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You Do the Math! Teaching with Trust

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Introduction

One of the key experiences that shaped the way I think about teaching and learning came in the early 1990s, while I was sitting in my office working with a first-year engineering student. We were going through the errors she had made on her first examination, including questions on the topic of acid-base chemistry.

Now before you decide to skip this article because I put the word math in the title and the word chemistry at the end of the first sentence, bear with me: it will be worth it. After all, I said it was a key experience. For the benefit of those who might have known this before and forgotten, one of the definitions for an acid is “proton donor.” To be a potential proton donor, all you need is to have a hydrogen atom in a molecular structure (hydrogen is symbolized by the letter “H”). Some protons are easier to donate than others, and some are downright impossible, but as long as there is an “H” in the molecular formula, then the donor-ability of the proton, that is, the acidity of that molecule, can be considered.

Here is my recollection of the conversation with my student. After 3 or 4 minutes of trying unsuccessfully to elicit the reasoning she used on an acid-base problem, I decided to back up and see if we were clear on the fundamental definitions.

Me: Can you tell me what an acid is?
Student [after a few moments of what looks like a student searching her mental note cards]: An acid is a proton donor.
Me [at this point I agree but do not indicate that]: OK, can you give me a list of, say, 10 examples of acids, then?
Student [somewhat shaken]: Ten? I don’t think so.
Me: OK, how about one example.
Student [same searching expression]: ummm… hydrochloric acid… HCl?
Me [at this point I agree but do not indicate that]: Go ahead and write that down. Can you think of another one?
Student [same searching expression]: HI. [she smiles; this is a simple move and replacement of an element in the same column of the periodic table… HF, HCl, HBr, and HI are all part of the same chemical family]
Me [at this point I agree but do not indicate that]: Go ahead and write that down. Can you think of another one?
Student [different thinking, it seemed; less recollection]: NaOH.
Me [at this point I do not agree but do not indicate that, either; in fact, I pretty much had an epiphany at that moment that I will attempt to explain and then make useful for you]: Go ahead and write that down.

All Protons Are Positive

Let me begin to explain something, here. While HCl (hydrochloric acid) and HI (hydroiodic acid) would be unquestioningly identified as acids, NaOH (sodium hydroxide) would widely and unequivocally not be identified as an acid, but instead it would be classified a base (the antithesis of an acid) because that is its chemical behavior. Note that the acidity of NaOH can be considered (it has an “H” in its chemical formula), but it is not

(chemically, experimentally) the kind of hydrogen for which one even unusually considers its acidity. Now, back to the story.

Me [reference to her list]: So all three of these are acids?
Student: Yes.
Me: And the definition of an acid you are using is…?
Student: An acid is a proton donor.
Me: And these are all proton donors?
Student: Yes.
Me: OK, would you point to the proton in the first example?
Student [pointing to the "H" in "HCl"]: Here.
Me: And in the second?
Student [pointing to the "H" in "HI"]: Here.
Me [I predicted, based on my hypothesis of the problem with acid-base chemistry that this student was having, that she would not point to the "H" in "NaOH"]: And in the third? Student [pointing to the "Na" in "NaOH"]: Here.
Me [pretty much glowing with self-satisfaction, which I try not to show, either]: That is the proton?
Student: Yes.
Me [me, double-checking, just to be sure]: Why is that the proton?
Student: Because protons are positive.

Let me review. The student used the language ("acids are proton donors") correctly, and generated a list that, if I had stopped after the second example or rejected her NaOH suggestion, would not have revealed the actual problem, namely, the student's misunderstanding that while all protons are positive, not every positive partner in a molecule is called a proton.

To her, positive and proton were exact synonyms. The rules and definitions that she and I were using for acidity, while actually different, overlapped for her first two examples. The student was not using my rules, though… ever, and yet she could still produce a substantial list of answers I would consider correct: this was my epiphany.

An Interesting Puzzle

When this student produced her list proton donors (HCl, HI, & NaOH), it was completely consistent with her understanding of "proton." And, when prompted to identify the proton, pointed to the H in HCl, the H in HI, and the Na in NaOH, all of which are the positive partners in these molecules. Without signaling my agreement (or disagreement) as she constructed her list, and by being patient, I learned something useful, which I call teaching with trust.

When listening to students explain their reasoning, I need to make a decision about apparent inconsistencies. Are the students simultaneously using correctly and incorrectly the proper rules, or are they using completely correctly and consistently some improper or inadequate rule? The "trust" in teaching with trust means that, as my default position, I assume that students who generate explanations and examples do so with a set of consistent rules.

The consequence of this decision was to understand that the best possible use of my time I could make available to students was charging them to show me their personally generated examples and/or explanations (Gobert & Clement, 1999). I can learn more about what a student is thinking from a list of compounds that they believe are acids, or from their own version of a set of lecture notes on the topic of acid-base chemistry, than I can from any number of other typical interactions we might have (going over problems, reviewing notes, etc.). In fact, the efficiency is remarkable, because it might take the serious student an hour or more to refine a document that they would be willing to show me, and then it takes me sometimes only seconds, thanks to my expertise in reading chemistry, to respond and learn some valuable insight into how the student is thinking about the subject. I am motivated to look at this kind of student work, because what might have previously been an exercise in arbitrating correct and incorrect (from my perspective) is now an interesting puzzle: what are the rules that one would use in order to have all of these examples be correct (from the student's perspective)?

The Basis of Errors

Whether it is working with undergraduate study group leaders, graduate teaching assistants, or new (or experienced) faculty, I have emphasized the idea that finding ways to help students understand the basis of their errors is more helpful than simply deciding how incorrect they are. I am certainly not inventing the concept that student-generated creative work better reveals student understanding. This is a principle that I understand well, and I appreciate greatly the advantages to which other disciplines put this strategy (e.g., writing, art, music, theater). I am a strong advocate for applying the studio (creative construction) concept in undergraduate science education (Coppola & Daniels, 1996; Coppola, 2002), so my notion of teaching with trust identifies an instructional attribute of the studio environment.

My challenge was to persuade people other than chemists to appreciate whatever subtleties are associated with this idea, which meant I needed to de-contextualize the concept and transfer it to a more accessible subject area. It only took a moment to select mathematics. I have used the simple query shown below in probably hundreds of different settings in order to get people to understand the importance of taking the learner's perspective when giving advice as a teacher (Hoffmann & Coppola, 1996).

> You have assigned a student the task of creating some multiplication problems, and these six examples are presented to you:
>
> | | | |
> |---|---|---|
> | 2 x 2 = 4 | -1 x 0.5 = -0.5 | 1.1 x 11 = 12.1 |
> | 3.5 x 1.4 = 4.9 | 2 x 4 = 6 | -3 x 0.75 = -2.25 |
>
> What advice do you give?

Like the example of acids that inspired this, the lesson from these multiplication problems points to why just checking for right and wrong answers may not be much help for revealing student's misunderstanding and for constructing relevant advice. In this list of student-generated multiplication problems, the most common advice given by audiences of novice teachers (particularly those who have not had the benefit of hearing the acids story) revolves around identifying the single incorrect-looking example ("2 x 4 = 6") and reinforcing the notion that 5 out of 6 were correct. Some audience members recommend positive reinforcement and encouragement ("you are really doing well, you might want to recheck your answers"), other wags are inclined differently ("you probably just pushed the wrong button on the calculator"), and others shake their heads with disbelief ("how can you get the hard ones right and then mess up on the easy one?").

Interestingly, all of these responses could be the worst advice to give.

Conclusion

This student may not understand multiplication at all, but has instead generated 6 examples that correctly and consistently apply the rules of addition! To suggest that one of them is wrong could add even more confusion to this student's actual misunderstanding, because from the student's perspective, he or she has acted completely faithfully within the context of a set of rules.

The leading question I have learned to ask myself, then, is whether there are conditions or assumptions under which a list of student-generated examples or explanations might be consistent because that might reveal student understanding (a phrase that we too often use only to mean "a student's correct understanding"). I now routinely offer to students that I will review a page or two of their examples or notes on a topic if they assure me that they have generated these as an honest representation of their understanding (and yes, I get them to think about how they might respond to the multiplication problems, too) (1).

This is one of a number of strategies I use to demonstrate to students the value of teaching in learning: because if they also learn to respond to their peers' questions with this perspective, it causes them to reflect more deeply on their own understanding (Coleman, Brown & Rivkin, 1997; Coleman, 1998). Not surprisingly, I have heard from students that they were able to self-correct their work by going through the exercise of writing down their generated examples or notes, and then deciding that they did not need to show them to me, after all.

References

Notes

1. These are not the only pairs of numbers that satisfy the mathematical relationship /xy/ = (/x/+/y/). Examples involving however many decimal places you wish can be generated in any spreadsheet by setting numerical values in one column to (-x/(1-x)) in another. I selected addition and multiplication for their greatest accessibility, and a sampling of whole numbers, single and double decimals, and negative signs, as distracters from the simplicity of the relationships, yet these were still multiplication problems that could be done quickly and mostly by inspection.

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