LEARNING SEMINAR ON DELIGNE'S *WEIL II* THEOREM
SUMMER 2016

Goal. This seminar is dedicated to Deligne's proof of the Riemann hypothesis part of the Weil conjectures, as given in [D]. Thus, the main goal is to understand the formalism of weights for ℓ-adic sheaves on varieties over finite fields, and, in particular, their stability under pushforwards.

References. Our primary reference will be the book [KW], which presents Deligne's argument after incorporating Laumon's simplifications based on the Fourier transform [L]; another very useful reference is [Ka].

Background. We will assume familiarity with the 6 functor formalism in étale cohomology (as explained in Math 731 in Fall 2015). It is *not* necessary to know the proofs of the main results in étale cohomology. Instead, it will be much more useful to have familiarity with basic sheaf-theoretic operations in the classical topology, and have the willingness to assume that similar statements hold in étale cohomology (as explained in say [FK] or [M]).

Plan of talks. Here is a tentative plan for the talks:

(1) **Overview:** review some aspects of étale cohomology (fundamental groups and lisse sheaves, Poincare duality, the Lefschetz trace formula), review the statement of the Riemann hypothesis part of the Weil conjectures from the sheaf-theoretic perspective, allocate talks.
(2) **Weil sheaves:** discuss the definition of Weil sheaves, the relation to classical étale sheaves, and the extension of the L-series formalism to the Weil setting ([KW, §1.1]).
(3) **Weights, part I:** discuss pure and mixed sheaves, the relation to the zeroes and poles of the L-function, and the semicontinuity of weights ([KW, §1.2, pages 13 — 20]).
(4) **Weights, part II:** discuss the sheaf-function correspondence, characterize the weight of a mixed sheaf in terms of the associated function in some cases ([KW, §1.2, pages 20 — 25]).
(5) **Monodromy:** describe Weil sheaves of rank 1, prove Grothendieck's theorem on semisimplicity of geometric monodromy groups of geometrically semisimple lisse sheaves, deduce consequences for determinental weights of wedge powers ([KW, §1.3]).
(6) **Real sheaves:** describe the Rankin-Selberg method, and deduce that real mixed sheaves are pure ([KW, §1.4]).
(7) **Fourier transform:** describe Artin-Schreier sheaves, define the ℓ-adic Fourier transform and prove its basic properties, including preservation of perversity ([KW, §1.5, §3.8]).

(8) **Weil conjectures for curves:** combine the preceding results to prove the Riemann hypothesis for local systems on curves ([KW, §1.6]).
(9) **Weil conjectures in general:** deduce the general case of the Riemann hypothesis from the case of curves, formulate and prove the preservation of mixedness under !-pushforwards at the level of derived categories ([KW, §1.7, pages 61—65 in §1.9]).

Optional talks. If time and energy permit, we will discuss a subset of the following additional closely related topics (assuming familiarity with basic formalism of perverse sheaves, as discussed in Math 731 in Fall 2015).

(1) Perverse sheaves, Gabber's purity theorem, the decomposition theorem
(2) Radon transform and Hard Lefschetz
(3) Radon transform and Hodge theory

REFERENCES

[BBD] A. Beilinson, J. Bernstein, P. Deligne, *Faisceaux pervers*
[D] P. Deligne, *La conjecture de Weil: II*
[FK] E. Freitag and R. Kiehl, *Etale Cohomology and the Weil conjectures*
[Ka] N. Katz, *L-functions and monodromy: four lectures on Weil II*
[KW] R. Kiehl and R. Weissauer, *Weil conjectures, perverse sheaves and l'adic Fourier transform*.
[L] G. Laumon, *Transformation de Fourier, constantes d'quations fonctionnelles et conjecture de Weil*
[M] J. Milne, *Etale cohomology*