Problem 1. Tarun decided to run a marathon. However, he started off way too fast and so his speed decreased throughout the race. Below is a table showing how many miles he had run at various times during the race:

<table>
<thead>
<tr>
<th>time (min.)</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>150</th>
<th>180</th>
<th>210</th>
<th>240</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance (miles)</td>
<td>5</td>
<td>9</td>
<td>12.5</td>
<td>15.5</td>
<td>18.5</td>
<td>21</td>
<td>23.5</td>
<td>25.5</td>
</tr>
</tbody>
</table>

Let $s(t)$ denote Tarun’s distance from the starting line (in miles) $t$ minutes after the beginning of the race.

(a) What is the practical interpretation of $s'(120)$ in the context of this problem? 

$s'(120)$ is Tarun’s speed (in miles per minute), 120 minutes into the race.

(b) Estimate $s'(120)$. (Show your work.)

$$s'(120) \approx \frac{18.5 - 12.5}{150 - 90} = 0.1 \text{ miles/min.}$$

(c) What is the practical interpretation of $s^{-1}(7)$ in the context of this problem?

$s^{-1}(7)$ is the time it takes for Tarun to run the first 7 miles of the race.

(d) Estimate $s^{-1}(7)$.

Interpolating linearly between the points $(5, 30)$ and $(9, 60)$, we can estimate

$$s^{-1}(7) \approx 30 + \frac{60 - 30}{9 - 5} \cdot (7 - 5) = 45 \text{ (min).}$$

(e) What does the derivative of $s^{-1}(x)$ at $x = 7$ represent in the context of this problem?

$(s^{-1})'(7)$ is Tarun’s running time per mile, based on his speed 7 miles into the race.

(f) Estimate the derivative of $s^{-1}(x)$ at $x = 7$.

$$(s^{-1})'(7) \approx \frac{60 - 30}{9 - 5} = 7.5 \text{ min./mile}$$
Problem 2. Write the limit definition of $f'(a)$.

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

[4 pts.]

Problem 3. Suppose $f$ is a function with the following properties:

- $f$ is continuous and has a derivative everywhere.
- $f'(x) < 0$ for all $x$ in $[1, 5]$.
- $f''(x) > 0$ for all $x$ in $[1, 5]$.
- $f(1) = 9$
- $f(5) = 3$

(a) Sketch a possible graph for $f$.

(b) What is the average rate of change of $f(x)$ on the interval $1 \leq x \leq 5$?

$$\frac{3 - 9}{5 - 1} = -1.5$$

[3 pts.]

(c) Which is greater, $f'(2)$ or $f'(4)$? Explain.

Since $f''(x) > 0$ on $[1, 5]$, we know $f'$ is increasing on this interval. Since $4 > 2$, it follows that $f'(4) > f'(2)$.

[3 pts.]

(d) What is the interval of all possible values for $f(3)$? Explain using the known properties of $f$.

Since $f'(x) < 0$ on $[1, 5]$, we know $f(x)$ is decreasing on this interval. Therefore, $f(3) > f(5) = 3$. However, since $f''(x) > 0$, we know $f(x)$ is concave up. Therefore, from $x = 1$ to $x = 5$, the graph of $f(x)$ lies below the secant line connecting $(1, 9)$ and $(5, 3)$. This implies that $f(3) < 6$.

To summarize, $3 < f(3) < 6$.

[4 pts.]
(e) What is the interval of all possible values for $f'(3)$? Explain. \[4 \text{ pts.}\]

We know that $f'(3)$ is negative. But because $f(x)$ is concave up on $[1, 5]$, we also know that $f'(3)$ is greater than the average slope of $f$ on $[1, 3]$. Since $f(3) > 3$, the average slope of $f$ on $[1, 3]$ is at least

$$\frac{3 - 9}{3 - 1} = -3.$$ 

To summarize, $-3 < f'(3) < 0$.

(This was by far the hardest part of the quiz. I suggest that you think about why all values between $-3$ and $0$ are possible.)