Agent-Based Computational Economics: Overview and Brief History [1]

Leigh Tesfatsion
Research Professor and Professor Emerita of Economics
Courtesy Research Professor of Electrical & Computer Engineering,
Iowa State University, Ames, IA 50011-1070 USA
Homepage: http://www2.econ.iastate.edu/tesfatsi/
Email: tesfatsi@iastate.edu

Abstract:

Key Words: Agent-based computational economics, modeling principles, history

JEL Codes: C6

Latest Revision: 11 February 2021

1 Introduction

The term *agent-based modeling (ABM)* refers to a class of modeling approaches designed for the study of systems whose dynamics are driven by successive open-ended interactions among heterogeneous entities. Such systems range from the particle systems studied in physics to the coupled human and natural systems studied in socio-ecology. Consequently, the pathways leading to the development of ABM cannot be depicted as a tree, or even as a gnarly bush, but instead must be envisioned as a forest of diverse trees supported by a complex interconnected network of roots.

Many previous authors have ably explored the various origins and meanings of ABM; see, for example, Arthur [1], Axtell and Farmer [3], Chen [5, 6, 7], Epstein [12], Gallegati [14], Kirman [17], Railsback and Grimm [20], and Wilensky and Rand [38]. The purpose of this essay is much more modest in scope: namely, to focus on the historical origin and development of one particular variant of ABM called *Agent-based Computational Economics (ACE)*.

ACE was named by me in Ames, Iowa, in August 1996, following my participation in a satellite meeting held at the end of the Second International Conference on Computing in Economics and Finance, in Geneva, Switzerland, June 26-28, 1996. A key purpose of this satellite meeting was to discuss how agent-based modeling could be promoted within the economics profession. I came away from this meeting determined to develop a website resource repository devoted to this objective, and I needed a name that would clearly convey to other economists that this approach differed in essential regards from then-standard economic modeling approaches.

[1] Invited perspective for the Shu-Heng Chen Festschrift. Portions of Sections 2-5 are adapted from Tesfatsion [27].

Through the years, however, I have come to realize that my conception of ACE modeling also differs in essential regards from other variants of ABM used by researchers in general and by economists in particular. For example, ACE agents can include physical, social, and institutional entities. Moreover, ACE models are *fully agent based*; roughly, this means that each event occurring within an ACE model must arise entirely from the actions of modeled agents, conditional on initially specified agent states. Thus, the current ACE website [32] now provides a precise definition of ACE modeling, expressed in the form of seven specific modeling principles.

Section 2 of this essay presents and interprets these seven ACE modeling principles. Major misconceptions expressed by some commentators about the ability of ACE agents to embody wide ranges of rationality and different forms of stochasticity are addressed in Sections 3 and 4, respectively. Four major strands of ACE research, delineated by objective, are identified in Section 5. Section 6 highlights potential major benefits of the ACE requirement that ACE models be fully agent based. A brief history tracing the origin and conceptualization of ACE modeling, as expressed through the ACE website [32] and accompanying online ACE news notes [33] distributed from 1996 through 2017, is given in the final Section 7.

2 ACE Modeling Principles

Roughly defined, ACE is the computational modeling of economic processes (including whole economies) as open-ended dynamic systems of interacting agents. The stress on *open-ended* dynamics fundamentally distinguishes ACE from all other currently mainstream economic modeling approaches.

More precisely, the ACE modeling approach is characterized by the seven modeling principles listed below. These principles reflect the primary goal of many agent-based modelers: namely, to be able to study real-world dynamic systems as historically unfolding sequences of events.

(MP1) *Agent Definition:* An *agent* is a software entity within a computationally constructed world capable of acting over time on the basis of its own *state*, i.e., its own internal data, attributes, and methods.
(MP2) *Agent Scope:* Agents can represent individuals, social groupings, institutions, biological entities, and/or physical entities.
(MP3) *Agent Local Constructivity:* The action of an agent at any given time is determined as a function of the agent's own state at that time.
(MP4) *Agent Autonomy:* Coordination of agent interactions cannot be externally imposed by means of free-floating restrictions, i.e., restrictions not embodied within agent states.
(MP5) *System Constructivity:* The state of the modeled system at any given time is determined by the ensemble of agent states at that time.
(MP6) *System Historicity:* Given initial agent states, all subsequent events in the modeled system are determined solely by agent interactions.

(MP7) *Modeler as Culture-Dish Experimenter:* The role of the modeler is limited to the setting of initial agent states and to the non-perturbational observation, analysis, and reporting of model outcomes.

Considered as a collective whole, modeling principles (MP1)–(MP7) embody the idea that an ACE model is a computational laboratory permitting users to explore how changes in initial conditions affect outcomes in a modeled dynamic system over time. This exploration process is analogous to biological experimentation with cultures in petri dishes. A user sets initial conditions for a modeled dynamic system in accordance with some purpose at hand. The user then steps back, and the modeled dynamic system thereafter runs forward through time as a virtual world whose dynamics are driven by the interactions of its constituent agents.

From a mathematical point of view, modeling principles (MP1)–(MP7) imply that ACE models are state-space models in initial value form. More precisely, an ACE model specifies how an ensemble of agent states dynamically evolves, starting from an initially given ensemble of agent states. However, these modeling principles further require an ACE model to exhibit essential real-world characteristics: namely, agent local constructivity, agent autonomy, system constructivity, and system historicity.

Modern economic theory also relies heavily on state-space models. However, these models typically incorporate modeler-imposed rationality, optimality, and equilibrium conditions that could not (or would not) be met by locally constructive agents interacting within an historical process. For example, rational expectations assumptions require ex ante agent expectations to be consistent with ex post model outcomes. Consequently, the derivation of rational expectations solutions is a global fixed-point problem that requires the simultaneous consideration of all modeled time periods without regard for local constructivity and historical process constraints.

Modeling principles (MP1)–(MP7) also permit ACE to be distinguished more clearly and carefully from other variants of agent-based modeling [7, Chpts. 1-2], and from important related modeling approaches such as econophysics [8], system dynamics [19], and microsimulation [21].

3 ACE Agent Rationality

For ACE researchers, as for economists in general, the modeling of decision-making agents is a primary concern. Here is it important to correct a major misconception still being expressed by some commentators uninformed about the powerful capabilities of modern computational tools: namely, the misconception that ACE decision-making agents cannot be as rational (or irrational) as real people.

To the contrary, the constraints on agent decision-making implied by the modeling principles (MP1)-(MP7) set out in Section 2 are constraints inherent in every real-world dynamic system. As demonstrated concretely in [22], the decision methods used by ACE agents can range from simple behavioral rules to sophisticated anticipatory learning algorithms for the approximate achievement of intertemporal objectives.

Extensive annotated pointers to reference materials on the implementation of learning and decision methods for ACE agents can be accessed at the ACE learning research repository [31]. Learning methods for computational agents covered by the materials posted at this repository include:

- reactive reinforcement learning, e.g., Roth-Erev reinforcement learning;
- belief-based learning, e.g., fictitious play, Camerer/Ho's EWA algorithm;
- anticipatory learning, e.g., Q-learning, adaptive dynamic programmming;
- evolutionary learning, e.g., genetic algorithms, genetic programming;
- connectionist learning, e.g. associative memory learning, artificial neural networks with multiple hidden layers (deep learning).

4 ACE Agent Stochasticity

Stochastic aspects are easily represented within ACE models. Agent data can include realizations for real-world random events, agent attributes can include beliefs based on probabilistic assessments, and agent methods can include *pseudo-random number generators (PRNGs)*.

A PRNG is a deterministic algorithm, initialized by a seed value, that is able to generate numerical sequences whose properties mimic the properties of random number sequences. PRNGs can be included among the methods of decision-making agents, thus permitting these agents to "randomize" their behaviors. For example, a decision-making agent can use PRNGs to choose among equally preferred actions or action delays, to construct mixed strategies in game situations to avoid exploitable predictability, and/or to induce perturbations in action routines in order to explore new action possibilities.

PRNGs can also be included among the methods of other types of agents, such as physical or biological agents, in order to model stochastic processes external to decision-making agents. For example, a Weather agent might use an empirically-based PRNG to generate a weather pattern for its computational world during each simulated year that in turn affects the actions of decision-making agents.

An additional important point is that ACE agents are *encapsulated*. That is, each agent's internal data, attributes, and/or methods can be partially or completely hidden from other agents, either by deliberate agent choice or by initial modeler specification. Thus, agents can be unpredictable to one another even if they make no use of real-world random event realizations, probabilistic assessments, or PRNGs.

Finally, ACE models are required to be *dynamically complete* virtual worlds. Consequently, ACE modelers must explicitly identify the *source* agents for any simulated stochastic shocks affecting events within their modeled worlds, not simply the *sink* agents that are affected by these shocks. The source agents can be individuals, social groupings, institutions, biological entities, and/or physical entities; and they can range from simply described "stub" agents to agents characterized by carefully articulated data, attributes, and methods.

Dynamic completeness thus forces ACE modelers to think carefully about the intended empirical referents for any simulated stochastic shock terms. This, in turn, should help to reduce or eliminate a reliance on ad hoc external shock terms as the source of dynamic persistence and the drivers of dynamic interactions.

5 ACE Research Objectives

Scientists seek to understand how real-world systems work, and how real-world systems could work better. The ACE modeling methodology facilitates the ability of researchers to pursue these goals specifically for economic systems.

One ACE research objective is *understanding of persistently observed empirical regularities*. An ACE model capable of generating an observed empirical regularity based on empirically credible agent specifications provides a candidate explanation for this regularity. As elaborated at [30], the empirical validation of this ACE model should ideally encompass four distinct aspects: validation of initially specified agent attributes; validation of initially specified agent methods; descriptive output validation (in-sample fitting); and predictive output validation (out-of-sample forecasting).

A second ACE research objective is *normative design*. How might existing economic systems be modified to work better? The ACE approach to normative design is akin to filling a bucket with water to determine if it leaks. An ACE researcher constructs a virtual world capturing salient features of a system operating under a proposed design. The researcher identifies a range of initial agent specifications of interest, including seed values for agent PRNG methods. For each such specification the ACE researcher permits the virtual world to develop over simulated time, driven solely by agent interactions. Recorded outcomes are then used to evaluate design performance.

A primary issue for ACE normative design researchers is the extent to which resulting outcomes are efficient, fair, and orderly, despite possible attempts by strategic decision-making agents to game the design for personal advantage. A related issue is a cautionary concern for adverse unintended consequences. *Optimal* design might not always be a realistic goal, especially for large complex systems; but ACE models can facilitate *robust* design for increased system resiliency, a goal that is both feasible and highly desirable.

A third ACE research objective is *qualitative insight and theory generation*. How can ACE models be used to study the potential future behavior of an existing economic system? Ideally, what is needed is the system's "phase portrait," i.e., a representation of its potential state trajectories starting from all feasible initial states. Phase portraits reveal not only the possible existence of equilibria but also the basins of attraction for any such equilibria. Phase portraits thus help to clarify which regions of a system's state space are credibly reachable, hence of empirical interest, and which are not. An ACE modeling of an economic system can be used to conduct batched runs starting from multiple feasible agent state configurations, thus providing a rough approximation of the system's phase portrait.

A fourth ACE research objective is *method/tool advancement*. How best to provide ACE researchers with the methods and tools they need to undertake theoretical studies of dynamic economic systems through systematic sensitivity studies, and to examine the compatibility of sensitivity-generated theories with real-world data? ACE researchers are exploring a variety of ways to address this objective ranging from careful consideration of methodological principles to the practical development of programming, visualization, and empirical validation tools.

6 ACE: A Mathematics for the Real World?

Science seeks to understand how real-world systems work. Models devised for scientific purposes must always simplify reality. However, ideally, scientists should be permitted to tailor these simplifications to purposes at hand. *Scientists should not be forced to distort reality in specific predetermined ways in order to apply a modeling approach.*

A key goal motivating the development in [27] of the seven modeling principles (MP1)-(MP7) set out in Section 2 was to adhere to this modeling precept for the study of real-world economic systems. An interesting question is the extent to which modeling principles (MP1)-(MP7) achieve this goal for real-world systems *in general*, not simply for real-world *economic* systems.

Any model adhering to (MP1)-(MP7) is an open-ended dynamic system of interacting agents representing physical, biological, and/or social entities, each characterized by its own state (data, attributes, methods). The interactions of these agents drive the dynamics of the system, starting from modeler-configured initial conditions. As a result of these interactions:

- each agent locally experiences "time" as an unfolding sequence of events;
- the dimension and content of agent states can change;
- agents can subsume other agents as components;
- agents can break apart into smaller component agents;
- new agents can be created; and
- existing agents can be destroyed.

This modeling flexibility permits the representation of a universe capable of supporting the evolution of perceptive self-conscious life.

Examples of state-changes for real-world agents include: changes in sensed surroundings; changes in physical attributes; belief changes; and belief-induced changes in action rules. Examples of real-world agent subsumption include: the formation of molecules through atomic bonding; the parasitism of one organism by another; the transition from prokaryotic to eukaryotic forms of organisms; the hiring of employees by corporate firms; and the acquisition of new members by existing organizations.

Examples of real-world agent creation and destruction include: volcanic eruptions; natural birth and death; the invention and obsolescence of products; and the establishment and disbanding of organizations. Creation and destruction events for populations of agents can be computationally modeled by means of evolutionary algorithms taking various forms.

Note, in particular, that models adhering to the modeling principles (MP1)-(MP7) set out in Section 2 permit the study of real-world systems that evolve from modeler-configured initial conditions with:

- no fixed "space" apart from persistent spatial agents (if any) that a modeler initially configures;
- no fixed "time process" apart from persistent event-scheduler agents (if any) that a modeler initially configures; and
- no fixed rules apart from persistent agent methods (if any) that a modeler initially configures.

The ability to model real-world systems without having to presuppose a fixed externally given "space" or "time process" permits the study of open perplexing questions in physics regarding the existence (or not) of these concepts as fundamental aspects of the physical universe.

Persistent agent methods that a researcher might want to initially configure for a modeled real-world system include methods that support self-organization and natural selection processes. These types of processes appear to be a basic driver of real-world agent interactions at all levels of agent encapsulation that humans can perceive. An interesting question is whether they also drive agent interactions at levels below human perception, such as at a quantum level.

7 Brief History

I first encountered agent-based modeling in a delightful 1983 *Scientific American* essay [15] by Douglas Hofstadter celebrating Bob Axelrod's work on *Iterated Prisoner's Dilemma (IPD)* tournaments [2]. Axelrod's key idea was to specify an initial population of computer programs, each implementing an IPD strategy, and to then let these programs engage in repeated round-robin play of PD games with or without evolution of their initially programmed strategies. The goal was to see under what conditions, and to what extent, cooperative play might be induced.

Two aspects of Axelrod's tournaments stood out for me in comparison with standard economic modeling approaches at the time. First, even in deterministic form, the tournaments involved sufficiently complex interactions that it was difficult to deduce long-run outcomes from initial conditions. Thus, as in real-world biological experiments with cultures in petri dishes, researchers could be "surprised" by tournament outcomes. Second, in repeated play, Axelrod's agents (computer programs) exhibited induced "social" behaviors with interesting "life-like" characteristics, such as trust, deception, reciprocity, and stance towards strangers.

In the mid-1980s I was heavily involved in the development of flexible least squares, a diagnostic method for model misspecification, as well as adaptive computation methods for nonlinear systems. It thus took me some time to redirect my research towards an exploration of Axelrod's intriguing agent-based approach for the possible study of economic systems.

Indeed, my first "agent-based" paper was not an economics study at all. Rather, it was a 1991 co-authored study [16] with Bob Kalaba on adaptive homotopy continuation for the solution of nonlinear systems of equations. In this study we replaced the usual homotopy continuation parameter traversing the real line from 0 to 1 by a "smart agent" able to adaptively makes its way by trial and error from 0+0i to 1+0i in the complex plane, avoiding regions where computation becomes ill-conditioned due to nearby singularities or bifurcation points.

During the early-to-mid 1990s I increasingly participated in ABM-related panel sessions at formally organized conferences. This participation included: the Artificial Life III Conference sponsored by the Santa Fe Institute, held at the Sweeney Center, Santa Fe, New Mexico, in June 1992; a session at the Economic Science Association (ESA) Meeting held in Tucson, Arizona, in October 1993; a session at the 1994 Summer Econometric Society Meeting held at Université Laval, Quebec City, in June 1994; the First International Conference on Computing in Economics and Finance (CEF1995), held at the University of Texas, Austin, May 21-24, 1995; the First Economic Artificial Life Conference held at the Santa Fe Institute, Santa Fe, New Mexico, May 26-29, 1995; an American Economic Association (AEA) panel session at the Annual Meeting of the Allied Social Science Associations held in San Francisco, CA, January 5-7, 1996; the UCLA Economic Simulation Conference held at the University of California, Los Angeles, on February 9, 1996; and the Fifth Annual Conference on Evolutionary Programming held in San Diego, California, in February 1996.

However, the most pivotal meeting for me, personally, was an informal meeting I organized on "agent-based economics," to be held immediately after the final conference sessions for the Second International Conference on Computing in Economics and Finance (CEF1996) in Geneva, Switzerland, June 26-28, 1996.[2] A key agenda item for this informal meeting was to consider how agent-based modeling might best be promoted to the economics profession at large. I left this meeting determined to develop a website resource repository devoted to this objective.

Exploiting the brand-new availability of web browsers, in particular Netscape Navigator,[3] I began my *Agent-Based Economics (ABE)* website in late July of 1996 on an Iowa State University (ISU) server. In August 1996, with important input from

[2] As indicated by a preserved sign-up sheet, the participants in this informal meeting were: Rob Axtell; Ann Bell; Chris Birchenhall; Kai Brandt; Thomas Brenner; Charlotte Bruun; Shu-Heng Chen; Michael Gordy; Sergei Guriev; Armin Haas; Esther Hauk; Gillioz Jean-Blaise; Alan Kirman; Bob Marks; Christian Rieck; Ernesto Somma; Leigh Tesfatsion; and Nick Vriend.

[3] Netscape Communications Corporation, founded in April 1994 by Marc Andreessen and James H. Clark, released Netscape Navigator in November 1994 as freely downloadable software. Netscape Navigator, a successor of Mosaic (co-developed by Andreesen), was among the first browser products released in support of the mid-1990s consumer Internet revolution.

Rob Axtell, I supplemented the ABE website with an ABE mailing list to be used for the distribution of occasional news notes.

However, microeconomists at ISU and elsewhere – Herman Quirmbach in particular – soon convinced me that "Agent-Based Economics" was a poor name choice for the website and mailing list since economic theorists could argue, with justification, that standard economic models focusing on the market interactions of consumers and firms were surely already "agent based". Consequently, as documented in the February 1997 ACE news notes [33],[4] I changed the names of my website and mailing list to *Agent-Based Computational Economics (ACE)* in August 1996 to stress computational implementation as a distinguishing feature of the proposed modeling approach.[5]

The website and mailing list name change from ABE to ACE turned out to be fortuitous. It immediately connected the ACE modeling approach to seminal work on "computational economics" being undertaken by Ken Judd and other participants in the Society for Computational Economics (SCE), founded in 1995. ACE was soon formally named an SCE Special Interest Group, thus permitting its consideration for panel session allotment at annual SCE meetings.[6]

A major ACE landmark occurred in the summer of 1997. As documented in my ACE news notes [33] distributed between February and May of 1997, Program Chair Ken Judd invited Blake LeBaron and myself to organize two contributed-paper sessions on ACE for the Third InternationalConference on Computing in Economics and Finance (CEF1997) to be held in July 1997 at Stanford University, plus a post-meeting satellite session devoted entirely to ACE topics.

A second major ACE landmark occurred in 1998: I was invited to guest-edit three special journal issues on ACE, one for the *Journal of Economic Dynamics and Control (JEDC)* [23], another for *Computational Economics (CE)* [24], and a third for the *IEEE Transactions on Evolutionary Computation (IEEE TEC)* [25]. As documented in my September 1998 ACE news notes [33], prospective authors for the JEDC special issue were asked to submit papers that addressed an issue of economic importance from an agent-based perspective. Prospective authors for the CE and IEEE TEC special issues were asked to submit papers with a strong agent-based computational component that addressed evolutionary economics issues.

These three ACE special issues all appeared in 2001. The research reported in these special issues demonstrated how ACE modeling permitted interesting ground-breaking extensions of then-standard economic modeling capabilities.

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[4] The formatting of the online ACE news notes [33] is ancient by browser standards. Although some formatting commands used in these news notes no longer compile properly using modern browsers, the news notes have been left in their originally released form in order to preserve their historical authenticity.

[5] As noted in my November 1997 ACE news notes [33], in Fall 1997 the website URL was changed from http://www.econ.iastate.edu/tesfatsi/abe.htm to http://www.econ.iastate.edu/tesfatsi/ace.htm and the mailing list address was changed from abelist@iastate.edu to acenewslist@iastate.edu in order to reflect these earlier website and mailing list name changes.

[6] The annual SCE meeting is officially referred to as the *International Conference on Computing in Economics and Finance (CEF)*.

For example, Chen and Yeh [9] develop an ACE stock market model consisting of a collection of stock market traders together with a ‘business school’. Each business school faculty member at any given time represents a particular ‘school of thought’ regarding the best stock market forecasting model. These various forecasting models are regularly subjected to a comparative review process (via a genetic programming algorithm) that results in model revisions. The traders can obtain access to these faculty forecasting models by choosing to visit the business school and attend faculty-offered classes. Once a trader attains access to a particular faculty member’s forecasting model, the trader can test whether this model outperforms the trader’s own current forecasting model. If so, the trader replaces its current model with the faculty member’s model and returns to market trading. The stock market traders thus evolve their forecasting models using a combination of individual learning (decision whether or not to visit the business school) and social learning (decision whether or not to adopt an accessed faculty forecasting model).

As a second example, Tesfatsion [26] develops an ACE labor market with an endogenous worker-employer matching process. Workers and employers match by means of a Gale-Shapley deferred acceptance mechanism. To implement this mechanism, the workers and employers must exchange messages with each other at event-triggered times regarding the receipt, acceptance, and refusal of work offers. During each labor market round the workers direct work offers to their most preferred employers, and the employers then accept work offers from their most preferred workers (refusing the rest). Once matched, a worker and employer engage in a work-site interaction modeled as a prisoner’s dilemma game. The outcomes of these games in each labor market round affect worker and employer preferences, hence who receives work offers and whose work offers are accepted or refused in the following round. The ACE labor market is thus a blend of matching theory with game theory.

A third major ACE landmark occurred in 2005. Ken Arrow and Mike Intriligator, general editors for the North Holland (Elsevier) Handbooks in Economics Series, invited Ken Judd and myself to edit an ACE handbook volume for this series. Potential lead authors, with co-authors of their own choosing, were invited to submit chapters on a wide variety of ACE-related topics.

Following a careful refereeing process, sixteen research chapters, seven perspective essays, and a resource guide for social science newcomers to agent-based modeling were accepted for the ACE handbook volume. The topic areas covered in the research chapters included: learning methods for economic agents; agent-based models and human-subject experiments; network formation among economic agents; agent-based computational finance; agent-based industrial organization; agent-based political economy; agent-based socio-economic modeling; agent-based software platforms for market design evaluation; and automated markets with trading agents. The ACE handbook volume [37] was published in 2006.

As documented at the ACE website [32], the number of books and articles making use of ABM/ACE has blossomed since 2006. ABM/ACE researchers are now publishing in a variety of existing and new economic journals with a welcoming inclusive methodological stance.[7] Research areas include: auction markets, automated markets, development economics, energy economics, financial economics, industrial organization, labor economics, macroeconomics, political economy, and technological innovation.

Another welcome development, stressed in recent reviews [1, 3, 27], is that ABM/ACE researchers are increasingly focusing on real-world applications as well as on conceptual advances. For example, as extensively documented at the ACE research repositories [34, 35] and in the survey articles [10, 11, 28], two fast-growing ACE application areas are macroeconomic policy and the design of electric power markets.

As a final note of optimism, consider the following. The new market design proposed for centrally-managed wholesale power markets in the 2021 Wiley/IEEE Press book [29] was developed and tested by means of an open-source ACE platform [4] that implements salient aspects of actual U.S. wholesale power market operations. This use of an ACE platform did not elicit any negative comments from the book's publisher or from its five anonymous referees. Indeed, based on extensive refereeing experience for power system journals, my assessment is that power market researchers now routinely rely on agent-based computational platforms to address the daunting complexity of actual real-world power market operations. Surely many real-world economic systems are at least as complex as power markets.

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[7] At present, the five primary economic journal outlets for ABM/ACE economic research are: *Computational Economics*; the *Journal of Economic Behavior and Organization*; the *Journal of Economic Dynamics and Control*; the *Journal of Economic Interaction and Coordination*; and the *Journal of Evolutionary Economics*. For a more extensive linked listing of welcoming journals, including financial journals, see [36].

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