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Ionisation cavity theory: a formal derivation of perturbation factors for thick-walled ion chambers in photon beams

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Abstract. Ionisation cavity theory for thick-walled chambers in photon beams is considered. Mathematically rigorous expressions based upon energy conservation are derived that equate the dose to the cavity to the product of the collision kerma, Bragg-Gray or Spencer-Attix stopping power ratios evaluated with idealised equilibrium spectra and correction factors. These correction factors account for photon attenuation and scatter and centre of electron production. They appear naturally in the derivation and differ only slightly from those used in the cavity-gas calibration factor, N_{gas}, in the AAPM *Protocol for the Determination of Absorbed Dose from High-Energy Photon and Electron Beams.* Additionally, a factor that corrects for the perturbation of the electron fluence by the cavity is derived. The expressions derived are formal only in the sense that analytical methods for calculating the fluences involved are not given. However, they may be evaluated in Monte Carlo calculations and they provide a framework for analytical calculations.

1. Introduction

The use of thick-walled ionisation chambers for dosimetric purposes has had a long tradition in medical physics. The response of these instruments to photon irradiation closely obeys the Bragg-Gray (Gray 1936) and Spencer-Attix (1955) expressions with small departures. These are due to photon scatter and attenuation in the chamber walls and the perturbative effect of the presence of the air cavity on charged-particle equilibrium. The emergence of powerful Monte Carlo techniques (Bond *et al* 1978, McEwan and Smyth 1983, Bielajew *et al* 1985) directed at calculating these correction factors has enabled one, at least in principle, to predict these correction factors with considerable accuracy. Yet a theoretical basis for these correction factors, directed towards Monte Carlo simulations, is lacking. Loevinger (1981) has presented an elegant formalism based upon the extraction of β, a correction factor that accounts for the transport of electrons downstream from the point of the interaction that set them in motion. Even so, we shall see that it is more natural for Monte Carlo simulations to group this correction together with another correction factor that accounts for the photon attenuation in the chamber walls.

The purpose of this paper is to develop a formal mathematical expression for the dose deposited in the gas cavity of thick-walled ionisation chambers subject to photon irradiation. A close connection will be made to the Bragg-Gray (Gray 1936) and Spencer-Attix (1955) cavity theories as applied to thick-walled ion chambers in photon beams. It will be found that correction factors appear naturally that account for the attenuation and scatter of the incident photons in the chamber walls as well as the electron transport away from the point of interaction and the perturbation of the

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electron fluence due to the presence of the gas cavity. The expressions that are derived may be evaluated directly by Monte Carlo methods or they may serve as a framework for analytical calculations. They also form a theoretical basis for the βA_{wall} coefficient, the photon attenuation and scatter, and the centre of electron production correction that has been calculated using Monte Carlo methods by Nath and Schulz (1981), McEwan and Smyth (1984) and Rogers *et al* (1985). These authors did describe the algorithms that they used for the calculation of the photon attenuation and scatter corrections, and their methods are shown to correspond very closely with the expressions that are developed within the course of the rigorous derivation given in this report. The βA_{wall} factors calculated by Nath and Schulz (1981) have been verified in an independent calculation by Rogers *et al* (1985) and are utilised in the AAPM protocol for the determination of the absorbed dose from high-energy photon and electron beams (Schulz *et al* 1983, 1985) for the determination of the cavity-gas calibration factor, N_{gas}.

The computation of βA_{wall} by Monte Carlo methods has been accomplished in two ways. Nath and Schulz (1981) make no allowance for the generation of δ rays in their simulations. This computational method is naturally associated with the application of Bragg-Gray theory to thick-walled ionisation cavities. The simulations of Rogers *et al* (1985) do allow for δ-ray transport and this is naturally associated with the Spencer-Attix formulation. It is not the purpose of this paper to discuss the relative merits of these two approaches, but rather to calculate the correction factors that measure the departure from the Bragg-Gray or Spencer-Attix theories, idealised for thick-walled ion chambers where the incident photon beam interacts but is neither attenuated nor scattered, and the presence of the cavity does not perturb the electron fluence. Once the Bragg-Gray or Spencer-Attix approach is accepted, the development proceeds as a consequence of the conservation of energy, and the derivation of the correction factors may be obtained in a straightforward fashion.

2. The Bragg–Gray approach

In this section a formal derivation of the βA_{wall} correction factor is given. It is suitable for Monte Carlo calculations where δ rays are not simulated, and total collision stopping powers are used to slow down the charged particles. This would apply, for example, to the technique of Bond *et al* (1978) used to calculate the βA_{wall} in the AAPM protocol (Nath and Schulz 1981, Schulz *et al* 1983). To accomplish this we develop an expression for the dose to the cavity of an ion chamber, and relate it to the collision kerma of the incident beam, a total collision stopping power ratio and correction factors. The collision kerma of the incident beam and the stopping power ratio are independent of the geometry of the chamber, while the correction factors account for photon attenuation, scatter, electron transport and electron fluence perturbation, which contain all the geometrical variation.

The dose to the gas cavity of the ion chamber may be written

$$D_{\text{g}} = D_{\text{g}}^{0} + D_{\text{g}}^{\text{s}} \tag{1}$$

where D_{g}^{0} is the dose due to the electrons set in motion by the first interaction of the primary photon beam, and D_{g}^{s} is the scatter component that contains the dose due to scattered photons as well as the dose due to bremsstrahlung, fluorescent and annihilation photons, whatever their origin.

The entire effect of the scatter component can be incorporated as a multiplicative correction factor, A_{scat}. That is,

$$D_{\mathrm{g}} = D_{\mathrm{g}}^{0} A_{\mathrm{scat}} \tag{2}$$

where A_{scat} is defined by

$$A_{\mathrm{scat}} \equiv 1 + D_{\mathrm{g}}^{\mathrm{s}} / D_{\mathrm{g}}^{0}. \tag{3}$$

(Henceforth, all quantities with a superscript 0 are related to the electron fluence that arises from the initial interactions of the incident photons.)

D_{g}^{0} may be written

$$D_{\mathrm{g}}^{0} = \sum_{i=1}^{N} \int \mathrm{d}E\ \Phi_{\mathrm{g},i}^{0}(E)(S_{\mathrm{col}}(E)/\rho)_{\mathrm{g}} \tag{4}$$

where $\Phi_{\mathrm{g},i}^{0}$ is the fluence, at the location of the cavity with the gas present, of the electrons arising from the initial interaction of the ith incident photon, and $(S_{\mathrm{col}}/\rho)_{\mathrm{g}}$ is the unrestricted collision mass stopping power for the gas. Strictly speaking, $\Phi_{\mathrm{g},i}^{0}(S_{\mathrm{col}}/\rho)_{\mathrm{g}}$ represents a sum over the charged-particle species, including positrons from pair creation and charged particles from photonuclear interactions. For brevity's sake this should be considered to be implicit in the following development.

Equation (4) is based on the assumption that the net effect of δ-rays is insignificant. However, equation (4) corresponds directly to the dose calculated by Monte Carlo codes where δ rays are not transported. The effects of δ rays may be important in some cases and the analogous equations including δ-ray effects are given in the next section.

We wish to undo the effect of photon attenuation on D_{g}^{0}. We may accomplish this by multiplying the electron fluence by the inverse of the attenuation probability of the photon that set the electron in motion. That is, we now consider the 'fictitious' case where the incident photon beam interacts but is not attenuated. This dose in the absence of photon attenuation, $\tilde{D}_{\mathrm{g}}^{0}$, is given by

$$\tilde{D}_{\mathrm{g}}^{0} = \sum_{i=1}^{N} \exp(\lambda_{\mathrm{g},i}) \int \mathrm{d}E\ \Phi_{\mathrm{g},i}^{0}(E)(S_{\mathrm{col}}(E)/\rho)_{\mathrm{g}} \tag{5}$$

where $\lambda_{\mathrm{g},i}$ is the number of mean free paths traversed by the ith incident photon in the walls or cavity of the chamber before its first interaction. (Henceforth, all quantities modified by a tilde, ~, are related to electron fluences with photon attenuation effects removed.) We note that $\lambda_{\mathrm{g},i}$ is not directly related to the position of the cavity. For photons that interact a large distance from the cavity, $\Phi_{\mathrm{g},i}^{0}$ is usually zero. The probability that $\Phi_{\mathrm{g},i}^{0}$ is non-zero increases with the proximity of the location of the first interaction and is greater if the interaction occurs on the upstream side than on the downstream side. The details of the calculational method relate the position of the interaction to the electron fluence in the cavity. This is implicit in equation (5). In Monte Carlo calculations one may easily associate the $\exp(\lambda_{\mathrm{g},i})$ factor with the electrons that may or may not deposit a dose to the gas in the cavity due to electrons arising from the initial interaction of the incident photon. Since this factor is associated with the point of interaction and not the position of the cavity, $\tilde{D}_{\mathrm{g}}^{0}$ is the dose to the gas in the cavity due to electrons arising from the initial interaction of the incident photon in the absence of photon attenuation, and the drift of the electrons away from the point of interaction is accounted for.

Now, consider a homogeneous medium, for example, an ion chamber with its gas cavity filled with wall material. If the walls surrounding the cavity (now filled with wall material) are thicker than the range of an electron having the maximum energy that can be imparted by the incident photons, and if we remove the effect of photon attenuation, then there must exist an exact charged-particle equilibrium in the location of the cavity. In this case we may equate $\tilde{D}_{\mathrm{w}}^{0}$, the dose deposited in the cavity filled with wall material arising from the first interaction of the incident photons, to the collision kerma, the total energy transferred to the electrons in the initial interaction that is dissipated by collision processes. This is a statement of the conservation of energy and it takes the form

$$\sum_{i=1}^{N} \exp(\lambda_{\mathrm{w},i}) \int \mathrm{d}E\ \Phi_{\mathrm{w},i}^{0}(E)(S_{\mathrm{col}}(E)/\rho)_{\mathrm{w}} = \Psi_0(\mu_{\mathrm{en}}/\rho)_{\mathrm{w}}^{0}. \tag{6}$$

$\Phi_{\mathrm{w},i}^{0}$ is the electron fluence arising from the initial interaction of the ith incident photon at the location of the cavity with the gas replaced by wall material, $\exp(\lambda_{\mathrm{w},i})$ is the inverse of the photon attenuation that is obtained when wall material is present in the cavity, Ψ_0 is the incident photon energy fluence at the location of the cavity with the chamber removed, and $(\mu_{\mathrm{en}}/\rho)_{\mathrm{w}}^{0}$ is the mass energy absorption coefficient for the wall material evaluated at the incident photon energy.

Equation (6) permits an exact rewriting of equation (4) as

$$D_{\mathrm{g}}^{0} = \Psi_0(\mu_{\mathrm{en}}/\rho)_{\mathrm{w}}^{0}\left(\sum_{i=1}^{N}\int \mathrm{d}E\Phi_{\mathrm{g},i}^{0}(E)(S_{\mathrm{col}}(E)/\rho)_{\mathrm{g}}\right)$$
$$\times\left(\sum_{i=1}^{N}\exp(\lambda_{\mathrm{w},i})\int \mathrm{d}E\Phi_{\mathrm{w},i}^{0}(E)(S_{\mathrm{col}}(E)/\rho)_{\mathrm{w}}\right)^{-1}. \tag{7}$$

Multiplying and dividing equation (2) by the right-hand side of equation (5), and using equation (7) gives

$$D_{\mathrm{g}} = \Psi_0(\mu_{\mathrm{en}}/\rho)_{\mathrm{w}}^{0}A_{\mathrm{scat}}\beta A_{\mathrm{att}}^{0}\left(\sum_{i=1}^{N}\exp(\lambda_{\mathrm{g},i})\int \mathrm{d}E\ \Phi_{\mathrm{g},i}^{0}(E)(S_{\mathrm{col}}(E)/\rho)_{\mathrm{g}}\right)$$
$$\times\left(\sum_{i=1}^{N}\exp(\lambda_{\mathrm{w},i})\int \mathrm{d}E\Phi_{\mathrm{w},i}^{0}(E)(S_{\mathrm{col}}(E)/\rho)_{\mathrm{w}}\right)^{-1} \tag{8}$$

where the attenuation correction and electron drift correction $\beta A_{\mathrm{att}}^{0}$ is defined by

$$\beta A_{\mathrm{att}}^{0} \equiv \left(\sum_{i=1}^{N}\int \mathrm{d}E\ \Phi_{\mathrm{g},i}^{0}(E)(S_{\mathrm{col}}(E)/\rho)_{\mathrm{g}}\right)$$
$$\times\left(\sum_{i=1}^{N}\exp(\lambda_{\mathrm{g},i})\int \mathrm{d}E\ \Phi_{\mathrm{g},i}^{0}(E)(S_{\mathrm{col}}(E)/\rho)_{\mathrm{g}}\right)^{-1} \tag{9}$$

$\beta A_{\mathrm{att}}^{0}$ completely accounts for the effects of the photon attenuation of the incident photon beam†.

We may multiply and divide equation (8) by the quantity

$$\sum_{i=1}^{N}\exp(\lambda_{\mathrm{w},i})\int \mathrm{d}E\ \Phi_{\mathrm{w},i}^{0}(E)(S_{\mathrm{col}}(E)/\rho)_{\mathrm{g}}$$

† This notation was chosen to be consistent with the AAPM protocol which defines $\beta A_{\mathrm{wall}} = \beta A_{\mathrm{att}}^{0}A_{\mathrm{scat}}$. β is the electron drift correction and in this analysis it is naturally associated with the attenuation correction.

with the result

$$D_g = \Psi_0(\mu_{en}/\rho)^0_w(\bar{s}^{BG}_{g,w})^0 A_{scat}\beta A^0_{att}\tilde{A}^0_{fl} \tag{10}$$

where the stopping power ratio $(\bar{s}^{BG}_{g,w})^0$ is defined by

$$(\bar{s}^{BG}_{g,w})^0 \equiv \left(\sum_{i=1}^{N} \exp(\lambda_{w,i}) \int dE\ \Phi^0_{w,i}(E)(S_{col}(E)/\rho)_g\right) \times \left(\sum_{i=1}^{N} \exp(\lambda_{w,i}) \int dE\ \Phi^0_{w,i}(E)(S_{col}(E)/\rho)_w\right)^{-1} \tag{11}$$

and the electron fluence correction factor, $\tilde{A}^0_{fl}$, is defined by

$$\tilde{A}^0_{fl} \equiv \left(\sum_{i=1}^{N} \exp(\lambda_{g,i}) \int dE\ \Phi^0_{g,i}(E)(S_{col}(E)/\rho)_g\right) \times \left(\sum_{i=1}^{N} \exp(\lambda_{w,i}) \int dE\ \Phi^0_{w,i}(E)(S_{col}(E)/\rho)_g\right)^{-1} \tag{12}$$

Both $\bar{s}^{BG}_{g,w}$ and $\tilde{A}^0_{fl}$ are calculated using the electron fluence arising from the initial interactions of the incident photons in the absence of photon attenuation, since A_{scat} and βA^0_{att} completely account for photon scatter and attenuation. From the definition given in equation (12), $\tilde{A}^0_{fl}$ can be interpreted as a correction for the replacement of the wall material by the gas in the location of the cavity.

Finally, equation (10) may be rewritten

$$D_g = (\tilde{D}^0_{BG})_g A_{scat}\beta A^0_{att}\tilde{A}^0_{fl} \tag{13}$$

where the Bragg-Gray dose $(\tilde{D}^0_{BG})_g$ is defined by

$$(\tilde{D}^0_{BG})_g \equiv \Psi_0(\mu_{en}/\rho)^0_w(\bar{s}^{BG}_{g,w})^0. \tag{14}$$

As indicated by the notation, this quantity is calculated with the effects of photon scatter and attenuation removed at exact charged-particle equilibrium. Since we are considering ionisation chambers with walls that have at least the full build-up thickness, $(\tilde{D}^0_{BG})_g$ is independent of the geometry of the chamber. It depends only on the materials that make up the chamber. All the geometrical variation is contained within the correction factors. $(\tilde{D}^0_{BG})_g$ is closely related to the Bragg-Gray dose to the gas in the cavity of an ion chamber (Gray 1936). The essential difference is that the stopping power ratio is evaluated using a fully equilibrated electron spectrum that arises from the initial interaction of an ideal, unattenuated, unscattered incident photon beam. The correction factors directly measure the departure from these assumptions. A_{scat}, given by equation (3), accounts for the photon scatter, βA^0_{att}, given by equation (9), accounts for the attenuation of the incident photon beam as well as the drift of electrons away from the point of initial photon interaction, and $\tilde{A}^0_{fl}$, given by equation (12), accounts for perturbation of the electron fluence due to the replacement of the wall material by the gas in the cavity.

$\tilde{A}^0_{fl}$ is expected to be very close to unity for similar wall-gas compositions, primarily due to effects described by the theorem of Fano (1954). Fano's theorem states that, under conditions of equilibrium in an infinite medium where the particle interaction rate is directly proportional to the density, the particle fluences will not be altered by changing the density of a given region. Since $\tilde{A}^0_{\mathrm{fl}}$ is calculated using primary electron fluences, the electron fluence in the cavity surrounded by full build-up walls simulates the infinite medium criterion. Inasmuch as the e^{λ} factors were required to produce an exact charged-particle equilibrium, the A^0_{fl} factor measures directly the departures from Fano's theorem whether they be due to different wall-gas composition or other density related differences.

3. The Spencer–Attix approach

In the previous section it was assumed that the electrons set in motion by photons did not create δ rays. The energy deposition by δ rays was effectively included in the unrestricted collision stopping power as an approximation. δ ray effects may become important for ion chambers with gas and walls of significantly different composition (e.g. aluminium-walled air chamber). The formalism of the previous section is easily extended to include δ-ray effects and relate the dose to the collision kerma of the incident beam, a restricted collision stopping power ratio and the correction factors. Monte Carlo codes that allow for δ-ray transport have already been used to study ion chamber response and A_{wall} corrections (Bielajew *et al* 1985, Rogers *et al* 1985).

The equations can be stated without proof. The derivation proceeds in the same manner as the previous section with the exception that the primary dose to the cavity is given by

$$D^0_{\mathrm{g}} = \sum_{i=1}^{N} \int \mathrm{d}E\ \Phi^{\Delta}_{\mathrm{g},i}(E)(L_{\Delta}(E)/\rho)_{\mathrm{g}} \tag{15}$$

where $\Phi^{\Delta}_{\mathrm{g},i}$ is the fluence of all electrons with kinetic energy greater than Δ set in motion directly or indirectly as a result of the first interaction of the ith primary photon and $(L_{\Delta}(E)/\rho)_{\mathrm{g}}$ is the restricted collision mass stopping power with cut-off Δ. Implicit in equation (15) is the dose deposited by the 'track ends'—particles with kinetic energy less than Δ. The contribution of track ends in similar equations has been discussed by Nahum (1978). It can then be shown that

$$D_{\mathrm{g}} = (\tilde{D}^0_{\mathrm{SA}})_{\mathrm{g}} A_{\mathrm{scat}} \beta A^0_{\mathrm{att}} \tilde{A}^0_{\mathrm{fl}} \tag{16}$$

where

$$(\tilde{D}^0_{\mathrm{SA}})_{\mathrm{g}} = \Psi_0(\mu_{\mathrm{en}}/\rho)^0_{\mathrm{w}}(\tilde{s}^{\mathrm{SA}}_{\mathrm{g,w}}(\Delta))^0 \tag{17}$$

$$(\tilde{s}^{\mathrm{SA}}_{\mathrm{g,w}}(\Delta))^0 = \left(\sum_{i=1}^{N} \exp(\lambda_{\mathrm{w},i}) \int \mathrm{d}E\, \Phi^{\Delta}_{\mathrm{w},i}(E)(L_{\Delta}(E)/\rho)_{\mathrm{g}}\right) \times \left(\sum_{i=1}^{N} \exp(\lambda_{\mathrm{w},i}) \int \mathrm{d}E\ \Phi^{\Delta}_{\mathrm{w},i}(E)(L_{\Delta}(E)/\rho)_{\mathrm{w}}\right)^{-1} \tag{18}$$

$$\beta A^0_{\mathrm{att}} = \left(\sum_{i=1}^{N} \int \mathrm{d}E\ \Phi^{\Delta}_{\mathrm{g},i}(E)(L_{\Delta}(E)/\rho)_{\mathrm{g}}\right) \times \left(\sum_{i=1}^{N} \exp(\lambda_{\mathrm{g},i}) \int \mathrm{d}E\ \Phi^{\Delta}_{\mathrm{g},i}(E)(L_{\Delta}(E)/\rho)_{\mathrm{g}}\right)^{-1} \tag{19}$$

$$\tilde{A}^0_{\mathrm{fl}} = \left(\sum_{i=1}^{N} \exp(\lambda_{\mathrm{g},i}) \int \mathrm{d}E\, \Phi^{\Delta}_{\mathrm{g},i}(E)(L_\Delta(E)/\rho)_{\mathrm{g}}\right) \times \left(\sum_{i=1}^{N} \exp(\lambda_{\mathrm{w},i}) \int \mathrm{d}E\, \Phi^{\Delta}_{\mathrm{w},i}(E)(L_\Delta(E)/\rho)_{\mathrm{g}}\right)^{-1} \quad (20)$$

and

$$A_{\mathrm{scat}} = 1 + D^{\mathrm{s}}_{\mathrm{g}}/D^0_{\mathrm{g}} \quad (21)$$

is unchanged.

Effectively, $\Phi^0_{m,i}(E)(S_{\mathrm{col}}(E)/\rho)_m$ is just replaced by $\Phi^{\Delta}_{m,i}(E)(L_\Delta(E)/\rho)_m$ in the integrals.

4. Interpretation and relationship between the correction factors

In the Bragg-Gray and Spencer-Attix expressions, $(\tilde{D}^0_{\mathrm{BG}})_{\mathrm{g}}$ and $(\tilde{D}^0_{\mathrm{SA}})_{\mathrm{g}}$, the stopping power ratios given by equations (11) and (19) are to be calculated using the electron fluences that arise from the initial interaction of the incident photon (including the δ-rays that are generated by the primary electron in the Spencer-Attix case) and also with the effects of photon attenuation removed. This is a departure from some calculations, for example those of Nahum (1978), who used physical fluences (including photon attenuation and scatter effects). Inasmuch as the definitions of the various correction factors are arbitrary, but recognising the fact that the derivation in this paper is exact, one may use either definition of the stopping power ratio and relate them to additional correction factors. For example, the Spencer-Attix stopping power ratio calculated with physical electron fluences is related to that given in equation (18) of this paper by a factor $(\tilde{s}^{\mathrm{SA}}_{\mathrm{g,w}}(\Delta))^0/s^{\mathrm{SA}}_{\mathrm{g,w}}(\Delta)$ where $s_{\mathrm{g,w}}(\Delta)$ is defined by

$$s_{\mathrm{g,w}}(\Delta) \equiv \left(\sum_{i=1}^{N} \int \mathrm{d}E\, \Phi^{\Delta}_{\mathrm{w},i}(E)(L_\Delta(E)/\rho)_{\mathrm{g}}\right)\left(\sum_{i=1}^{N} \int \mathrm{d}E\, \Phi^{\Delta}_{\mathrm{w},i}(E)(L_\Delta(E)/\rho)_{\mathrm{w}}\right)^{-1} \quad (22)$$

and $\Phi^{\Delta}_{\mathrm{g},i}$ now includes the electrons set in motion by scattered photons. The effects both of utilising only those electrons set in motion by the initial interaction and of removing photon attenuation, effectively shift the electron fluence slightly towards a higher energy. Since stopping power ratios are known to vary only slowly with energy (Nahum 1978), the factor $(\tilde{s}^{\mathrm{SA}}_{\mathrm{g,w}}(\Delta))^0/s^{\mathrm{SA}}_{\mathrm{g,w}}(\Delta)$ should be very close to unity. Unlike $\tilde{s}^0_{\mathrm{g,w}}(\Delta)$, $s^{\Delta}_{\mathrm{g,w}}(\Delta)$ depends slightly on geometry, due to the inclusion of the scatter component. As an example, for a typical minimum build-up thickness carbon-walled chamber with a 2 mm deep cylindrical cavity of radius 1 cm, subject to a broad parallel beam of ^{60}Co photons incident on the flat planar front, a Monte Carlo calculation of the ratio $\tilde{s}^0_{\mathrm{air,C}}(10\,\mathrm{keV})/s_{\mathrm{air,C}}(10\,\mathrm{keV})$ gave 1.0002. (The Monte Carlo code used was a slightly modified version of that used previously by Bielajew *et al* (1985) and Rogers *et al* (1985).) This represents a negligible correction if a stopping power ratio incorporating physical electron fluences (equation (22)) is used in the formalism.

The photon attenuation factor βA^0_{att}, as defined in this paper by equations (9) and (19), is calculated using only the electrons that arise directly (primary electrons) or indirectly (δ rays from primary electrons) from the initial interaction of the incident photon. This differs from the algorithm of Rogers *et al* (1985) and apparently with that of Nath and Schulz (1981) where total electron fluences were used. βA_{att} calculated using the total electron fluence is given in the 'Bragg-Gray' case by

$$\beta A_{\mathrm{att}}=\left(\sum_{i=1}^{N}\int \mathrm{d}E\ \Phi_{\mathrm{g},i}(E)(S_{\mathrm{col}}(E)/\rho)_{\mathrm{g}}\right)$$
$$\times\left(\sum_{i=1}^{N}\exp(\lambda_{\mathrm{g},i})\int \mathrm{d}E\ \Phi_{\mathrm{g},i}(E)(S_{\mathrm{col}}(E)/\rho)_{\mathrm{g}}\right)^{-1}. \tag{23}$$

However, the scatter contribution to the total dose is at most only a few per cent for most practical ion chambers (Nath and Schulz 1981, Rogers *et al* 1985) and the distribution of initial photon interaction points for the scatter component of the dose to the gas in the cavity should not differ greatly from that of the primary component. Therefore, the ratio $\beta A^0_{\mathrm{att}}/\beta A_{\mathrm{att}}$ should be very close to unity. Consider a chamber with carbon walls of minimum full build-up thickness with, typically, a 2 mm deep cylindrical chamber of 1 cm radius which is subject to a broad parallel beam of ^{60}Co photons, incident on the front planar face. Based on the simplifying assumptions that the scatter component (2.3% of the total dose as calculated in a Monte Carlo simulation) reaches the cavity equally from all directions, and that the primary component reaches the cavity from only the front face, an upper bound on the factor $\beta A^0_{\mathrm{att}}/\beta A_{\mathrm{att}}$ is 1.0003. This is a negligible correction if equation (23) is used in the formalism and is well within the statistical limits quoted by Nath and Schulz (1981) and Rogers *et al* (1985).

The correction factor A_{scat}, given by equations (3) or (21), corresponds directly to the algorithm of Rogers *et al* (1985) and apparently to that of Nath and Schulz (1981).

The electron fluence perturbation correction $\tilde{A}^0_{\mathrm{fl}}$ may also be related to an analogous term involving physical electron fluences, and a further correction factor may be obtained from procedures similar to those given above. However, $\tilde{A}^0_{\mathrm{fl}}$ is independent of wall thickness and needs only to be calculated once for a given cavity shape. For ion chambers with gas and wall material of similar composition (except for density), $\tilde{A}^0_{\mathrm{fl}}$ is expected to be small, as indicated by Fano's theorem (Fano 1954). For highly dissimilar compositions $\tilde{A}^0_{\mathrm{fl}}$ may represent a significant correction. In this case, δ-ray effects may also be significant, and the Spencer-Attix formulation of § 3 should be used.

5. Conclusion

In this report, the dose to the gas of an ion chamber, subject to photon irradiation as calculated with and without δ rays, has been related to the incident collision kerma and total collision or restricted collision stopping power ratios. It has also been related to correction factors that account for photon attenuation and scatter in the chamber walls, electron transport away from the point of interaction and electron fluence perturbation due to replacement of the wall material by the gas in the cavity. It has been found that the dose to the gas cavity of a thick-walled ion chamber exposed to photons can be written exactly as

$$D_{\mathrm{g}}=\Psi_{\mathrm{o}}(\mu_{\mathrm{en}}/\rho)^0_{\mathrm{w}}(\tilde{s}^{\mathrm{SA}}_{\mathrm{g,w}}(\Delta))^0 A_{\mathrm{scat}}\beta A^0_{\mathrm{att}}\tilde{A}^0_{\mathrm{fl}} \tag{24}$$

and we restate the meaning of the various components as follows.

A_{scat}: this factor completely accounts for the effects of photon scattering in the chamber walls. This same factor has been used by other authors (Nath and Schulz 1981, Rogers *et al* 1985). This quantity is defined in equations (3) or (21).

βA^0_{att}: this factor completely accounts for the attenuation of the primary photon beam in the chamber walls, and electron transport away from the point of interaction. Scatter effects are already removed by A_{scat}. This quantity is defined in equations (9) and (19). It differs from that used by other authors who included scatter (Nath and Schulz 1981, Rogers *et al* 1985) by only a few parts in 10^4.

$\tilde{A}^0_{\rm fl}$: this factor completely accounts for the perturbation, due to the presence of the cavity, on the electron fluence arising from the primary, unattenuated photon beam. The quantity is defined in equations (12) and (20). Scatter and attenuation effects are already accounted for by $A_{\rm scat}$ and $A^0_{\rm att}$. This factor, although expected to be small for similar wall-gas compositions, may be significant for highly dissimilar materials.

The above three factors completely account for any geometrical dependence. The remaining components of equation (24) are: as follows.

$(\bar{s}^{\rm SA}_{\rm g,w}(\Delta))^0$: this is the average restricted collision mass stopping power ratio as defined in equation (18). The unrestricted analogue is defined in equation (11). This quantity involves integrals over the electron fluence arising from the initial photon interaction in the absence of photon attenuation and scatter and as calculated with the perturbation effect of the cavity removed. This quantity is completely independent of the geometry and depends only on the beam quality. (It is understood that Δ should be of the appropriate size—electrons with kinetic energy Δ should not have a range greater than the average dimension of the cavity.) In this sense it is different from similar quantities quoted in the literature but the effect of including attenuation and scatter have been shown to be small.

$\Psi_0(\mu_{\rm en}/\rho)^0_{\rm w}$: this is the incident collision kerma (in the wall material), given by the product of the incident photon energy fluence and mass energy absorption coefficient (for the wall material). For non-monoenergetic incident photons this quantity may be replaced directly by an integral over the incident photon spectrum.

Although some of the equations derived in this report are somewhat unwieldly, they are all mathematically rigorous and they all lend themselves to ready interpretation as correction factors which measure the size of the departures from the idealised Bragg-Gray or Spencer-Attix theories. The requirement that the perturbation factors be small may be considered to be a fundamental restriction on the usefulness of this theory. However, for thick-walled ion chambers it should be noted that $A_{\rm scat}$ and $A^0_{\rm att}$ largely cancel each other, because primary-photon attenuation is the source of the photon scatter component. This has been verified by calculation (Nath and Schulz 1981, Rogers *et al* 1985). Therefore, this theory is useful for virtually any practical chamber wall thickness above minimum full build-up.

It is encouraging that the theory can be constructed around a stopping power ratio that is independent of the geometry of the chamber, photon attenuation and scatter effects. This allows the direct theoretical connection to the Bragg-Gray and Spencer-Attix applications to ion chamber response in photon fields. Thus, the new theory states that stopping power ratios need only be calculated once for any given wall-cavity composition.

Finally, an exact mathematical expression was derived for $\tilde{A}^0_{\rm fl}$, the correction factor that measures the perturbation on the charged-particle equilibrium due to the presence of the cavity. For chambers with highly dissimilar wall-cavity compositions, or for very large cavities, this factor may become significant. More investigation and calculation of the size of this factor is warranted.

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Résumé

Théorie de la cavité: formulation pour les facteurs de perturbation des chambres d'ionisation à paroi épaisse utilisées dans les faisceaux de photons.

L'auteur considère la théorie de la cavité pour des chambres à paroi épaisse utilisées dans des faisceaux de photons. Avec une grande rigueur mathématique et à partir du principe de la conservation de l'énergie, l'auteur établit des formules pour la dose dans la cavité, considérée comme égale au kerma de collision, les rapports des pouvoirs de ralentissement selon l'hypothèse de Bragg-Gray ou de Spencer-Attix évalués pour un spectre idéal d'électrons en équilibre vrai, et les facteurs de correction. Ces facteurs de correction tiennent compte de l'absorption et de la diffusion des photons, ainsi que du centre de production des électrons. Ils se déduisent facilement de la formulation et différent peu de ceux inclus dans le facteur d'étalonnage exprimé en termes de dose dans le gaz de la cavité N_{gas} et qui est proposé dans le protocole AAPM de détermination de la dose absorbée dans les faisceaux d'électrons et de photons de haute énergie. En outre, l'auteur a déduit un facteur corrigeant pour la perturbation de la fluence électronique dûe à la cavité. Les expressions présentées dans ce travail font intervenir des fluences pour lesquelles il n'est pas donné de méthode analytique de calcul. Cependant, elles peuvent être évaluées par la méthode de Monte Carlo et elles fournissent une base pour les calculs analytiques.

Zusammenfassung

Hohlraum-Ionisationstheorie: eine formale Herleitung von Störfaktoren für dickwandige Ionisationskammern in Photonenstrahlen.

In der vorliegenden Arbeit wird die Hohlraumtheorie für dickwandige Kammern in Photonenstrahlen näher betrachtet. Auf der Grundlage der Energieerhaltung wurden mit mathematischer Genauigkeit Gleichungen hergeleitet für die Dosis im Hohlraum, die gleichgesetzt wird mit der Kerma und die Bragg-Gray- oder Spencer-Attix-Verhältnisse für das Bremsvermögen mit idealisierten, voll angepaßten Elektronenspektren und Korrektionsfaktoren. Diese Korrektionsfaktoren berücksichtigen Schwächung und Streuung der Photonen, sowie das Zentrum der Elektronenproduktion. Sie kommen ganz von selbst in den Ableitungen vor und unterscheiden sich nur leicht von den Faktoren, dje beim Kalibrierungsfaktor für das Hohlraum-Gas N_{gas} im AAPM-Protokoll für die Bestimmung der Energiedosis bei Hochenergiephotonen- und Elektronenstrahlen verwendet wurden. Zusätzlich wird ein Faktor hergeleitet, der die Störung der Elektronenfluenz durch den Hohlraum korrigiert. Die hergeleiteten Gleichungen sind nur in dem Sinne formal, Daß analytische Methoden zur Berechnung der Fluenzen nicht angegeben werden. Sie können aber durch Monte Carlo-Rechnungen bestimmt werden und liefern somit Rahmenbedingungen für analytische Berechnungen.

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