Problem 1. The population of lemmings in a remote region of Norway can be modeled by means of the “logistic map.” According to this (somewhat simplistic) model, the current year’s lemming population $P$ (in thousands of lemmings) is the sole input determining next year’s lemming population, which is therefore a function of $P$; we call this function $\ell(P)$. Some values of $\ell(P)$ are given by the following table:

<table>
<thead>
<tr>
<th>$P$</th>
<th>0</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
<th>300</th>
<th>350</th>
<th>400</th>
<th>450</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell(P)$</td>
<td>0</td>
<td>162</td>
<td>288</td>
<td>378</td>
<td>432</td>
<td>450</td>
<td>432</td>
<td>378</td>
<td>288</td>
<td>162</td>
<td>0</td>
</tr>
</tbody>
</table>

(Notice that the higher $P$ is, the higher $\ell(P)$ is, up to a point. If $P$ is too large, then competition for resources drives the population down in the next generation.)

(a) Estimate $\ell(\ell(150))$. Show your work. 

(b) Interpret your answer from part (a) in terms of lemming population.

(Warning: Pay close attention to the nature of the input and output of $\ell$.)

(c) Estimate the derivative of $\ell(P)$ at $P = 150$. Show your work.

(d) Interpret your answer from part (c) in terms of lemming population.

(e) Estimate the derivative of $\ell(\ell(P))$ at $P = 150$. Show your work.

(f) Interpret your answer from part (e) in terms of lemming population.
In case you need it for the next problem, here’s the quotient rule:
\[
\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}.
\]

**Problem 2.** Let \( h(x) = (\ln x)/x \).

(a) Determine the intervals on which \( h(x) \) is (i) increasing, (ii) decreasing. Use the techniques of calculus; show your work, and do not round off the endpoints of your intervals. [9 pts.]

(b) Determine the intervals on which \( h(x) \) is (i) concave up, (ii) concave down. (Same provisos as above.) [9 pts.]

**Problem 3.** Using your results from Problem 2, find the \( x \)- and \( y \)-coordinates of the highest point on the graph of \( y = x^{1/x} \) (over the domain \( x > 0 \)). Do not round off.

(Warning: Attempting to differentiate \( x^{1/x} \) is probably a bad idea. Look for another approach.) [4 pts.]