Meta-analysis of Factor Analyses: An Illustration Using the Buss-Durkee Hostility Inventory

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Among the many topics considered by meta-analysts, one that has received relatively little attention is the statistical integration of multivariate outcome data. This article focuses on a procedure proposed by Kaiser that has been used to integrate the results from different factor analyses. After giving a geometrical description of Kaiser's procedure, the authors illustrate its use with factor analytic studies of the Buss-Durkee Hostility Inventory. The results show that the seven subscales of the Buss-Durkee Hostility Inventory measure two dimensions of aggressiveness, one that can be called overt *and one that can be called* covert. *The need for further development and application of multivariate data synthesis procedures is also discussed.*

Traditionally, meta-analytic techniques have been used to integrate univariate outcome data. Less attention has been paid to the statistical integration of multivariate outcome data, such as those from factor analysis, even though there is a similar need to synthesize such data. This lack of attention can be attributed to several causes. One is that procedures for synthesizing multivariate data are relatively new, and less well known, in comparison with procedures for synthesizing univariate data (Olkin, 1990). Another is that as the complexity of primary data analysis procedures increases, so does the complexity of associated meta-analysis procedures. Still another is that the reporting problems that plague the statistical integration of studies (e.g., missing values, lack of standardization, nonindependent data sets) can be even more debilitating with multivariate research than with univariate research.

Statisticians have made more progress in developing methods for integrating data from factor analysis than for other multivariate procedures. Several methods have been proposed for comparing factor structures from different samples of subjects (e.g., Cliff, 1966; Jöreskog, 1971; Schonemann, 1966; Ten Berge, 1977; Ten Berge & Knol, 1984; see also Gorsuch, 1983, chap. 13). This article will focus on a method developed by Kaiser (1960; Kaiser, Hunka, & Bianchini, 1969, 1971). We chose to concentrate on Kaiser's method because it uses data that typically are available to the meta-analyst.

The method proposed by Jöreskog (1971) is perhaps the best for comparing factor structures from different samples. It can be used to test some interesting hypotheses. One hypothesis is that the covariance matrices underlying the factor solutions are equivalent. Testing this hypothesis is analogous to testing the hypothesis that a group of univariate effect sizes is homogeneous (Hedges & Olkin, 1985). A second hypothesis is that there is an equal number of common factors in each solution. A third hypothesis is that the factor pattern is the same for each solution. The primary disadvantage of this method is that it requires data that are typically not available to the meta-analyst, such as the covariance or correlation matrices and the standard deviation vectors for each group. Kaiser's method, in contrast, requires only the factor loadings for the variables. These data are routinely provided in factor analytic study reports.

A brief geometrical description of Kaiser's method is given in the next section (for a mathematical description, see Kaiser et al., 1971). Kaiser's method compares factor solutions from two samples of subjects. If there are more than two factor solutions, the procedure is re-

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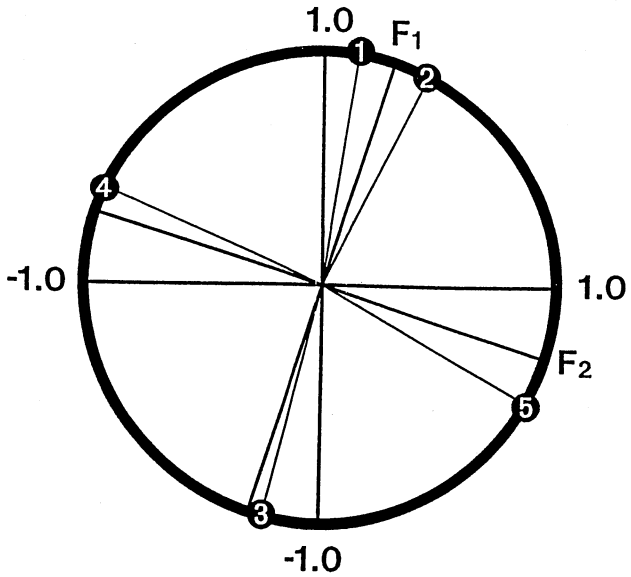

Figure 1 Hypothetical data for Sample 1. F1 = Factor 1 axis, F2 = Factor 2 axis. The five variables are numbered 1 through 5.

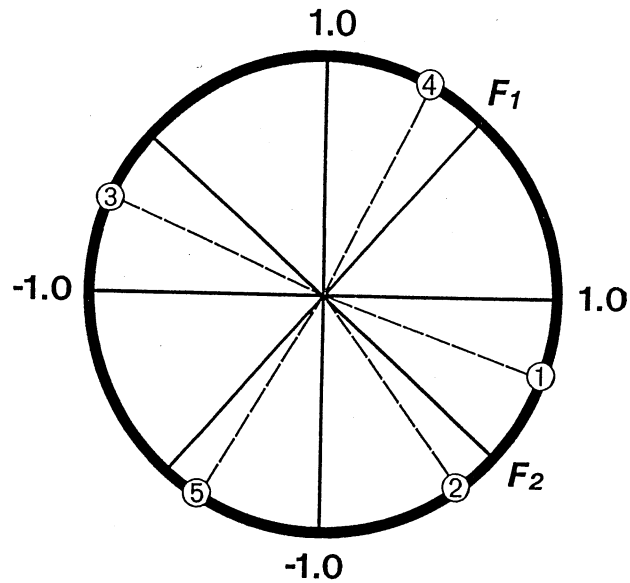

Figure 2 Hypothetical data for Sample 2. F1 = Factor 1 axis, F2 = Factor 2 axis. The five variables are numbered 1 through 5.

peated for all pairs of solutions. In our description we will assume that the two factor analyses were performed on the same set of variables[1] and that the factor vectors are orthogonal.

KAISER'S METHOD FOR COMPARING FACTOR SOLUTIONS

Kaiser's method involves three steps. First, the variable vectors from both samples are normalized to have Length 1. This is done to ensure that each variable makes an equal contribution in determining the final position of the factor axes.[2] We have illustrated this step graphically in Figures 1 and 2, using hypothetical data for two samples. Note that both samples have five variables and two factors and that all variable vectors have been normalized to Length 1.

In the case where the two samples contain the same number of factors, the variables from one of the samples are first located in vector space by means of their factor loadings. The variables from the other sample are then represented in this same space. In the case where the two samples do not contain the same number of factors, the variables from the sample containing the smaller number of factors are represented in the vector space of the sample containing the larger number of factors. In the common space all vectors have the same origin and the same factor orientation. This step is illustrated in Figure 3 for our hypothetical data. The vectors for Sample 2 are represented in the same space as the vectors for Sample 1. The common vector space is two-dimensional because both studies have two factors. The common origin is indicated by point A. Note also that the factor axes for the two studies are parallel.

Next, the factor axes are rigidly rotated until a maximum degree of overlap between matching variables is obtained. That is, the factor axes are rotated until the sum of products of loadings on corresponding factors is maximized. The rotation yields a unique solution. After the solution is obtained, the cosines of angles between all pairs of factor vectors are measured. According to Kaiser and his colleagues, these cosines can be taken as "a measure of relationship between the factors represented, and a measure which certainly can be interpreted in the same way as a correlation coefficient" (Kaiser et al., 1971, p. 411). Two factor vectors are identical if the cosine of the angle between them equals 1 (i.e., the angle between them equals 0°) and orthogonal if the cosine of the angle between them equals 0 (i.e., the angle between them equals 90°). This final step is shown in Figure 4 for our example. The angle between the factors for the two studies is given by β, and the angles between matching variables are given by $\alpha_1, \alpha_2, \ldots, \alpha_5$.

The mean cosine between matching variables is also calculated.[3] According to Gorsuch (1983), "If the mean cosine is low, it is not possible to relate factors at all because the relationships among the variables are different" (p. 284; also see Barrett, 1986).

Veldman (1967) has written a FORTRAN computer program to carry out the procedures described above.[4]

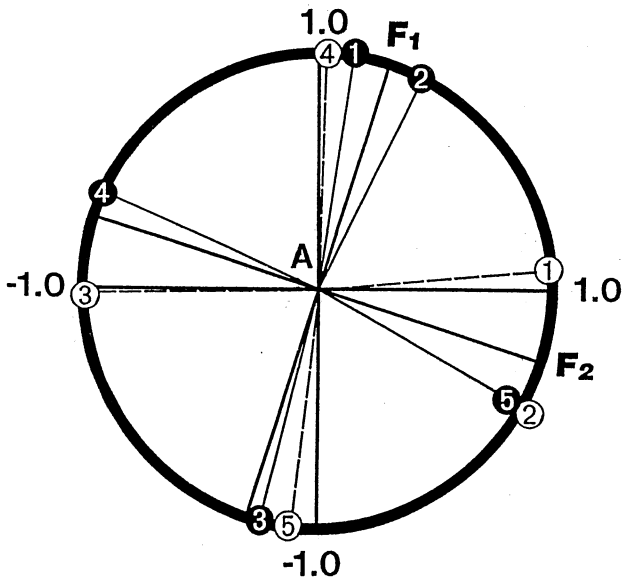

Figure 3 Sample 1 and Sample 2 factor vectors in common vector space. F1 = Factor 1 common axis, F2 = Factor 2 common axis, A = common origin.

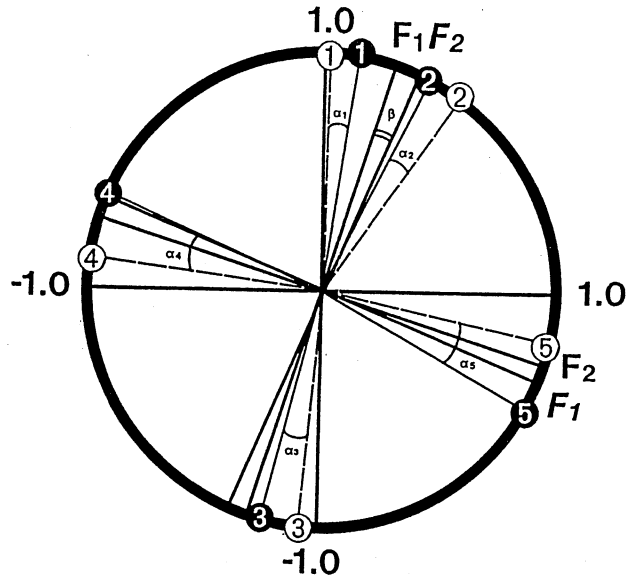

Figure 4 Final position of rotated factor axes. β = angle between factor axes, α_1 through α_5 = angles between variables 1 through 5.

Although Kaiser has extended his method to compare oblique factor structures, Veldman's program handles only orthogonal factor structures. The output from the program includes the cosines of angles between factors and the cosines of angles between variables.

Social scientists have used Kaiser's procedure to integrate the results from different factor analytic studies. The results of these efforts are described briefly in the next section.

PAST EFFORTS TO INTEGRATE THE RESULTS OF FACTOR ANALYTIC STUDIES USING KAISER'S METHOD

Hans and Sybil Eysenck have used Kaiser's method to compare the male and female factor solutions of the Eysenck Personality Questionnaire (EPQ) from England with the male and female factor solutions of the EPQ from other countries. Cross-cultural comparisons have recently been summarized for 24 countries (Barrett & Eysenck, 1984; Eysenck, Barrett, & Eysenck, 1985). The results of the review showed that the same EPQ factors that were found in the England sample (i.e., Extroversion, Neuroticism, Psychoticism, and Social Desirability) were also found in the samples from other countries.

Thompson (1989) used Kaiser's method to compare factor solutions of the Bem Sex-Role Inventory. His review included seven factor analyses from five research reports. The results showed that the inventory's primary factors, Femininity and Masculinity, were reproduced in most of the samples.

Kaiser's method has also been used to examine the stability of other inventories, such as the College Self-Expression Scale (Galassi & Galassi, 1979), the Comfrey Personality Scales (Vandenberg & Price, 1978), the Reducer-Augmenter Scale (Kohn, Hunt, Cowles, & Davis, 1986), and the Survey of Ethical Attitudes (Johnson, Hogan, Zonderman, Callens, & Rogolsky, 1981). In the next section we will illustrate the use of this procedure on factor analytic studies of the Buss-Durkee Hostility Inventory.

AN EXAMPLE OF META-ANALYSIS OF FACTOR ANALYSES USING THE BUSS-DURKEE HOSTILITY INVENTORY

The Buss-Durkee Hostility Inventory (BDHI)[5] contains 66 true/false items, which are divided, on theoretical grounds, into seven subscales. The subscales were developed to measure different types of hostility. A brief description of each subscale is given below.

Assault: Physical violence against others (e.g., "If somebody hits me first, I let him have it").

Indirect Hostility: Both roundabout and undirected aggression (e.g., "I sometimes spread gossip about people I don't like.").

Irritability: A readiness to explode with negative affect at the slightest provocation (e.g., "I often feel like a powder keg ready to explode").

Negativism: Oppositional behavior usually directed against authority (e.g., "When someone is bossy, I do the opposite of what he asks").

Resentment: Jealousy and hatred of others (e.g., "Almost every week I see someone I dislike").

Suspicion: Projection of hostility onto others (e.g., "I know that people tend to talk about me behind my back").
Verbal Hostility: Negative affect expressed in both the style and content of speech (e.g., "If someone annoys me, I am apt to tell him what I think of him").

Method

Literature search procedures. Two sources were used to obtain factor analytic studies of the BDHI: PsycINFO (1967-1989) and the *Social Science Citation Index* (1960-1989). Any study that included the descriptor *Buss-Durkee Hostility Inventory* in the title or abstract was retrieved from PsycINFO, and any study that referenced the original Buss and Durkee (1957) article was retrieved from the *Social Science Citation Index.* In all, 64 studies were obtained from PsycINFO and 242 studies were obtained from the *Social Science Citation Index.* The abstracts of these studies were then examined to identity those that employed factor analysis. In all, 21 factor analytic studies were located.

Criteria for relevance. Thompson's (1989) criteria were employed to determine whether a factor analytic study should be included in the present review. First, studies that included variables other than the seven subscales of the BDHI in the factor analysis were excluded. The authors of these studies were contacted in order to determine whether they had also examined the factor structure of the BDHI separately. Second, only studies that extracted factors using some variation of principal factor analysis and rotated factors using orthogonal methods (e.g., Varimax) were considered.[6] Third, studies had to report factor loadings for all seven aggression scales to be included in the review.[7]

Five factor analyses met the criteria above. These factor analyses were obtained from two published studies (Buss, Fischer, & Simmons, 1962; Holland, Levi, & Beckett, 1983). The Buss et al. (1962) study included subsamples of men and women who were psychiatric patients. The factor solutions for both subsamples contained two factors. However, the authors did not indicate how the number of factors was determined. The Holland et al. (1983) study included subsamples of White, Mexican-American, and Black male offenders from a corrections institution. The factor solutions for all three subsamples contained two factors. The authors retained all factors with eigenvalues greater than 1.

One unpublished factor analytic study by Bushman and Heisler (1989) was added to the pool of studies. This study included subsamples of male and female college students. The female solution contained two factors, whereas the male solution contained three factors. Following Guttman's (1954) recommendation, all initial factors with eigenvalues greater than 1 were extracted.[8]

TABLE 1: Factor and Variable Cosines for Samples That Contained Two-Factor Solutions

	Sample Number					
	1	*2*	*3*	*4*	*5*	*6*
1	—	.99	.95	.99	1.00	1.00
2	.88 (.92)	—	.99	1.00	1.00	1.00
3	.86 (.92)	.76 (.91)	—	.99	.97	.96
4	.90 (.91)	.80 (.94)	.95 (.97)	—	1.00	1.00
5	.98 (.98)	.85 (.91)	.82 (.91)	.83 (.90)	—	1.00
6	.90 (.94)	.96 (.96)	.88 (.96)	.89 (.96)	.88 (.95)	—

NOTE: The values above the diagonal represent the cosines between factor vectors. The values below the diagonal represent the mean cosines for the seven aggression subscales of the Buss-Durkee Hostility Inventory. Thompson (1986) recommends using Fisher's z' transformation on the cosines prior to calculating their mean, and then transforming the z' back to a cosine value (see Note 3). These values are in parentheses. Sample 1 = 33 men (Buss, Fischer, & Simmons, 1962); Sample 2 = 50 White men (Holland, Levi, & Beckett, 1983); Sample 3 = 53 Mexican-American men (Holland et al., 1983); Sample 4 = 48 Black men (Holland et al., 1983); Sample 5 = 63 women (Buss et al., 1962); Sample 6 = 62 women (Bushman & Heisler, 1989).

A study by Kazdin, Rodgers, Colbus, and Siegel (1987) was also analyzed separately. Kazdin and his colleagues derived a hostility inventory for children from the BDHI. The inventory is completed by the child's parent or guardian. The 38 items of the Children's Hostility Inventory were taken from the BDHI but were reworded so that they were applicable to children. The Kazdin et al. study included subsamples of boys and girls, but the data were combined for the factor analysis. The factor solution contained two factors with eigenvalues greater than 1.

Results

Six of the seven factor solutions compared had two factors. The results from these six factor analyses are shown in Table 1. The elements above the diagonal are the cosines between factors, whereas the elements below the diagonal are the mean cosines for the seven aggression subscales. For example, the factor cosine comparing men and women from the Buss et al. (1962) study (i.e., Sample 1 vs. Sample 5) is 1.00. The mean cosine for this comparison is .98.

The male factor solution for the Bushman and Heisler (1989) study contained three factors and was therefore not included in Table 1. The highest loadings on Factor I were Suspicion (.86) and Resentment (.76), the highest loadings on Factor II were Indirect Hostility (.91) and Verbal Hostility (.84), and the highest loading on Factor

TABLE 2: Comparison Between the Bushman and Heisler (1989) Male Sample and the Samples With Two-Factor Solutions

Comparison Study	*Item Mean*	*Factor*		
		I	*II*	*III*
Buss, Fischer, & Simmons (1962) men	.74 (.83)	.92	.62	.74
Holland, Levi, & Beckett (1983) White men	.85 (.89)	.96	.92	.38
Holland et al. (1983) Mexican-American men	.72 (.82)	.86	.47	.93
Holland et al. (1983) Black men	.77 (.85)	.80	.43	.90
Buss et al. (1962) women	.73 (.77)	.96	.65	.74
Bushman & Heisler (1989) women	.84 (.92)	.99	.80	.59

NOTE: Entries for Factors I, II, and III are analogous to the above-diagonal entries of Table 1, and entries in the column "Item Mean" correspond to the below-diagonal entries of Table 1. The Bushman and Heisler (1989) male sample size was 24.

III was Assault (.94). Table 2 shows the relationship between this sample and the samples with two factors.

The Kazdin et al. (1987) factor analysis was also compared with the other seven factor analyses. The factor cosines between this sample and the samples with two-factor solutions were: Buss et al. (1962), men, 1.00; Buss et al., women, 1.00; Holland et al. (1983), Black men, .99; Holland et al., Mexican-American men, .94; Holland et al., White men, .99; and Bushman and Heisler (1989), women 1.00. The cosines between the Kazdin et al. sample and Factors I, II, and III of the Bushman and Heisler (1989) male sample were .97, .81, and .54, respectively. Overall, the mean variable cosines ranged from .82 to .98.

DISCUSSION

The primary purpose of this article was to introduce social and personality psychologists to a procedure that has been used to integrate multivariate outcome data. We illustrated this procedure using factor analytic studies of the Buss-Durkee Hostility Inventory. These data were obtained from widely different samples of subjects. The subjects in the Holland et al. (1983) study were offenders who were undergoing felony presentence evaluation, the subjects in the Buss et al. (1962) study were psychiatric patients from an acute treatment center, and the subjects in the Bushman and Heisler (1989) study were college students. Nevertheless, there appears to be a high degree of correspondence among the factor structures for the different samples. Kaiser's method shows that factor solutions that ma appear to be different are in fact similar when they are represented in the same vector space.

The results show that the seven subscales of the Buss-Durkee Hostility Inventory measures two dimensions of aggressiveness, one that can be called *overt* and one that can be called *covert*. The overt factor consists primarily of the Assault and Verbal Hostility subscales, whereas the covert factor consist primarily of the Resentment and Suspicion subscales. In the sample that had a three-factor solution, the covert aggressiveness factor emerged and the overt aggressiveness factor was split into two separate factors.[9]

There are at least two issues to consider when using Kaiser's method. The first issue concerns the criterion used to determine the number of extracted factors. Results may be difficult to compare if different criteria were used by different researchers. For instance, the third factor in the Bushman and Heisler (1989) study had an eigenvalue of 1.10. This factor met the 1.00 minimum eigenvalue criterion, but it may not have met another criterion. A second issue is how to interpret factor and variable cosines. It is easy to interpret cosine values of 1.00 or 0.00; it is much more difficult to interpret values between these two extremes. Although several researchers have offered arbitrary conventional values (e.g., Barrett, 1986; Kaiser et al., 1971), we are aware of no test to determine whether a cosine is significantly greater than zero. Computer simulation work in this area would be desirable.

It is hoped that the present article will stimulate meta-analysts to think about integrating multivariate outcome data. There is much room for progress in this area.

NOTES

1. Kaiser's procedure can be used when the two factor analyses include different variables, although some difficulties arise in these situations (see Kaiser, Hunka, & Bianchini, 1971).

2. Alternatively, instead of allowing all variables to make an equal contribution in determining the solution, the meta-analyst could allow the variables to influence the solution in proportion to their commonalities so that the variables having little common variance have little influence. In this case, the raw loadings can be used rather than the noramlized ones.

3. Thompson (1986, 1989) recommends using Fisher's z' transformation on the cosines prior to calculating their mean, and then transforming the z' back to a cosine value. We have some reservations about this practice, however. It is, of course, less conservative than computing the mean of the cosines. In addition, as Kaiser and others have noted, the cosines *between* studies are "not correlations in the usual sense (because there are no common individuals on which to base a correlation)" (Kaiser, Hunka, & Bianchini, 1971, p. 411). The fact that cosines are analogous to correlations does not imply that they have the same sampling distributions. Fisher's z' transformation is used to normalize and stabilize the variance of random variables that have the same distribution as the correlation coefficient. There is no reason to assume that it will be useful for cosines.

4. Veldman's (1967) program was written FORTRAN II, which is now obsolete. An updated version of the program, compatible with

FORTRAN 77, can be obtained by writing to the first author. Appreciation is extended to Wayne Churchill, Craig Pepmiller, and Bruce Thompson for their assistance with the computer program.

5. The original scale by Buss and Durkee (1957) included a *Guilt* subscale and was called the Hostility-Guilt Inventory. The Buss-Durkee Hostility Inventory, however, does not include the *Guilt* subscale.

6. Buss and Durkee (1957) originally extracted factors using Thurstone's centroid method and rotated the axes to the same simple structure. Later, Buss, Fischer, and Simmons (1962) reanalyzed these data and rotated factors by means of the Varimax procedure. However, personal communication with Buss (June 1989) indicated that these data were no longer available. Consequently, the male and female factor solutions from the original Buss and Durkee (1957) article were not included in the review.

7. Initially, we had planned to examine studies that factor-analyzed the items of the BDHI (e.g., Velicer, Govia, Cherico, & Corriveau, 1985). However, because there were very few of these studies, we resorted to using studies that factor-analyzed the subscales of the BDHI. This procedure, though not ideal, provides a nice illustration of Kaiser's method.

8. The sample sizes for all studies are small relative to the number of items analyzed (see Tables 1 and 2 for exact values). Although this creates instability in factor solutions across studies, Kaiser's method is blind to underlying sample sizes.

9. The reader should note that certain constraints may operate when a two-factor solution is compared with a three-factor solution. In this case, one factor from the two-factor solution may always match one factor from the three-factor solution, and the second factor will either have a match or reveal split loadings. This constraint may not hold when the numbers of factors in the studies become more disparate (e.g., two factors vs. four factors). To our knowledge, there is no analytical work on this question at the present time.

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