s|)]$; (iii) the effect of electron drift through the chamber walls is not considered having already been split off into a multiplicative factor K_{cep}; (iv) the electrons travel through the cavity in straight lines, (v) the electrons deposit energy in proportion to their track length through the cavity, (vi) the angular distribution of the electrons entering the cavity is isotropic, (vii) the scattered photon direction is colinear with the incident photon direction, and (viii) electrode and guard areas are ignored.

Assumption (i) is justified since wall correction factors vary only very slightly with distance for practical measurement distances, especially for spherical chambers.[11] Assumption (ii) is valid for small chambers where the corrections are small perturbations, such as those discussed in this report. Crude second-order estimates have shown this approximation to be good to about 0.1%. Assumption (iii) strictly applies only for equilibrium thicknesses, but allowing the electron-drift dependence to be separated permits one to define an extrapolation to zero wall thickness. Note that for complete generality, K_{cep} can assume different values depending on the geometry of the chamber and the quality of the radiation. If one attempted to construct a theory of ion chamber response valid for the nonequilibrium region, the electron-drift factor would have to assume an explicit dependence on wall thickness. In this report, K_W is defined only for the quasiequilibrium region owing to the assumed independence of the electron-drift factor on wall thickness. However the extrapolation to zero wall thickness in Eq. (1) is still well defined physically and mathematically. It can be interpreted as the relative ion chamber response assuming that charged-particle equilibrium exists. Assumption (iv) is valid if the cavity is small for most of the electrons that enter the cavity. This assumption breaks down for the low-energy electrons whose range approaches the characteristic size of the cavity. Assumption (v) neglects the increased stopping power of lower energy electrons entering the cavity but it should not matter if assumptions (iv) and (vi) strictly hold. Assumption (vi) is not fully justified except at extremely low energies where photoelectric interactions and strong multiple scattering cause the electron distributions to be nearly isotropic. This assumption has been shown to be an oversimplification for accurate work involving nonspherical geometries.[9] Assumption (vii) is justified at high energies (including ^{60}Co and ^{137}Cs) where recoil Compton photons scatter predominantly into the forward direction. This assumption also ignores scatter contributions that are not produced directly upstream of the cavity. The data of Shortt and Ross[2] for a cylindrical cavity (diameter 1.58 cm, length 1.61 cm, Fig. 1) show a much reduced effect from the end cap. A more accurate theory should include more realistic photon scatter distributions. However contradictory assumptions (vi) and (vii) are, the present theory is meant to provide insight only, avoiding an elaborate mathematical development. It will be argued that wall correction factors should be calculated by Monte Carlo methods and the theory presented herein may serve as substantiating that argument. Assumption (viii) is not a fundamental restriction, but it makes the mathematical decomposition of Eq. (1) realizable. As long as the volume of the guard areas or electrodes are small compared to the volume of the cavity, this assumption should be valid.

III. APPLICATION TO SPHERICAL CHAMBERS

Spherical geometry encompasses a large, important class of chambers, in particular, the NIST (formerly NBS) exposure standards.[1]

Within the assumptions adopted, K_W may be shown to be

$$K_W^{-1}(w) = 1 - \mu_{\text{eff}} w f_s(\alpha) , \quad (2)$$

where μ_{eff} is the effective attenuation coefficient, $\alpha \equiv r_s/w$, w is the wall thickness (assumed to be uniform), r_s is the radius of the cavity and $f_s(\alpha)$ is the slope correction function for spheres. This slope correction function depends only on α and is given by

$$f_s(\alpha) = [1 + (1 + 1/2\alpha)\log(1 + 2\alpha)]/2 , \quad (3)$$

and is plotted in Fig. 3. Equations (2) and (3) are derived in the Appendix starting from Eq. (1).

The Monte Carlo calculations of ion chamber response and correction factors employed the EGS4 code system[12] enhanced by the PRESTA electron-transport algorithm used in its default configuration[13] that has been shown to calculate absolute ion chamber response reliably for carbon-walled chambers. The EGS4 user code employed in this report, CAVSPH, is a descendant of CAVITY, which has been used previously to calculate scatter and attenuation correction factors.[5–7] The reliability of the Monte Carlo calculations is considered to be within the stated uncertainties, which were estimated by dividing the calculation into ten batches and calculating the estimated variance of the mean. A minimum of about 2.5×10^6-incident photon histories was used for each simulation. Each incident photon was forced to interact at least twice in the chamber (a standard variance reduction technique, see, for example Rogers and Bielajew[14]). The simulations were performed on an IBM 3090 and the typical simulation time was about 1 h CPU time per chamber per wall thickness. Realistic source energy distributions were

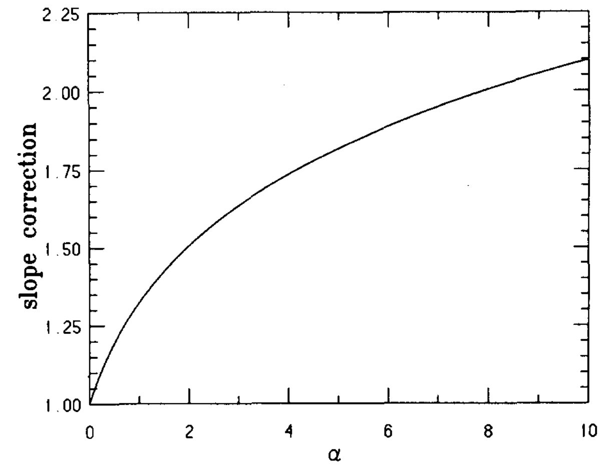

FIG. 3. Extrapolation slope correction function for a spherical chamber as a function of $\alpha = r_s/w$ where r_s is the cavity radius and w is the wall thickness.

employed. For the ^{60}Co calculations, the distribution used was obtained from a Monte Carlo simulation of a ^{60}Co therapy head[15] while the ^{137}Cs spectrum employed was measured.[16]

A comparison of the wall correction factors K_W obtained by linear extrapolation and Monte Carlo calculation for the NIST spherical chambers[1] for irradiation by ^{60}Co is given in Table I and for ^{137}Cs in Table II. The measured values are multiplied by K_{cep} factors of 0.995 ± 0.002 and 0.999 ± 0.001 for ^{60}Co and ^{137}Cs, respectively. (All estimated uncertainties are quoted at the 95% confidence level.) The average discrepancy between measured and Monte Carlo calculated K_W's is 1.0090 ± 0.0010 for ^{60}Co and 1.0096 ± 0.0009 for ^{137}Cs. The $K_W^{\text{nonlinear}}/K_W^{\text{linear}}$ ratios were determined from the experimental data. Since this is a ratio, the experimental uncertainies are correlated resulting in a small experimental uncertainty. In addition, there is a systematic uncertainty associated with the approximations inherent in the nonlinear technique. This is estimated to be about 0.2% although it is not included in the tables where only experimental uncertainties are quoted. It is interesting to note that NIST applies an additional correction factor for the small 1-cm^3 chamber of 0.9970 for ^{60}Co and 0.9962 for ^{137}Cs to bring it into line with the weighted mean of all the chambers.[1] (No other explanation is given.) An explanation for this extra correction is given by the ratio $K_W^{\text{nonlinear}}/K_W^{\text{linear}}$, which is the ratio of wall correction factors

TABLE I. A comparison of measured and Monte Carlo calculated wall correction factors for the NIST chambers in ^{60}Co.

Chamber	Experimental	Monte Carlo	Ratio $\left(\frac{\text{Monte Carlo}}{\text{experimental}}\right)$	$K_w^{\text{nonlinear}}/K_w^{\text{linear}}$
1	1.0117 ± 0.0036	1.0215 ± 0.0012	1.0097 ± 0.0037	1.0058 ± 0.0011
10	1.0165 ± 0.0022	1.0247 ± 0.0006	1.0081 ± 0.0022	1.0072 ± 0.0003
30	1.0169 ± 0.0022	1.0263 ± 0.0006	1.0092 ± 0.0022	1.0073 ± 0.0003
50-1	1.0176 ± 0.0022	1.0261 ± 0.0012	1.0084 ± 0.0025	1.0079 ± 0.0003
50-2	1.0267 ± 0.0022	1.0367 ± 0.0012	1.0097 ± 0.0025	1.0081 ± 0.0003
50-3	1.0335 ± 0.0022	1.0432 ± 0.0014	1.0094 ± 0.0025	1.0080 ± 0.0002

TABLE II. A comparison of measured and Monte Carlo calculated wall correction factors for the NIST chambers in ^{137}Cs.

Chamber	Experimental	Monte Carlo	Ratio $\left(\frac{\text{Monte Carlo}}{\text{experimental}}\right)$	$K_w^{\text{nonlinear}}/K_w^{\text{linear}}$
1	1.0189 ± 0.0036	1.0268 ± 0.0020	1.0078 ± 0.0037	1.0068 ± 0.0011
10	1.0250 ± 0.0022	1.0333 ± 0.0012	1.0081 ± 0.0018	1.0086 ± 0.0003
30	1.0239 ± 0.0022	1.0344 ± 0.0012	1.0103 ± 0.0018	1.0083 ± 0.0003
50-1	1.0262 ± 0.0022	1.0364 ± 0.0016	1.0099 ± 0.0021	1.0087 ± 0.0003
50-2	1.0374 ± 0.0022	1.0489 ± 0.0020	1.0111 ± 0.0023	1.0089 ± 0.0002
50-3	1.0457 ± 0.0022	1.0566 ± 0.0036	1.0104 ± 0.0037	1.0087 ± 0.0002

with the nonlinear and linear fitting procedures applied to the experimental data. The nonlinearity is 0.2%–0.3% smaller for the 1-cm^3 chamber compared to the larger chambers.

The extrapolation data and the fits to the NIST 50-cm^3 chambers, as well as Monte Carlo data are plotted in Fig. 4 for ^{60}Co and Fig. 5 for ^{137}Cs. The Monte Carlo calculations plot the relative values of K_W^{-1} normalized to unity at the wall thickness corresponding to the 50-1 chamber, 0.632 g cm^{-2}, where the experimental data are normalized. Ion chamber theory implies[10] that all the variation in response is due entirely to K_W in the quasi-equilibrium region. The relative values of K_W^{-1} were depicted because the typical calculated uncertainty on K_W is less than 0.1% for the calculation compared to about 0.5%–0.8% for the raw-calculated responses. The relative calculated values of K_W^{-1} agree well with the measured responses. The linear fits to the experimental data (shown with solid lines) model equally well the experimental and calculated data in the vicinity of the experimental data. Indeed, to within the calculated and experimental uncertainties the data exhibit no patent nonlinearity in this region. The ^{137}Cs calculated data suggest that if the measurements were extended to smaller wall thicknesses (but still larger than full buildup), the nonlinearity would have been apparent. A linear fit to all the Monte Carlo calculated data reduces the disagreement to within 0.5%, although the data are modeled better by the nonlinear fit. When the nonlinear theory is applied, the extrapolated value is greater by about 1% than that obtained by a linear fit to the experimental data.

The Monte Carlo calculated wall corrections and the extrapolated wall correction factors employing the nonlinear corrections agree quite well, to within the 0.2%–0.4% statistical uncertainty presented in Tables I and II and the 0.2% estimated systematic uncertainty of the nonlinear extrapolation method. The average increases in K_W due to the nonlinear extrapolation are 1.0077 ± 0.0001 for ^{60}Co and 1.0087 ± 0.0001 for ^{137}Cs which should be compared to the average discrepancy between measured and Monte Carlo calculated and linearly extrapolated K_W's of 1.0090 ± 0.0010 for ^{60}Co and 1.0096 ± 0.0009 for ^{137}Cs. The remaining difference motivates the estimate of systematic uncertainty to 0.2% for spherical chambers.

IV. DISCUSSION AND CONCLUSIONS

A theory of extrapolation to zero wall thickness for thick-walled spherical ion chambers subject to photon beams has been developed. Use of this theory resolves most of the dif-

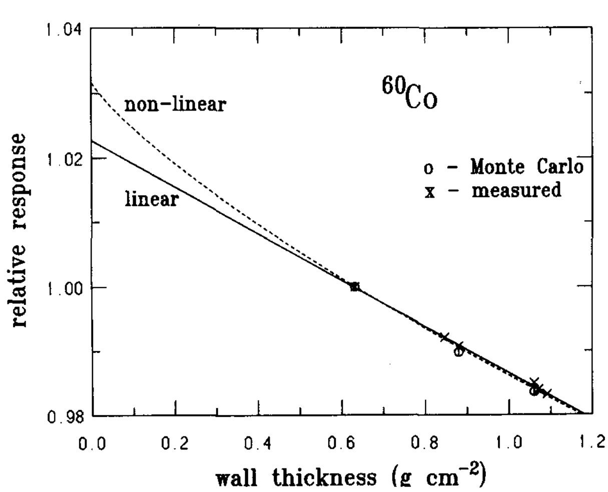

FIG. 4. The NIST extrapolation data for their 50 cm^3 chambers exposed to ^{60}Co. Measured and Monte Carlo calculated data are shown. The linear fit (solid line) and first-order nonlinear fit (dashed line) represent the data equally well in the measurement region. However, the extrapolation to zero wall thickness differs by almost 1%.

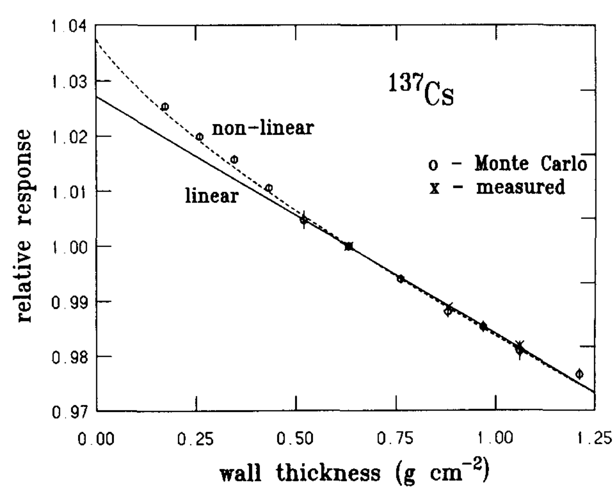

FIG. 5. The NIST extrapolation data for their 50 cm^3 chambers exposed to ^{137}Cs. Measured and Monte Carlo calculated data are shown. The linear fit (solid line) and first-order nonlinear fit (dashed line) represent the data equally well in the measurement region. However, the extrapolation to zero wall thickness differs by almost 1%.

ferences of the order of 1% between experiment and Monte Carlo calculations of wall correction factors of the NIST spherical chambers. Notwithstanding the excellent and possibly fortuitous results for the NIST spherical chambers, the theory is too approximate to be utilized at better than 0.2% systematic uncertainty in spherical geometries and is not recommended for nonspherical chambers without a more complete modeling as indicated below.

No detailed comparison with cylindrical chambers was carried out owing to a paucity of experimental data for chambers where the end caps and the electrode can be ignored. The theory may be extended to include end caps and electrodes at the expense of mathematical complication. In addition to including electrodes, the electron isotropy assumption (vi) and the photon colinearity assumption (vii) would have to be relaxed owing to the loss of spherical symmetry. The nature of the extrapolation changes as well if extrapolation on the end caps and walls is performed separately or together. The symmetry of the spherical geometry allowed the employment of the above approximations and further theoretical investigations on other geometries are not pursued. Rather, another publication compares all available extrapolation data for all geometries and compares extrapolated and Monte Carlo wall correction factors.[7] Certainly the strong nonlinearity in the spherical case throws doubt upon the linear extrapolation technique for all chambers except possibly for guarded plane parallel chambers where the contribution from the curved wall does not cause appreciable nonlinearities.

It has been determined that the NIST exposure standard should be increased by 0.90% for ^{60}Co and 0.96% for ^{137}Cs. Given that the simplest nonlinear theory resolves most of the differences between Monte-Carlo-calculated wall correction factors and experiment for spherical chambers, and that there are potential ambiguities for other geometries, the universal employment of Monte Carlo calculated K_W factors is advocated. Moreover, the Monte-Carlo-calculated wall correction factors contain corrections for electron drift and can be obtained with accuracies of the order of 0.1% or better, exceeding even the most careful experiments where the extrapolation extends over the wide nonequilibrium region.

APPENDIX: DERIVATION OF THE SLOPE CORRECTION FACTOR

The inner integral of Eq. (1) may be expressed as

$$\int dV \frac{\cos \eta}{u^2} = \frac{\mathbf{r}_s}{r_s} \cdot \int dV \frac{\mathbf{r}_s - \mathbf{r}_v}{|\mathbf{r}_s - \mathbf{r}_v|^3} = 2\pi \int_0^{\pi} \sin\theta \, d\theta \int_0^{r_s} r_v^2 dr_v \frac{r_s - r_v \cos\theta}{(r_s^2 - 2r_s r_v \cos\theta + r_v^2)^{3/2}}, \tag{A1}$$

where $\mathbf{r}_s$ is a vector from the cavity center to the surface, $\mathbf{r}_v$ is a vector from the cavity center to the cavity-volume element dV, and θ is the angle between the two vectors. Equation (A1) may be evaluated straightforwardly with the result $4\pi r_s/3$. Incorporating this result into Eq. (1) gives,

$$K_w^{-1} = 1 - \frac{\mu_{\text{eff}}}{A_{\text{cav}}} \int dA \, |\mathbf{R} - \mathbf{r}_s|, \tag{A2}$$

where A_{cav} is the surface area of the cavity. Equation (A2) is still interpreted as the average value of $|\mathbf{R} - \mathbf{r}_s|$ but simplified to an integral over just the cavity surface. The photon pathlength through the chamber wall may be shown to be $|\mathbf{R} - \mathbf{r}_s| = \sqrt{R^2 - r_s^2 \sin^2\theta} - r_s|\cos\theta|$, where θ is now the angle between $\mathbf{r}_s$ and the incident photon beam direction. Substituting $R = w + r_s$ and employing the ratio, $\alpha = r_s/w$, Eq. (1) is transformed to

$$K_W^{-1} = 1 - \mu_{\text{eff}} \, w \int_0^{\pi/2} d\theta \sin\theta \times (\sqrt{1 + 2\alpha + \alpha^2 \cos^2\theta} - \alpha\cos\theta). \tag{A3}$$

The integral is elementary and evaluates to the form $f_s(\alpha)$ given in Eq. (3).

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