MY TEACHING PHILOSOPHY

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> "I never teach my pupils; I only attempt to provide the conditions in which they can learn."
>
> Albert Einstein

What is mathematics about? Ask an undergraduate, and you may hear several answers: Math is about functions and equations; math is about solving problems; or, more loftily, math is the language of the sciences.

But when my undergraduates ask *me*, I tell them that mathematics is a martial art. Just as martial arts cultivate physical self-reliance, so does mathematics teach mental self-reliance—the ability to decide what is true, false, or merely ill-posed, using only one's brain. Its satisfactions derive not only from getting the right answers to difficult problems, but from *knowing*, with absolute clarity, why those answers are correct.

Thus it may seem a cruel irony that, of all subjects, mathematics can inspire so much self-doubt in students—so much anxiety to have their answers confirmed by an authority. Yet in this doubt, I see the seed of mastery. Doubt shows concern for certainty. Mathematicians, too, crave authority in the form of a proof; the difference is that they have the conceptual framework, habits of inquiry, and self-confidence to access this authority by themselves. These are the skills I wish to plant in my students.

First things must be first, though: confidence comes after competence, and competence after practice. I believe that mathematics is learned by doing, not by observing. For this reason, I integrate problem-solving into class. When teaching calculus, for example, I have begun each day by assigning a warm-up problem, which the students discuss in groups of three or four. This problem is meant to introduce the day's topic in a self-contained way, beginning with easy steps that build confidence, but leading my students up to a "mystery" which requires them to make a conjecture or a conceptual leap.

These warm-up problems serve several purposes. They get the students involved right away, inculcating that they are responsible for their own learning. The warm-ups get them to explain their ideas to one another (an act which often benefits the explainer as much as the listener), and they leave the students in a state of suspense which the remainder of the day's activities—a combination of discussion, lecture, and more group work—are designed to relieve. An example creates the appetite for an explanation.

I use lecture in a supporting role. To the extent that is practical, I prefer to nudge my students toward recognizing the key ideas "by themselves"; but they do sometimes need a model to follow, an experienced voice to underline take-away lessons and expose them to the conventions of mathematical usage. The trick is to accomplish all this, but not displace the students' authority with my own. I do this by interrupting my own lecture to ask my students questions, by explicitly tying "new" definitions and theorems to our earlier discussions, and by hitting the ball back into their court (with a new problem) as soon as possible.

The danger of teaching through problem-solving is that it can train students to mimic techniques without understanding why they are doing so. This becomes apparent when they are asked a question in an unfamiliar form; if their problem-solving skills are not backed by a robust conceptual structure, they cannot make a connection to what they have learned. To fight this tendency, I establish a norm that solving a problem *means* being able to explain every step of the solution. When a student presents an answer in class, my first response is not “That’s right” or “Try again,” but “How do you know?” On written assignments, I assess how well my students argue their solutions in plain, grammatical English. They are expected to explain all choices they made as to method and notation, as if teaching their peers.[1] As a by-product, this requirement helps them to realize that every word and symbol of their work *is* a choice.

I also vary the appearance of problems. This makes students uncomfortable at first, since they are used to seeking repetition as a guide to what is important. Yet this very habit, which often does more harm than good, is turned to advantage in my classroom. For although the problems do not repeat, the basic concepts which arise in solving them *do* repeat; and so it is these concepts which end up being reinforced.

The responsibilities I ask of my students are new to many of them. Confidence does not come to them right away: indeed, the path to independence affords my students many vistas of how much they do not yet know. To support them, I marry strict standards for their work to a conspicuous willingness to help them improve. I give my students voluble feedback on every page they submit, and I encourage them to ask for help—my only requirement is that they show me their previous efforts and try to articulate a question.[2] I find that most students will work to meet high expectations, especially when they feel that the teacher is in their corner. I make a point of being friendly, patient, and good-humored with my students; I feel that these traits are fundamental to persuading my students to accept the high demands I make of them. I am proud of the rapport I have had with my classes.

Because there are many ways to learn, I believe that I am never finished learning how to teach. My “side projects,” from mentoring gifted pupils at Stanford University Math Camp to helping students in California’s AVID program struggle through algebra, have brought me into contact with a wide variety of learners. Every such encounter challenges me to extend my range of techniques—and the reach of my empathy for everything that makes math a “hard subject” for so many people. I feel that this is my most invaluable professional training.

[1] In fact, that is exactly what they are doing: I distribute copies of the best written work as models to the class.

[2] More than once, I have not had to say a word: some questions, when articulated, answer themselves.