

Comparing Modeling Methodologies

[Formerly titled: Modeling Security Issues of Central Asia]

By

Robert Axelrod
Gerald R. Ford School of Public Policy
University of Michigan
Ann Arbor, MI 48109

axe@umich.edu

March 18, 2004

Prepared for CMT International - Project on "Security in Central Asia"
June 2004. US Govt. Contract # 2003*H513400*000

1. Introduction
 - a. Purpose of This Report
 - b. The Security Issues of Central Asia
 - c. Goals of Modeling Security Issues of Central Asia
 - d. The Law of the Hammer
2. Modeling Approaches
 - a. Game Theory
 - b. Simulation Modeling
 - i. Agent-Based Modeling
 - ii. Equation-Based Modeling Including Dynamical Systems
 - iii. Systems Dynamic Diagrams
 - c. Data-Driven Modeling
3. Rationality in Modeling
4. Comparison of Modeling Approaches
5. Conclusions
 - a. Making the Whole Greater than the Sum of the Parts
 - b. Challenges for Modeling Security Issues of Central Asia

Appendices

- I. Appropriateness of Modeling Methodologies
- II. Nine Relevant Issues Already Addressed by Agent-Based Models

References Cited

1. INTRODUCTION

a. Purpose of this Report

This report supplements the September 2003 Workshop on “Modeling the Security Issues of Central Asia.” Its purpose is: to convey my inputs to the Workshop, to present my views on the appropriateness of the various modeling methodologies for the analysis of the critical issues, and to report my understanding of the salient challenges in developing a system of models appropriate for the security of central Asia.¹

b. The Security Issues of Central Asia

Central Asia encompasses a very wide range of security issues including:

- * Terrorist organizations with connections to global terrorism,
- * Great power rivalry,
- * Islamist challenges,
- * Drug networks,
- * Environmental degradation,
- * Corruption,
- * Smuggling,
- * Social cleavages (e.g., urban/rural; rich/poor),
- * Tribal competition, and
- * Insurgent opposition to authoritarian governments.

I refer the reader to the Workshop briefings and papers for an exposition of these security issues in central Asia, and their historical context.

c. Goals of Modeling the Security Issues of Central Asia

While central Asia is important in its own right, the main goal of the present modeling effort is to test the value of modeling for a wide range of topics of interest to intelligence analysts. From this perspective, central Asia is merely an example. The main point is to learn lessons about the kind of help that modeling can provide to analysts. Contributing to these lessons is the reason for this report.

To start with, it pays to keep in mind the potential uses of modeling. Schrodtt (2003) provides a list of three potential uses of a model: heuristic, predictive, and prescriptive. The following list draws on Axelrod’s (1997) somewhat different and more comprehensive list of uses of modeling.

1. Prediction. A simulation is able to take complicated inputs, process them by taking hypothesized mechanisms into account, and then generate their consequences as predictions. For example, if the goal is to predict interest rates in the economy three months into the future, simulation modeling can be the best available technique.

¹ The author thanks the participants in the Workshops, and especially Nazli Chourcr, for their contributions and suggestions. The remaining errors are all mine.

Unfortunately, in the area of security issues, it would be unrealistic to expect any model to provide predictions that are both precise and accurate. It is important to understand models of social and political phenomena are not likely to provide precise predictions of specific events such as the assassination that set off World War I, or the date at which the next Asian currency crisis will start. What good models can, however, provide are predictions of some *pattern* of events such as the dangerous instability among the Great Powers in 1914, or the growing vulnerability of a currency as speculative holdings increase.² In addition, some models can provide indications of the comparative likelihood of different kinds of outcomes. The modeling efforts represented in this project use the broader definition of “prediction” as indications of tendencies or probabilities, rather than dates of particular events or the like. To highlight this broad use of the term in this report, the word “prediction” will be kept in parentheses.

2. Performance. Models can also be used to perform certain tasks. This is typically the domain of artificial intelligence. Tasks to be performed include medical diagnosis, speech recognition, function optimization, and pattern recognition. In the realm of security issues, data-driven models have potential for pattern recognition. Schrodt (2003, p. 10) proposes a case-based retrieval model that would take a description of a situation and find the closest match in a database of historical events. For example, a case-based model could take a revolution in Kazakhstan and find the best match among the revolutions from around the world over the last 50 years. For this purpose, the performance of the model would be judged by its performance at the task of identifying historical analogies that proved useful to the analyst.

3. Training. Many of the earliest and most successful simulation systems were designed to train people by providing a reasonably accurate and dynamic interactive representation of a given environment. Flight simulators for pilots are an important example of the use of simulation for training. The military often use elaborate simulations as part of “command post” exercises to provide a synthetic environment for the exercise.³ Similarly, a political-military role playing could be used to explore the security problems of central Asia. For example, a model dealing with the conditions for a successful revolution can help the umpire in a role-playing exercise.

4. Entertainment. From training, it is only a small step to entertainment. Flight simulations on personal computers are fun. So are simulations of completely imaginary worlds. To the extent that analysts are entertained by interacting with a simulation model, they might be motivated to learn and take advantage of a particular model.

5. Education. From training and entertainment, it is only another small step to the use of simulation for education. A good example is the computer game SimCity. SimCity is an interactive simulation allowing the user to experiment with a hypothetical city by changing many variables, such as tax rates and zoning policy. For educational purposes, a simulation need not be rich enough to suggest a complete real or imaginary world. The main use of simulation in education is to allow the users to learn relationships and principles for themselves.

² Choucri, North and Yamakage (1992) illustrate this point with an econometrics simulation of Japan’s going to war.

³ For example, the DoD’s flagship training and modeling simulation program for the warfighter is JWFC JSIMS. See <http://www.jsims.mil/>

6. Proof. Simulation can be used to provide an existence proof. For example, Conway's Game of Life (Poundstone, 1985) demonstrates that extremely complex behavior can result from very simple rules. Gabbay's report (2003) points out that models using dynamical systems can be placed on a firm mathematical footing that allows general statements to be proved about the issues such as the stability.

7. Heuristic use. The heuristic value of modeling security issues of central Asia is the potential to enhance a users "feel" for a topic. A good model can help the user see some of the implications of making certain assumptions about how things work. Perhaps the most commonly used model in the social sciences is the iterated Prisoner's Dilemma. Among the results from this model is that when the "shadow of the future" is strong enough, even egoists can develop cooperation based on reciprocity (Axelrod, 1984). While no model is correct, and the Prisoner's Dilemma model is an *extremely* simplified view of the world, it can help the user to understand better the very common tension between helping someone else and looking out for number one. This tension certainly exists at many levels in central Asia, from individual honor to respect for international agreements. To the extent that a model captures one or more fundamental features of the issue at hand, it can have heuristic value by giving the user a deeper understanding of the implications of these features. A model need not be a plausible representation of the entire situation in order for it to help a user see how a fundamental process might operate in that situation. The models discussed in the papers for the Workshop all include among their most important goals heuristic value for the user.

d. The Law of the Hammer

There is an old adage that if you give a kid a hammer, everything looks like a nail. The more general form of the adage would be: every kid has one and only one tool for attacking any problem. At this workshop, the modelers were presented with the problems of central Asia, and each proposed the kind of model with which he or she was most familiar. This is natural, and to be expected.

I was not surprised that they all responded to the task by proposing models of the type with which they were most experienced. What *did* surprise me was that, almost without exception, the other modelers had only the barest acquaintance with each other's tools, let alone the capacity to pick from a variety of tools as a problem might require.

Thus, a major challenge is to harness the capacities of the entire modeling tool kit when everyone knows how to use only one tool. The problem is to organize the teams so that the "whole is greater than the sum of the parts." Ways to meet this challenge are discussed in the Conclusion.

We now turn to specific modeling approaches, and their comparative strengths and weaknesses.

2. MODELING APPROACHES

For the purposes of this report, there are three main types of models: game theory models, simulation models (both agent-based and equation-based) and data-driven

models. Game theory models assume forward-looking calculating players with fixed goals that can be combined into a single utility function. The other types of models need not assume a fixed utility function. A game theory model is sometimes embedded in another kind of model. Among the other kinds of models, the distinction between simulation and data-driven models is based on two factors. First, simulation models explicitly represent the flow of time, and are therefore dynamic models, while data-driven models typically do not. Second, simulation models typically do statistical analysis on the artificial histories generated by the model, whereas data-driven models typically do their statistical analysis on empirical data.

Within the category of simulation models, three types were discussed at the Workshop: agent-based models (Robert Axelrod, Ian Lustick, and William Reynolds/David Dixon), dynamical systems (Michael Gabbay), and the systems dynamics (Khalid Saeed). Within the category of data-driven models were the suggestions offered by Philip Schrodt. This report provides ways to compare all these modeling approaches, to assess the appropriateness of each for exploring the relevant security issues, and to see how the modeling approaches can work together.

a. Game Theory

Game theory allows a very rich way of analyzing what will happen in a specific strategic context. To specify a game, one needs to specify the players, the choices, the outcome as determined jointly by the choices, and the payoff to each player associated with that outcomes.⁴ One more thing is needed, namely a way to determine how the players will make their choices. Traditionally, game theory has calculated what players will do by assuming the players are rational, that they know the other players are rational, and that everyone has the ability to do unlimited calculation. Game theory therefore looks for equilibrium “solutions” based on one of many different solution concepts. A good example of game theory applied to a problem relevant to security in central Asia is the model of Fearon and Laitin (1996) exploring how one ethnic group might discipline its own members to avoid conflict with another ethnic group.

b. Simulation Modeling

i. Agent-based models

Agent-based models are already available to explore a wide range of issues. I have identified nine issue areas relevant to security in central Asia for which agent-based models already exist. For descriptions and citations, see the Appendix II. The issues are:

- * Politics of Closed Regimes,
- * Institutionalization of Identity,
- * Nationality Formation,
- * Instigation of Communal Violence,
- * Suppression of Rebellion and Communal Violence,

⁴ Game theory as a modeling technique should not be confused with role-playing games.

- * Escalation from Demonstrations to Revolution,
- * Policing Corruption,
- * Intergroup Competition Over Territory, and
- * Alignments Between and Within Nations

An excellent introduction to agent-based modeling is provided by Rauch (2001) and is available online.

To illustrate how an agent-based model can be applied to security issues of central Asia, I will use my own model developed with Ross Hammond to explore in-group/out-group behavior (Axelrod and Hammond, 2003).

First, it is important to specify what the model can refer to in the central Asian context. Originally, we had in mind only the group distinctions based on ethnicity. However, the model can just as easily apply to any type of distinction that may interest the analyst in which individual group membership is visible and stable. In-groups in central Asia are based on not only ethnicity, but also clan ties and loyalty based on locale. Since the present form of the Axelrod-Hammond model does not take into account individual mobility, it works best when there is little long-distance movement. Since one's ethnic affiliation is relatively stable and typically visible to one's neighbors, ethnicity can serve to illustrate the Axelrod-Hammond model of in-group/out-group behavior. According to one of the area experts (Atkins 2003, p. 10), "there have been inter-ethnic clashes in central Asia in recent years and ... various groups play on ethnic animosities for their own advantages..." Certainly, ethnicity is weaker and more complex in central Asia than in Europe. However, one should keep in mind that ethnicity is just one kind of social distinction that can be analyzed with the Axelrod-Hammond model of in-group/out-group behavior.

In the Axelrod-Hammond model, the agents are individuals.⁵ Each agent has a "color" which can be interpreted as its ethnicity or other group membership. Each agent also has a two-part strategy. One part of the strategy specifies whether the agent cooperates or not when interacting with a neighbor who has the same color. The other part of an agent's strategy specifies whether the agent cooperates or not when interacting with a neighbor who has a *different* color. As in all agent-based models, the rules for the agent's interactions and behavior were set, and then a computer simulation was used to generate artificial histories of the resulting process. For example, in Axelrod and Hammond's in-group/out-group model, the interaction between neighboring agents is a one-move Prisoner's Dilemma. To focus on intergroup competition, other factors were deliberately left out – including the possibility of reciprocity or reputation. The original design goal was to understand the conditions under which the population will eventually become dominated by people who provide costly help members of their own group, but refuse to provide such help with members of other groups. The problem is hardly trivial because in the short run, each agent is better off providing costly help to no one.

Since the purpose of the model was heuristic, a major design criterion was simplicity. The Axelrod-Ross model itself can be completely described in 125 words, as follows.

Start with an empty 50x50 space with 4 cell neighborhoods.

⁵ In this model, the agents could also be interpreted as small collections of families such as villages with only one ethnic group.

Repeat 2000 times:

1. Immigration. An immigrant agent with random genes enters at a random empty site.
 - An agent has three genes:
 - Tag: one of four colors.
 - Choice when meeting an agent with its color: help or not.
 - Choice when meeting an agent of another color: help or not.
2. Interaction. Each agent starts the period with Potential to Reproduce (PTR) = 12%. Each adjacent pair of agents plays a one-move Prisoner's Dilemma:
 - if help: PTR decreases 1%
 - if get help: PTR increases 3%
3. Reproduction. In random order, each agent, with probability PTR, reproduces into an adjacent empty cell, if available, with mutation/gene = 0.5%
4. Death. Each agent has a 10% chance of dying.

The basic finding from running this model is that ethnocentric behavior can be expected to occur in a wide range of situations even with minimal assumptions about the cognitive capacities of the agents. For example, the agents need not be rational for ethnocentrism to emerge and prove stable. A surprising result is that when the environment is relatively austere, the ability to distinguish between the in-group and the out-groups leads to higher levels of cooperation than would occur if the agents were "color blind."

Although developed for academic purposes, the Axelrod-Hammond model of ethnocentric behavior can be applied to help the user gain insights into the dynamics of inter-group conflict in central Asia.

1. To start with, it is easy to use contemporary or historical data on central Asia to initialize the model. An example would be to use a map of the distribution of ethnic groups to assign particular kinds of agents to corresponding places on the model's map. The model could then be run from that initial condition to see how and where ethnic conflict might be anticipated. Since the model is very simple, one would not expect a high degree of accuracy in such "predictions", but they might still be of interest in identifying some potential zones of conflict.

2. The model could easily be adapted to explore the effects of specific aspects of modernity, such as the growth of mass media. This could be done by adding a single mechanism to the model so that an agent is affected by long-range broadcasts as well as by neighbors. One could ask, for example, what changes might be expected as the media became increasingly influential. One could ask what might happen if the provincial media became strong enough to challenge the dominance of national media.

3. The model could easily be adapted to explore the effects of changing perceptions of what differences really matter. Among the questions that could be explored are: what would happen in Kazakhstan if the divisions among the Moslems became unimportant relative to the divisions between the Muslims and the ethnic

Russians? Conversely, what would happen if a myriad of local tribal loyalties tended to dominate?

4. One could also ask about the potential effects of multiple and crosscutting loyalties, such as rich/poor, urban/rural, Christian/Muslim differences as well as local and linguistic affiliations.

As Ian Lustick (2003) points out in one of his reports, simulation models, including agent-based models, are easy to use for asking “what if” questions. In fact, because they typically have a stochastic (probabilistic) element, by running a simulation many times, an entire *distribution* of a counterfactual can be generated. Distributions are useful to see whether a particular kind of outcome would be expected to be likely or rare.

ii. Dynamical Systems

Dynamical systems are a form of equation-based modeling. To understand how equation-based modeling works, a good place to start is with the classic Lotka-Volterra model of predators and prey. In a central Asian context, the predators and prey can be thought of as border police and smugglers. The model postulates that the more smugglers the police find, the more the police force will grow due to increased political support. The ability of the policy to catch smugglers depends on the chance that a police officer will encounter a smuggler, with the chance this will happen being proportional to both the size of the police force and the number of smugglers. The model also postulates that the larger the police force the greater pressure there is to reduce it. Finally, it assumes that the number of smugglers would grow exponentially if none were caught. To explain this equation-based model, I adapted following explanation from a Wolfram Web site.⁶

The Lotka-Volterra equations describe an ecological police-smuggler model which assumes that, for a set of fixed positive constants A (the growth rate of smugglers), B (the rate at which police catch the smugglers), C (the rate of decline in the police force), and D (the rate at which police force increases when a smuggler is caught), the following conditions hold.

1. A smuggler population x increases at a rate $dx = Ax dt$ (proportional to the number of smugglers) but is simultaneously destroyed by police at a rate $dx = -Bxy dt$ (proportional to the product of the numbers of smugglers and police).
2. A police population y decreases at a rate $dy = -C y dt$ (proportional to the number of police), but increases at a rate $dy = Dxy dt$ (again proportional to the product of the numbers of smugglers and police).

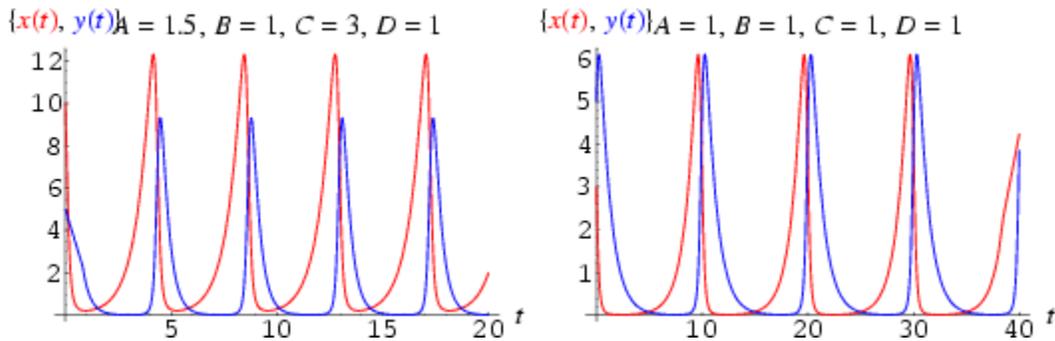
This gives the coupled differential equations

$$\frac{dx}{dt} = Ax - Bxy$$

⁶ <http://mathworld.wolfram.com/Lotka-VolterraEquations.html>

$$\frac{dy}{dt} = -Cy + Dxy,$$

solutions of which are plotted below, where smugglers are shown in red, and police in blue. The lines can also be distinguished by the fact that the smuggler curve always leads the police curve.



To me, the striking result of this police-smuggler model is the cycling it produces. Suppose we start with few police. Then the smugglers will grow in numbers, and eventually their numbers will increase so much that the police will have an easier time catching some of them. As the police are successful, their numbers will be increased by the government, and the police will then be effective enough to dramatically reduce the number of smugglers. Then, as smugglers become rare, the police catch fewer of them resulting in the police force being reduced to its original size. The whole process will then repeat itself – not what one might have expected if only part of the cycle were anticipated.

The Workshop example of an equation-based model using dynamical systems was presented by Michael Gabbay (2003). Gabbay’s model explores opinion change in a group. The idea is that each actor has an opinion that is influenced by neighboring actors, as well as outside influences. As each person’s opinion changes, its influence on the opinion of others changes as well. With one equation for each actor, the course of changing opinions in the group can be mathematically analyzed. A typical result is that if the outside influences are much stronger than the inside influences, then the group will not converge to a uniform opinion. Instead, the opinions will tend to oscillate back and forth.

iii. Systems Dynamics Representation of a Cognitive Map

The workshop report of Khalid Saeed (2003) provides a good example of the systems dynamics style of modeling applied to represent a user's beliefs on a given topic. For the purposes of this report, the most valuable part of the systems dynamics approach is not the model per se, but rather the diagram of causal relationships upon which an actual model could be based. In his illustration (Figure 4, p 127), the causal relationships are represented as a graph with nodes (points) as variables and arrows as causal effects from one node to another. This graph can be thought of as someone’s “cognitive map” of

relationships between the variable and causal effects being represented.⁷ In Saeed's example, "defensive investment" increases "manifest govt. power" which in turn causes actions against extremists resulting less "extremist power." This causal pathway continues with "extremist power" leading to "anti-state violence" which increases the "threat to govt." which in turn leads back to "defensive investment" to form one complete feedback loop. Altogether, Saeed's diagram includes 14 variables and 21 causal effects forming 6 different feedback loops. The graphical representation makes it easy to see the relationships between the variables, and the six feedback loops that result from these relationships.

The simplicity of Saeed's systems dynamics formulation comes from considering only two kinds of causal effects, positive and negative, i.e.: the more A, the more B, or the more A the *less* B respectively.⁸ Thus, each causal arrow is labeled as just plus or minus, depending on whether an increase in the variable at the tail of the arrow tends to increase or decrease the variable at the head of the arrow. To determine whether a given loop provides positive or negative feedback, it is only necessary to count whether the number of negative arrows in a given loop is even or odd.

A major advantage of using a system dynamics diagram is that it provides a useful way of representing beliefs. Because the diagram idea of system dynamics can be taught in less than an hour, typical users could construct and use such diagrams for themselves. A user begins with a few variables of interest, such as "extremist power" and "defensive investment" and then writing down the causes and effects of these variables. This is done in terms of signed arrows to and from these original variables, adding new variables as necessary. Then the user is then invited to think about the causes and effects of these new variables, and write down additional variables and causal links between them. The user needs to exercise a certain degree of restraint by discarding ideas that go off in entirely different directions, and sticking to variables and causal pathways that might eventually lead back to the original items of focal interest, such as "extremist power" and "defensive investment."

Once the user has represented his or her ideas as points (variables) and arrows (causal links between variables), the resulting graph is likely to reveal some relationships that the user may not have previously considered. In Saeed's example, there is not only a *negative* feedback loop from defensive investment to extremist power going through reduced threats to the government, there is also a *positive* feedback loop between defensive investment to extremist power going through reduced economic investment.

Asking a user to represent his or her beliefs in the form of a graph of points and arrows can be helpful in several different ways:

1. By formulating his or her beliefs in a structured and visual manner, the analyst may well come to see some relationships that were not obvious before. Using Saeed's example, the analyst who was well aware of one of these feedback loops might see that there was a competing feedback

⁷ The method of representing beliefs as cognitive maps of variables and causal paths linking them was developed by Robert Axelrod (1976). An interesting finding of Axelrod's study was that the arguments used by policy makers from Britain, Germany, and Japan in closed meetings rarely had feedback loops.

⁸ Systems dynamics can also be used in a quantitative manner if the links between variables are described in functional form. When used that way, the system dynamics is essentially a specific kind of nonlinear dynamic model as described in Gabbay's report.

loop that he or she had not taken into account. The analyst might then start to think about which of the two causal pathways will dominate in determining whether defensive investment will increase or decrease extremist power in some particular setting.

2. Thinking about what is needed to answer such questions that arise from the diagram might lead to new intelligence requirements.
3. Several analysts could work together to develop a single graph. This exercise could help the analysts identify their differences, and see which of these differences represent some factor that one analyst had not even considered, and which differences represent differences of beliefs worthy of discussion.
4. A cognitive map can be a useful way to communicate a complex set of beliefs in visual terms. As Saeed's example shows, even 14 variables and 21 causal mechanisms can be encompassed in a single cognitive map that is easy to visualize and analyze.

iii. Data-Driven Models

Philip Schrodts report (2003) discusses how a database of events from around the world can be used to extract information relevant to security problems of central Asia. The basic idea is to identify those events that are most similar to the event of interest to the user, and then perform a statistical analysis of the data using that limited set of events. For example, suppose the analyst wants to investigate the probability of success of a military coup in Tajikistan might assume that success depended in part on the rank of the leader of the coup attempt. In particular, the analyst might want to find out how the rank of the leader of a coup attempt has affected the coup's chance of success in actual coup attempts in the past. The statistical model might assume that the relationship is linear: coup success is directly proportional to rank of the leader. Statistical analysis could then provide the best fit using the set of relevant military coups in the database. A typical finding might be that for every increase in rank (say major to lieutenant colonel), there is a ten percent increase in chance of success.

As Schrodts points out, the value of data-driven models depends on not only the breadth and quality of the data, but also on the variables and weights that are used to measure the similarity between each relevant case and the target case. For example, if the analyst tells the modeler that only coups in Islamic countries are relevant to understanding a potential coup in Tajikistan, then relatively few historical cases will be considered relevant.

3. RATIONALITY IN MODELING

What do these modeling approaches assume about rationality? To answer this question, one must first distinguish two kinds of rationality. I will call them broad rationality and narrow rationality. Broad rationality is close to what the layperson thinks of when asking if someone, like Kim Jong Il, is rational. Thus, broad rationality includes

an assessment of the individual's goals, as well as the means chosen to pursue them. For example, a commentator would judge Kim Jong Il not to be rational (in the broad sense) if the commentator regards pursuit of national self-sufficiency at the price of mass starvation to be an irrational goal. Narrow rationality, on the other hand, accepts the person's goals as given and just evaluates whether the person is pursuing them efficiently.

Modeling techniques almost never consider the question of broad rationality. Instead, they take the individual's goals as given, whether or not they seem rational in the broad sense of the term.⁹ If the goals can be combined into a single overall goal, then the individual is said to have a utility function. Rational choice theory, and classical game theory not only assume that the individuals have utility functions, but also assume that the individual is rational in the still more arrow sense of making choices that maximize the utility function the individual is assumed to have. In this sense, a game theorist might say that Kim Jong Il is rational (in the narrow sense) if the choices Kim makes are consistent with a single unchanging utility function – no matter how aberrant that utility function might be.

Rationality in the narrow sense is so widely assumed in social science modeling that it constitutes the mainstream understanding of what models are about. Economists have led the way in showing how the assumption of (narrow) rationality allows useful game theoretic models to be built and studied. Political scientists have followed this lead, in such areas as deterrence theory and voting behavior. Typically, narrow rationality is defended either by saying that people come sufficiently close to it, or that the deviations from rationality might cancel each other out as in a large market.

Recently, even economists are recognizing the empirical problems with assuming narrow rationality. The trend started with the concept of bounded rationality introduced by Simon (1957) and developed by March (1978). Bounded rationality takes into account the limited time to make a decision, limited ability to calculate all the implications of a given choice, and so on. More recently, some economic have become fascinated with the implications of various psychological findings, especially those of Daniel Kahneman and Amos Tversky. In fact, both Simon and Kahneman have received Nobel Prizes in Economics for their contributions in undermining the assumption of narrow rationality. (Tversky would presumably also have won one had he lived long enough.)

The major alternative to the assumption of (narrow) rationality is adaptation. In recent years, even mainstream game theorists have begun to relax the assumption of rational actors, and studied various forms of adaptive behavior.¹⁰ The difference is that rational choice models assume that an individual makes a choice by looking *forward* to calculate expected consequences of a choice (as chess players do), whereas adaptive models assume that individuals look *backward* and change their behavior in light of what seems to have worked well in the past. Four important forms of adaptation are used in modeling, namely learning, imitation, social influence and selection.¹¹

⁹ Among the models exploring how and when goals change is Axelrod and Cohen (1984).

¹⁰ See Dixit and Skeath (1999) for an introduction to how game theory can incorporate adaptive decision making.

¹¹ For policy implications for the design of adaptive organizations, see Axelrod and Cohen (2001).

1. Learning is perhaps the simplest example of an adaptive model. In a simple Bush-Mosteller learning model, the individual (possibly a lab rat) is constantly adjusting its probability of making a given choice in accordance with how often that choice has been rewarded or punished in the past.
2. Another form of adaptation is to imitate the choices of someone who is more successful than oneself. For example, if a terrorist group sees that car bombings are effective elsewhere, the group might imitate that kind of behavior and try car bombing itself. Here is another example of imitation. If a central Asian nation seems to be doing well with the military assistance America gives in exchange for basing rights, the other central Asian nations may look to the United States for a similar deal.
3. Social influence is illustrated by Gabbay's model of opinion change, although in his model all neighboring agents have the same influence regardless of their success.
4. Selection deals with a changing population of agents. Suppose there are many variants of Shiite Islam, each advocated by certain clerics. Those variants that attracted followers would survive, while those that failed to attract followers would decline and eventually change or disappear. This process could be modeled like a Darwinian process of evolution by natural selection. The Axelrod-Hammond model of ethnocentric behavior is an example of a model that uses selection, with more successful agents getting more offspring.

4. COMPARISON OF MODELING APPROACHES

Appendix I compares the modeling approaches in terms of six criteria. The table does not include mathematical rigor that is discussed below. The table also does not include predictive value since none of the modeling approaches, in my opinion, hold much promise for substantial predictive value in the context of security issues in central Asia.¹² Finally, the table does not include heuristic value because one model using the a particular approach might have a great deal of heuristic value, but another model using the same approach might have very little. Among the factors that affect the heuristic value of a model is its transparency.

Transparency is one of the criteria of appropriateness included in the comparison of modeling approaches provided in Appendix I, but some elaboration on its meaning and value may be helpful at this point. An analyst is unlikely to give credence to the results of a model unless the assumptions it uses are clear. An analyst is likely to be especially skeptical of any model that yields a result that might be used to support a "prediction" or policy that is contrary to the analyst's own prior judgment. In such cases, the analyst will look for places where the results might have been "built into" the model to support the modeler's own policy preconceptions. If the analyst can not completely understand the

¹²The Workshop did not consider macroeconomic models. Such models might hold promise for predicting some strictly economic variables in central Asia.

workings of the model, he or she might dismiss any unexpected or unwanted results of the model. For this reason, transparency is a valuable attribute of any model designed for use by analysts or decision makers.

The extent to which a particular model is transparent to a user depends on four factors: the methodological training of the user, the complexity of the model itself, the quality of the user interface, and the experience of the user with this particular model or with others very similar to it. If the user understands differential equations, and/or has had experience with coupled oscillators, then a simple dynamical systems model will be transparent. Thus, analysts with backgrounds in such fields as physics, chemistry, physiology and electronics are likely to be able to understand and appreciate models using dynamical systems. Unfortunately, a user with little mathematical background, or with only statistical training, will have a difficult time understanding a system of nonlinear differential equations well enough to be able to assess whether or not some unexpected result is worth serious consideration.

Agent-based models typically require little or no mathematical training to appreciate since the assumptions can usually be stated in logical form such as “if A happens then B happens.” An assumption of an agent-based model can be simple enough to be transparent, and still have thousands of agents whose interactions and adaptations generate complex results. This is the reason agent-based models are widely used for the study of complexity, and especially for the study of Complex Adaptive Systems. For example, the Axelrod-Hammond model has thousands of agents, but only three mechanisms: migration, reproduction, and death. The migration and death mechanisms are almost trivial, and the details of the reproduction mechanism are not too hard to grasp even without mathematical aptitude or training.

In comparing the appropriateness of an agent-based approach to an equation-based approach a major consideration is whether the issues to be explored are best treated as choices by individual agents, or as forces that affect all agents of the same kind in the same way. According to an excellent article by Parunak and Riolo (1998) on the comparison between agent-based models (ABMs) and equation-based models (EBMs), “The difference in representational focus between [an] ABM and [an] EBM has consequences for how the models are modularized. EBMs represent the system as a set of equations that relates observables whose values are affected by the actions of multiple individuals, so the natural representation often crosses boundaries among individuals. ABMs represent the internal behavior of each individual [agent].” Actually, the particular EBM proposed by Gabbay (2003) is very unusual in that its equations are replicated for each agent, making it very much like an agent-based model. More typically, EBMs are very much like formal versions of a system dynamic model in which each causal arrow is represented as an equation, and each node represents a different group property (such as per capita income) as the average over all individuals. Thus, an EBM is typically more appropriate if all the agents of a certain kind can be represented by a single “average” agent. If not, then ABMs are likely to be more appropriate since each agent in an ABM can have its own traits (such as “color”), and its own strategy.

The Workshop considered two kinds of agent-based models, distinguished by how much detail is required to represent a single agent. The agents in the Axelrod-Hammond model are very light in that they have each only three traits, and make a binary choice that takes account of only one piece of information, namely the

neighboring agent's "color." In contrast, the Policrash model tool kit was designed primarily for models that can only be developed by many days of interviews with a subject matter expert (Reynolds and Dixon, 2003, p.5). In fact, a design criterion of Policrash is to help users manage massive amounts of information. The Policrash system is able to handle heavy agents that can take into account a wide variety of information, a large number "if-then" rules for decision making, and great deal of memory. In sum, light agents are relatively good for heuristic value and for transparency, while heavy agents are relatively good for embodying substantial amounts of subject matter expertise.

Finally, consider the question of mathematical rigor. As Gabbay's report points out, dynamical systems can be thought of as the very mathematics of stability. Although a set of partial differential equations that used to formulate a partial dynamical systems model typically can not be solved explicitly, it is still sometimes possible to prove rigorously some abstract propositions about a particular model. For example, it may be possible to prove that a particular model will produce a certain kind of path dependence called hysteretic (Gabbay, p. 8). The extent to which such abstract properties are helpful to the user remains an open question.

Unfortunately, only the relative simple equation-based models can be solved with mathematical rigor. In addition, virtually all agent-based models are impossible to solve with rigor. For both equation-based models that can not be solved explicitly, and for virtually all agent-based models computer simulation provides a way of generating results for specified conditions. If some of the parameters are not known with precision, the models have to be run over a range of plausible values of these parameters. If there is a stochastic (random) element in the model, the analysis of the outcomes typically requires simple statistical analysis.

5. CONCLUSIONS

a. Making the Whole Greater than the Sum of the Parts

There are several different methods for developing a system of models to address security issues of central Asia. To explain the alternative of linking different models, let us look at the simplest case in which there are only two models, A and B. For the sake of concreteness, let us suppose that model A deals with the mobilization and suppression of oppositional political activity, and B is some other model, perhaps about the same thing.

Method 1. Models A and B can provide alternative perspectives for the same phenomena. For example, Model A might use mainly political variables to explore and analyze oppositional political activity, while Model B might try to account for the same thing using mainly economic variables such as tax rates and the government's ability to pay its troops. The different perspectives taken by the two models might provide a richer understanding of the issue in question than either could provide alone. Even more valuable would be if one could identify the conditions under which the two models led to different conclusions. For example, if a variant of the scenario included financial subsidies from the U.S., the economic approach used in Model B might predict the survival of the government once it could pay its troops. On the other hand, the political perspective of Model A might suggest that if some elements of the population viewed the

acceptance of outside support as delegitimizing the government, then accepting the subsidy might actually hurt the government. Using both perspectives to analyze the same historical, current, or hypothetical scenario can provide insight into the implication of viewing the situation one way (say in political terms) versus viewing it in another way (say in economic terms). In this way, a user comfortable with the economic perspective might come to appreciate when the political perspective might give a different result. Likewise, a user comfortable with the political perspective might come to appreciate when the economic perspective offers new insights, and sometimes even different “predictions”. From a management point of view, using alternative models to explore the same issue requires that the models be compatible in two different ways, namely their time scales, and their definition of the issue at hand.

Method 2. The output of model A can be used as the input to model B. For example, in a given scenario model A might predict that there would be an escalating wave of anti-government activity, resulting in the government’s need to choose between accommodation and suppression. Now suppose that model B dealt with the internal dynamics of the ruling elites. The output (“predictions”) from model A could then be used as inputs to model B. In this way, a model of oppositional politics could provide the initial conditions for a model of stresses within the governing elite. To make this work, at least some of the output variables in model A would have to match with corresponding variables in Model B. For example, model A might predict the trend in the magnitude of public demonstrations, and model B would use this information on demonstrations as part of the input to a model of elite decision making under stress. From a management point of view, the two modelers or teams of modelers would have to agree on the meaning of each linking variable, such as magnitude of public demonstrations). In our example, this would mean that the magnitude of public opposition was measured in the same way (say as the log of the number of demonstrators, or as the product of the demonstration’s numbers and intensity). Using one team’s model as input into another team’s model makes coordination relatively easy. The gain would be a combined sequential model that encompassed more than either team alone was modeling.

Using a data-driven model to provide input to a simulation model is another promising possibility. For example, the strength of a hypothesized relationship could be measured using a data-driven model. If the results are statistically significant, the data-driven model would help establish the existence of the hypothesized relationship, and perhaps even supply an empirical estimate of the value of the corresponding parameter in the simulation model.¹³ On the other hand, if the result of the data analysis were no different than chance, there may not be any causal link between the two variables. To return to the example of a potential coup in Tajikistan, if the rank of the leader of relevant historical coup attempts had no significant correlation with the probability of success after controlling for other factors, then the rank of the leader may not matter in the current situation. In that case, the simulation model should not include rank as a cause of success, unless there were good reasons to think that the current situation in Tajikistan is sufficiently atypical of the set of relevant military coups in the database.

An alternative use a data-driven model is to identify just the one or two military coups in the database whose antecedents most resemble the potential military coup the

¹³ In practice, a statistical model usually “controls for” possible confounding effects such as the type of government at which the coup is directed.

user is interested in understanding. In effect, this process would identify the closest analogy to the current situation. The construction of the simulation model could then benefit from an understanding of the driving forces in the historical case since the historical case was selected as the one most similar to the case at hand.

Method 3. Models A and B can be designed so that they can be integrated. The most ambitious method is to develop a single unified outline of a comprehensive model, and assign different parts to different teams. Under this method, the sponsor would probably have to assign one team as the leader to guarantee that the separate pieces would fit together to make a single model. This method of linking two or more models is substantially more ambitious than merely requiring that one model's output to serve as another model's input, since the comprehensive method requires that the parts work together in many different ways. One way to manage the interfaces would be to have one team model a process that was represented as a single causal element of a larger model. Another way to keep the management relatively simple would be dividing the comprehensive model into two or more virtually autonomous parts that interacted through a single feedback loop. For example, the comprehensive model might be one large feedback loop in which one model represented part of the loop, and the other model represented the rest. Each of the shared variables would have to be coordinated. The real difficulty would be in designing the comprehensive model so that it could be broken into several almost autonomous parts.

b. Challenges for Modeling Security Issues of Central Asia

Here are several observations and recommendations for the next phase of the project on Modeling Security Issues in central Asia. The recommendations are based on the September 2003 workshop, and the reports written before and after that workshop.

1. The workshop demonstrated that the security problems of central Asia are amenable to modeling, at least in principle. In addition, the breadth and adaptability of the models at the workshop suggest that similar techniques could be applied in the future to the different security issues in other geographic regions.
2. The state of the art of modeling security issues is still largely based on academic goals and standards. The project is not sufficiently advanced to be judged by how helpful the models are to users. Therefore, the next phase of the project should be viewed as a continuation of the pilot phase.
3. It seems to me that the other researchers demonstrated at the Workshop and in their papers that each had competence in, or even understanding of, only one approach to modeling. Therefore, it might be best to accept these limits for the next phase by letting each modeler or team use the type with which they are adept.
4. There is a variety of methods for getting different models or even modeling approaches to complement each other. If this is to happen, however, some care needs to be taken that the various modeling efforts will be compatible in the required ways. If the sponsor or lead team

desires help to assure that the total result is worth more than the sum of the parts, I am available for consultation.

5. The modelers should be encouraged to develop good user interfaces so that the users can easily change the input to the model, and immediately see the output in visual form. This would allow the users to play with the model and try out a number of “what if” scenarios.
6. The modelers should be encouraged to make their models as transparent as possible so that users can understand not only the results, but also the reasons for results. Otherwise, users are likely to dismiss any results that differ from their current understanding as having been “built into” the model.
7. The next phase of the project should explore the many different ways models can prove helpful to analysts. The Workshop focused mainly on using models help anticipate the distribution and likelihood of future possibilities. Prediction of the timing and nature of specific events is far beyond the state of the art for the security issues of central Asia. Among the goals that are more realistic in the short run are helping an analyst see relationships, such as feedback loops, that might not have been salient before; helping several analysts communicate with each other on the causal beliefs that underlie their different understanding of a situation; and identifying new intelligence requirements.

Appendix I
 Appropriateness of Modeling Methodologies:
 Low, Medium or High on Six Criteria

	Constru- c-tion time	User Pre- requisites	Learning Time	Flexi- bility	Repertory size	Trans- parency
Game Theory	L	L-H	L-H	M	H	L-H
Agent-Based Modeling with Light Agents	M	L	L	M	M	H
Agent Based Modeling with Heavy Agents	H	M	M	H	L	M
Dynamical Systems	M	H	H	M	M	M
Systems Dynam. Diagrams	L	L	L	H	L	H
Data-driven Models	M*	M	L	M*	M	H

*Excluding construction or expansion of the database.

The Criteria

Ideally, the first three criteria would be Low, and the last three criteria would be High.

Construction Time. Time and effort needed for a modeler skilled in this methodology to build a useful model with input from users.

User Prerequisites. Amount of technical background needed by the user to understand as well as use the model.

Learning Time. Time and effort for a typical user with the necessary prerequisites needs to learn a specific model.

Flexibility. Ease with which the modeler can modify the model to incorporate a new variable.

Repertory size. The number of published models of this type with features that could be adapted for use as part of a model on issues relevant to security in central Asia.

Transparency. The ease with which the user can discover anything in the model that might bias the results. See also Section 4.

Other criteria considered in the text are mathematical rigor, predictive value and heuristic value.

Appendix II

Nine Relevant Issues Already Addressed by Agent-Based Models

1. Politics of Closed Regimes

The regimes of the central Asian nations are authoritarian. Therefore understanding the politics within the small ruling elite is important for understanding what a government might do in a given situation. Rick Riolo, Ravinder Bhavnani and David Becker (2003) have an agent-based model of the elite politics within a closed regime such as Syria. Their model includes an example of a very good user interface that allows an intelligence analyst to “play” with the model.

2. Institutionalization of Identity

An important theme in central Asia is the competition over loyalties. These loyalties are often driven by how individuals see themselves. For example, an individual may have competing identities as an Islamic fundamentalist, a citizen of their country, and an elder of their tribe. It is important to know how these alternative (or complementary) identities are mobilized or minimized. Ian Lustick has studied the factors that evoke and transform personal identity, and the institutional implications of these processes. A published example is his model of collective identity formation (Lustick 2000). This agent-based model builds on constructivist theory by representing agents as having a repertoire of identities. The agents are located in a two-dimensional space, and are influenced by their neighbors. Lustick’s model includes entrepreneurs represented as agents who have a relatively large influence on their neighbors, and a large repertoire of possible identities. Among the issues Lustick explores is the effect of exclusivist identities on other members of the population.

3. Nationality Formation

The nations of central Asia face struggles over the very definition of state and society. In other words, they have a problem of building national identities. Lars-Eric Cederman (1995) has an agent-based model of how competing identities develop into national identities that are more or less inclusive. His model explores center-periphery relations, political mobilization, and the interaction between material and cultural factors. For an equation-based model dealing with the same issue, see Arfi (2000).

4. Instigation of Communal Violence

In south Asia, central Africa, and the Balkans one ethnic/religious group has attacked and murdered members of another such group. Salient examples are the Hindu-Moslem riots that have occurred in a number of Indian cities, the massacres in Rwanda and Burundi, and the “ethnic cleansing” in the former Yugoslavia. The potential for this type of violence exists in central Asia as well. Ravi Bhavnani (2003) has developed two different agent-based models aimed at understanding the dynamics of communal violence. One model deals with the dynamics of mobilization within one country such as Burundi. The other deals with how the communal violence in one country can affect the likelihood and severity of communal violence in a neighboring country such as Rwanda.

5. Suppression of Rebellion and Communal Violence

Joshua Epstein (2002) has two related agent-based models dealing with the suppression of violence within a nation. The first deals with the dynamics of a central government trying to suppress a decentralized rebellion. The second deals with the government trying to suppress violence between two ethnic groups. The paper for the Workshop by Reynolds and Dixon (2003) sketched how to go about building a two-actor model of regime suppression of the populace.

6. Escalation from Demonstrations to Revolution

Susanne Lohmann (1994) has an agent-based model that explores when anti-government demonstrations will grow until they overwhelm the government’s will or ability to resist. Her empirical example is the series of escalating weekly demonstrations in Leipzig that were instrumental in the overthrow of the German Democratic Republic in 1991. The model deals with how each individual changes his or her opinion about the risks of demonstrating based on how many others demonstrated last time. The resulting “information cascade” can take off once enough people peacefully demonstrate to convince others that the risks of demonstrating are low enough for them to join the next round. For what amounts to an equation-based model of the same issue, see Granovetter (1978).

7. Policing Corruption

Where corruption is endemic the legitimacy and authority of state is undermined, and the economic development is hindered. The roles of drugs and smuggling in many central Asian countries raise the importance of corruption to the level of national security. Ross Hammond (2000) has an agent-based model of fighting corruption. The dynamics of the model deal with people who might try to bribe an official if the chances of being

caught are not too great, and with police who can use a variety of tactics to suppress bribery. An early version of this model has been described in the *Atlantic* (Rauch, 2001).

8. Intergroup Competition over Territory

In central Asia, with its limited agricultural resources, competition for land and water can be intense. The basis for this competition might be tribal, ethnic, religious or national. Because the Axelrod-Hammond model (2003) described earlier in this report uses a geographic map, it can be used to explore intergroup competition for territory.

9. Alignments Between And Within Nations

In central Asia, alignments are important at two levels: interstate and subnational. The important interstate alignments deal not only with relationships among the central Asian states themselves, but also between each of them and the outside states with an interest in central Asia such as China, Russia, and the United States. Alignments between these nations might take the form of traditional security alliances, but it is more likely that alignments between nations will not strong or durable, and may even be overlapping. An example of an alignment between nations is the Shanghai Cooperation Organization that brings together China, Russia, Kazakhstan, Kyrgyzstan, Tajikistan and Uzbekistan. Alignments at the subnational level are also important in central Asia. For example, which local leaders align with which others can have a major impact on the politics of a nation. Moreover, alignments between groups across different nations (for example between drug cartels or between leaders of Islamic organizations) can have a major impact on the politics and stability of one or more countries. Robert Axelrod and Scott Bennett (1993) provide an agent-based model to predict which actors (at any given level such as either nations or religious groups) will be most likely to align together. While almost any alignment of nations in and relevant to central Asia will make strange bedfellows, the Axelrod-Bennett predicts that the alignments that actually do form will tend to *minimize* the “strangeness of bedfellows.” For example, the alignment of the Shanghai Cooperation Organization includes China, Russia and four central Asian states, but deliberately leaves out the United States.

References Cited

- Arfi, Badredine, “‘Spontaneous’ Interethnic Order: The Emergence of Collective, Path-Dependent Cooperation.” *International Studies Quarterly*, vol. 44, 2000, 563-590.
- Atkins, Muriel, “Review: MIT Workshop on Central Asia Security,” July 2003.
- Axelrod, Robert, *The Evolution of Cooperation* (New York: Basic Books, 1984).
- _____ (ed.), *Structure of Decision* (Princeton, NJ: Princeton University Press, 1976).
- _____ and D. Scott Bennett, "A Landscape Theory of Aggregation," *British Journal of Political Science*, vol. 23, 1993, pp. 211-33.
- _____. “Advancing the Art of Simulation in the Social Sciences,” in Rosario Conte, Rainer Hegselmann and Pietro Terna (eds.), *Simulating Social Phenomena* (Berlin: Springer, 1997), pp. 21-40. Reprinted in Robert Axelrod, *The Complexity of Cooperation* (NY: Basic Books, 1997). Updated 2003 version is available at <http://www-personal.umich.edu/~axe/research/AdvancingArtSim2003.pdf>
- _____ and Michael D. Cohen, "Coping with Complexity: The Adaptive Value of Changing Utility" *American Economic Review*, 74 (March 1984), pp. 30-42.
- _____ and Michael D. Cohen, *Harnessing Complexity* (Basic Books, 2001).
- _____ and Ross A. Hammond, “The Evolution of Ethnocentric Behavior” Paper presented at the annual meeting of the Midwest Political Science Convention, Chicago, IL, April 2003. Available at http://www-personal.umich.edu/~axe/research/AxHamm_Ethno.pdf
- Bhavnani, Ravinder, “Essays on Intraethnic and Interethnic Violence,” Ph.D. Dissertation, Department of Political Science, University of Michigan, Ann Arbor, 2003.
- Cederman, Lars-Erik, “Competing Identities: An Ecological Model of Nationality Formation,” *European Journal of International Relations*, vol. 1, 1995, pp 331-365. Reprinted in Lars-Erik Cederman, 1997. *Emergent Actors in World Politics: How Nations Develop and Dissolve* (Princeton, NJ: Princeton University Press, 1995), Chapter 8.
- Choucri, Nazli Robert C. North, and Susumu Yamakage. 1992. *The Challenge of Japan Before World War II and After: A Study of National Growth and Expansion* (London: Routledge, 1992).
- Dixit, Avinash and Susan Skeath, *Games of Strategy* (New York: Norton, 1999).
- Epstein, Joshua M., ”Modeling Civil Violence: An agent-based Computational Approach,” *Proceedings of the National Academy of Sciences*, vol. 99 (Suppl. 3): 2002, pp. 7243-7250. Available by license at http://www.pnas.org/cgi/reprint/99/suppl_3/7243.pdf
- Gabbay, Michael. “Nonlinear Dynamic Modeling and Central Asia,” 2003.
- Granovetter, Mark, 1978. "Threshold Models of Collective Behavior," *American Sociological Review*, vol. 83, 1994, pp. 1420-42.
- Hammond, Ross A. “Endogenous Transition Dynamics in Corruption: An Agent-Based Computer Model,” Brookings Institution, Center on Social and Economic Dynamics, Working Paper 19, December 2000. Available at <http://brookings.edu/dybdocroot/es/dynamics/papers/ross/ross.pdf>

- Lohmann, Susanne, "The Dynamics of Informational Cascades: The Monday Demonstration in Leipzig, East Germany, 1989-91," *World Politics*, vol. 47:42-101.
- Lustick Ian S., "Counterfactuals and Central Asian Futures," October 11, 2003.
- March, James G., "Bounded Rationality, Ambiguity and the Engineering of Choice," *Bell Journal of Economics*, vol. 9. 1978, pp. 587-608.
- Parunak, H. Van Dyke and Rick L. Riolo, "Agent-Based vs. Equation-Based Modeling: A Case Study and Users Guide," in *Proceedings of the Workshop on Modeling Agent-Based Systems (MABS98)*, 1998, p 11. Available at <http://www.pscs.umich.edu/education/CSCS-courses/cscs530/W02/Txt/mabs98.pdf>.
- Rauch, Jonathan, "Seeing Around Corners," *Atlantic*, April 2001. Available at <http://www.theatlantic.com/issues/2002/04/rauch.htm>
- Reynolds, William N. and David S. Dixon, "Modeling Central Asia: Heavy Agents, Causalities and Policrash," October, 2003.
- Riolo, Rick, Ravinder Bhavnani and David Becker, "A Hybrid Model of Decision Making in Closed Regimes," in *Social Agents: Ecology, Exchange and Evolution*, Proceedings of the Agent 2002 Conference. pp. 283-304. Available at <http://www.agent2003.anl.gov/proceedings/2002.pdf>
- Saeed, Khalid, "Preliminary Workshop on Security Issues in Central Asia," August 2003.
- Schrodt, Philip A. "Data-Driven Models for the Analysis of International Security in Central Asia." 2003.
- Simon, Herbert A. *Models of Man* (New York: Wiley, 1957).