Problem 1. Let $f(t)$ stand for the number of students in the Polka Dot Hills school district $t$ years after Labor Day, 2000. Also, suppose $g(t)$ is the number of teachers at that time.

Let $h(t)$ stand for the annual funding per student provided by the state at that time. Let $k(t)$ stand for the average annual salary per teacher in the district at that time. Both are given in thousands of dollars.

(a) The student-to-teacher ratio, measured in students per teacher, is given by a function $R(t)$. Write a formula for $R(t)$ in terms of the functions above.

(b) The total funding from the state ($t$ years after Labor Day, 2000) is given by a function $I(t)$. Write a formula for $I(t)$ in terms of the functions above.

(c) The total payroll for all teachers ($t$ years after Labor Day, 2000) is given by a function $E(t)$. Write a formula for $E(t)$ in terms of the functions above.

Now suppose the district releases a report in 2011 which contains these data:

\[ f(11) = 700 \quad g(11) = 28 \quad h(11) = 2 \quad k(11) = 36 \]
\[ f'(11) = 53 \quad g'(11) = 1 \quad h'(11) = ??? \quad k'(11) = 5 \]

(d) Evaluate $R(11)$ and $R'(11)$. Is the sign of $R'(11)$ good news or bad news for parents who are concerned about classes being too big?

(e) Evaluate $E'(11)$.

(f) The district doesn’t know the value of $h'(11)$, because the state will decide that in its next budget. What would $h'(11)$ have to be in order to make funding ($I(t)$) increase just as fast as salary expenses ($E(t)$)? If this is the district’s goal, what request should the district make to state legislators (who may not understand fancy words like “derivative”)?

Problem 2 (Honolulu revisited). On Team Homework #2, you came up with a formula for a function $T(t)$ predicting the temperature in Honolulu, Hawaii as a function of the time since midnight on January 1.

(a) Revisit your work: taking my comments into account, do you stand by your original answer? If so, just copy the formula into your current homework for reference. If not, please make any needed corrections and write the new formula here.

(b) Find the derivative, $T'(t)$. You should get two sinusoidal terms. What does each term represent? One term should have much larger amplitude than the other. Why do you think this is the case?

Problem 3. The U. S. Census Bureau has published the following estimates of mid-year world population since 1950:

<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
<th>Year</th>
<th>Population</th>
<th>Year</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950</td>
<td>2,556,506,575</td>
<td>1975</td>
<td>4,090,586,151</td>
<td>2000</td>
<td>6,094,669,571</td>
</tr>
<tr>
<td>1955</td>
<td>2,781,208,967</td>
<td>1980</td>
<td>4,453,473,910</td>
<td>2005</td>
<td>6,479,962,284</td>
</tr>
<tr>
<td>1960</td>
<td>3,042,445,758</td>
<td>1985</td>
<td>4,859,510,682</td>
<td>2010</td>
<td>6,868,528,206</td>
</tr>
<tr>
<td>1965</td>
<td>3,350,250,014</td>
<td>1990</td>
<td>5,291,102,471</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1970</td>
<td>3,712,969,501</td>
<td>1995</td>
<td>5,703,456,064</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Let \( P(t) \) stand for the mid-year world population \( t \) years after 1950.

(a) Plot \( P(t) \) and \( P'(t) \) for the interval \( 0 \leq t \leq 60 \). Make sure to scale your graphs appropriately so that the trends in world population are apparent. Describe these trends.

A semi-logarithmic plot of a function is a graph on which the values on the \( y \)-axis are distributed according to their logarithm. Here is an example (snarfed from the Internet):

![Semi-logarithmic plot example](image)

Notice that the lines \( y = 10, y = 100, \) and \( y = 1000 \) are equally spaced, since the base-10 logarithms of 10, 100, and 1000 are 1, 2, and 3 respectively. (The natural logarithms are 2.3, 4.6, and 6.9, which are also equally spaced.)

(b) In the example, the horizontal grid lines between \( y = 10 \) and \( y = 100 \) correspond to all multiples of ten (\( y = 20, 30, \ldots, 90 \) ). Why are they spaced closer and closer together as \( y \) increases? (Note: Above \( y = 100 \), only multiples of 100 are marked.)

Making a semi-logarithmic plot of \( f(x) \) is not hard; you don’t need special graph paper (although semi-log graph paper does exist and is quite well-known among engineers!). You just copy an ordinary graph of \( y = \ln f(x) \), then relabel the \( y \)-coordinates according to the actual values of \( f(x) \).

(c) Make a semi-logarithmic plot of \( P(t) \), scaling appropriately so that the shape of the graph can be clearly seen.

(d) Use the fact that your plot is just a relabeled graph of \( \ln P(t) \), together with the chain rule, to explain why the slope of this plot at \( t = a \) is \( P'(a)/P(a) \).

(e) Given that \( P'(t) \) is the absolute rate of change of the population per year in the year 1950 + \( t \), what does \( P'(t)/P(t) \) represent? (Hint: It’s also a kind of “rate” which we have discussed before.)

(f) Describe the trends in \( P'(t)/P(t) \) since 1950. Using your answer from part (e), say what these trends mean in terms of world population.

(g) What kind of function appears as a straight line on a semi-logarithmic plot? If you extrapolated world population into the future using a tangent line on the semi-logarithmic plot, what assumption would you be making? Is this assumption valid?