1. Let \( f : X \to Y \), and \( g : Y \to Z \) be covering spaces. Assume that \( g^{-1}(z) \) is finite for each \( z \in Z \). Show that \( g \circ f \) is a covering space.

2. Let \( f : X \to Y \) be a covering space, and let \( \alpha : Z \to Y \) be any map. Show that the projection map \( X \times_Y Z \to Z \) is a covering space; here \( X \times_Y Z \) is the fibre product of \( f \) and \( \alpha \).

3. Let \( f : X \to Y \) and \( g : Y \to Z \) be two maps. Assume \( f \) and \( g \circ f \) are covering spaces. Prove that \( g \) is a covering space.

4. Let \( f : X \to Y \) be a covering space.
   (a) Show that \( f \) is an open map.
   (b) Show that \( f \) is a local homeomorphism, i.e., for each \( x \in X \), there is an open neighbourhood \( x \in U \subset X \) such that \( f \) induces a homeomorphism \( U \simeq f(U) \).

5. Let \( \alpha_n : S^1 \to S^1 \) be the map \( z \mapsto z^n \). Let \( m \) and \( n \) be coprime positive integers. Using covering space theory, show that there is no continuous map \( f : S^1 \to S^1 \) such that \( \alpha_m \circ f = \alpha_n \).

6. Show that all continuous maps \( \mathbb{R}P^2 \to S^1 \times S^1 \) are null-homotopic.

7. Let \( G \) be a (discrete) group acting on a topological space \( X \). Assume that for each \( x \in X \), there is an open neighbourhood \( x \in U \subset X \) such that \( e \neq g \in G \). Show that the quotient map \( X \to X/G \) is a covering space.

8. For a finite group \( G \), construct a path-connected space \( X \) with fundamental group \( G \).

9. Let \( G \) and \( H \) be path-connected topological groups, and let \( f : G \to H \) be a covering space that is also a group homomorphism. Show that \( \ker(f) \) is abelian.

10. Using the path/homotopy lifting property, show that any simply connected space \( Y \) has no non-trivial covering spaces, i.e., if \( f : X \to Y \) is a covering space, then \( X = \sqcup \{ Y_i \} \), with \( f \) inducing a homeomorphism \( Y_i \to Y \).

The next set of exercises describe the classification of covering spaces in terms of groupoids. Let \( Y \) be a topological space, and assume that \( Y \) is locally simply connected, i.e., there is an open cover \( \{ U_i \} \) of \( Y \) with each \( U_i \) simply connected.

11. Let \( f : X \to Y \) be a covering space. Show that the association \( y \mapsto f^{-1}(y) \) extends to a functor \( \Phi_f : \tau_{\leq 1}Y \to \text{Set} \) using the path/homotopy lifting property; here \( \text{Set} \) is the category of sets.

12. Let \( F : \tau_{\leq 1}Y \to \text{Set} \) be a functor. Define \( X \) to be the set of pairs \( (y, \eta) \), where \( y \in Y \), and \( \eta \in F(y) \); let \( \Psi_F : X \to Y \) be the projection on the first factor. Endow \( X \) with a topology to make \( \Psi_F \) into a covering space.

13. For a functor \( F : \tau_{\leq 1}Y \to \text{Set} \), show that \( \Phi_{\Psi_F} \simeq F \).

14. For a covering space \( f : X \to Y \), show that \( \Psi_{f*} \simeq f \); here two covering spaces \( X_1 \to Y \) and \( X_2 \to Y \) are said to be isomorphic if there is a homeomorphism \( X_1 \to X_2 \) commuting with the map down to \( Y \) from either factor.