PROBLEM SET 10 (MATH 631)

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All the numbered problems are from §II.7 in Hartshorne’s book.

(1) Using the criterion for a projective morphism to be a closed immersion, check that the Veronese map is a closed immersion.

(2) Describe $P_X(E)$ explicitly, where $X = \text{Spec}(k[x])$ is the affine line over a field $k$, and $E$ corresponds to a $k[x]$-module of the form $k[x]/(x^n)$ for $n \geq 1$.

(3) Let $k$ be a commutative ring, let $R = k[x_0, \ldots, x_n]$ with the standard grading, and let $I = (x_0, \ldots, x_n)$. Show that $\text{Bl}_I(\text{Spec}(R))$ identifies with $V(\mathcal{O}_{\text{Proj}(R)}(1))$, and that the exceptional divisor of the blowup identifies with the zero section of $V(\mathcal{O}_{\text{Proj}(R)}(1)) \to \text{Proj}(R) = \mathbb{P}^n_k$. (Bonus: generalize this to the case where $R = \oplus_{n \geq 0} \Gamma(X, \mathcal{O}_X(n))$, where $X \subset \mathbb{P}^n_k$ is a projective scheme over a field $k$.)

(4) Let $k$ be a field. Explicitly describe the blowups of:

(a) $\text{Spec}(k[x,y]/(x^2 - y^3))$ along $(x,y)$.

(b) $\text{Spec}(k[x,y]/(xy))$ along $(x,y)$.

(c) $\text{Spec}(k[x,y,z]/(xy - z^2))$ along $(x,y,z)$.

(5) II.7.2

(6) II.7.4

(7) II.7.11 (a) and (b)

(8) II.7.12