Physics 240 Fall 2005: Final Exam

Please print your name:___________________________________________________

Please list your discussion section number:___________________________________

Please list your discussion instructor:________________________________________

Form #1

Instructions

1. Fill in your name above
2. This will be a 1.5 hour, closed book exam. The exam includes 20 questions.
3. You may use a calculator, please do not share calculators
4. You may use four 3”x5” note cards (three from the earlier exams, plus one new one) with notes and equations you think may be useful. You can write on both sides of the card if you like.
5. You will be asked to show your University student ID card when you turn in your exam.

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Table of constants:
\[ \varepsilon_0 = 8.85 \times 10^{-12}\, \text{C}^2/\text{Nm}^2 \]
\[ k = 1/(4\pi\varepsilon_0) \]
\[ q_{\text{electron}} = -1.6 \times 10^{-19}\, \text{C} \]
\[ q_{\text{proton}} = 1.6 \times 10^{-19}\, \text{C} \]
\[ m_{\text{electron}} = 9.1 \times 10^{-31}\, \text{kg} \]
\[ m_{\text{proton}} = 1.67 \times 10^{-27}\, \text{kg} \]
\[ \mu_0 = 4\pi \times 10^{-7}\, \text{Tm/A} \]
\[ G = 6.67 \times 10^{-11}\, \text{Nm}^2/\text{kg}^2 \]
Topics:
1. Electric charge
2. Insulators and conductors
3. *Coulomb’s law
4. Electric field
5. *Electric flux and Gauss’ law
6. Shielding
7. *Electric potential energy and potential
8. *Equipotential surfaces and electric field
9. *Capacitors and dielectrics
10. Energy stored in capacitors
11. Electric current and current density
12. *Resistance and Ohm’s law
13. Energy and power in circuits
14. *Resistors in series and parallel
15. *Kirchoff’s Laws
16. *Circuits with capacitors
17. RC circuits
18. Ammeters and Voltmeters
19. Magnetic field
20. *Magnetic force on a charge
21. Magnetic force on a current
22. Loops and magnetic torque
23. Ampere’s law
24. Magnetic flux
25. *Faraday’s law
26. *Lenz’s law
27. Electrical generators and motors
28. Inductance
29. RL circuits
30. *Energy stored in the magnetic field
31. *Transformers
32. Alternating currents and voltages
33. Capacitors in AC circuits
34. RC circuits
35. *LC oscillators
36. Inductors in AC circuits
37. *RLC circuits
38. Resonance in AC circuits
39. Production of EM waves
40. *Propagation and the EM Spectrum
41. Energy and momentum in EM waves
42. Polarization
43. Reflection
44. *Refraction
45. Dispersion
1: In the figure below, two batteries generate current through a network of three resistors. The batteries supply voltages $\xi_1=5.0\text{V}$ and $\xi_2=12.0\text{V}$. The resistors have values $R_1=6.0\Omega$, $R_2=10.0\Omega$, and $R_3=4.0\Omega$. What is the current $i_3$? Please give a positive answer if the current flow in the direction shown by the arrow, and negative if it flows in the direction opposite the arrow.

a) $-0.52\text{A}$
b) $-1.9\text{A}$
c) $-0.34\text{A}$
d) $0.18\text{A}$
e) $*-1.0\text{A}$

\[ V_1 - I_1R_1 + I_3R_3 = 0 \]
\[ V_2 - I_2R_2 + I_3R_3 = 0 \]
\[ I_1 + I_2 + I_3 = 0 \]
\[ V_1 - V_2 + I_2R_2 - I_1R_2 = 0 \]
\[ I_2 = 1/R_2*(V_2 - V_1 + I_1R_1) \]
\[ I_1 + 1/R_2*(V_2 - V_1 + I_1R_1) + I_3 = 0 \]
\[ I_3 = (V_1/R_1 + I_3(R_3 / R_1)) * (1 + R_3/R_2) \]
\[ + (V_2 - V_1)/R_2 + I_3 = 0 \]
\[ (V_1 + I_3R_3)(1 + R_3/R_2) \]
\[ + (V_2 - V_1)(R_1/R_2) + I_3R_1 = 0 \]
\[ V_1 + I_3R_3 + I_3(R_3/R_2) + V_2(R_1/R_2) \]
\[ + I_3R_1 = 0 \]
\[ I_3 = -(R_2V_1 + R_1V_2) / (R_1R_2 + R_1R_3 + R_2R_3) \]

2: A 400$\mu\text{F}$ capacitor is connected to a 350 milli-Henry inductor as shown in the circuit below. While the switch is open, the capacitor is charged until the potential difference across it is 45 Volts. When the switch is closed, the energy in this circuit oscillates back and forth between the capacitor and the inductor. What is the maximum current which appears in the inductor during this process?

a) $0.24\text{A}$
b) $*1.52\text{A}$
c) $0.61\text{A}$
d) $0.04\text{A}$
e) $0.37\text{A}$

Energy goes from being stored in the capacitor to the inductor:
\[ 1/2CV_{\text{max}}^2 = 1/2LI_{\text{max}}^2 \]
\[ I_{\text{max}} = V_{\text{max}}(C/L)^{1/2} = 1.52\text{A} \]
3: What is the total electrical potential energy of the arrangement of charges shown below? You may take the potential energy when the charges are infinitely far apart to be zero.

\[
PE = kQ_1Q_2/r_{12}
\]

\[
PE = -2kQ^2 / r - 2kQ^2/2r + 4kQ^2/3r + 4kQ^2/r - 8kQ^2/2r - 8kQ^2/r
\]

\[
PE = -11kQ^2/r + 4kQ^2/3r = -29kQ^2/3r
\]

\[
PE = \frac{12kQ^2}{r}
\]

\[
PE = -\frac{4kQ^2}{r}
\]

\[
PE = -\frac{kQ^2}{3r}
\]

\[
PE = \frac{9kQ^2}{4r}
\]

\[
PE = -\frac{29kQ^2}{3r}
\]

4: A narrow laser beam shines through a 3.0 cm thick pane of glass at an angle of incidence of 40° as shown in the figure. How large is the offset distance Δy denoted in the figure? The index of refraction of this glass is n=1.65.

\[
sin40 = 1.65sin\theta_g\quad so\quad \theta_g = 23°
\]

Length of path through glass:
\[
L = 0.03 \text{ m} / \cos23 = 0.032
\]

Offset = Lsin(40-23) = 0.0094 = 0.94cm
5: What is the equivalent capacitance of the network of identical capacitors shown in the picture if the capacitance of each capacitor is C?

- a) $7C/8$
- b) $4C$
- c) $5C/12$
- d) $C/6$
- e) $3C/8$

Reduce the part in the middle, on the left it is $C/2$, on the right $2C$. Those are in parallel, so they’re equivalent to $2.5C$. Then you have three in series:

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{2}{(5C)} + \frac{1}{C} = \frac{12}{5C}$$

$$C_{eq} = \frac{5C}{12}$$

6: A series RLC circuit is constructed from a 12 Ω resistor, an 0.15 milli-Henry inductor, and a 180 nanoFarad capacitor. This circuit is then driven by an AC voltage source with an amplitude of 100V. If we drive this circuit at a frequency of 50,000 Hz, what will be the amplitude of the current through the resistor?

- a) 0.26 A
- b) 0.71 A
- c) 1.33 A
- d) 2.99 A
- e) *3.15 A

What’s the impedance? $\omega = 2\pi f = 3.14 \times 10^5 \text{ rad/s}$

$$Z = (R^2 + (\omega L - 1/\omega C)^2)^{1/2} = 17.7 \Omega$$

$$I_{\text{max}} = \frac{V_{\text{max}}}{Z} = 3.15 \text{ A}$$
7: Consider an insulating rod of length $L$. The rod has a total excess charge $Q$, which is spread uniformly along the rod.

![Image showing a rod with charge $Q$ and a small charge $q$ at distance $d$ from the end of the rod.]

Find an equation for the total Coulomb force between this rod and a small charge $q$ located along the axis of the rod at a distance $d$ from the end of the rod.

\[ F = \int_{d}^{d+\delta L} \frac{kqQ}{Lr^2} \, dr = \frac{kqQ}{L} \int_{d}^{d+\delta L} \frac{dr}{r^2} \]

\[ F = \frac{kqQ}{L} \left( -\frac{1}{r} \right)_{d}^{d+\delta L} = \frac{kqQ}{L} \left( -\frac{1}{d+L} + \frac{1}{d} \right) \]

\[ F = \frac{kqQ}{L} \left( d + L - d \right) \frac{d}{d(d+L)} = \frac{kqQ}{d(d+L)} \]

8: A 2.0 kg rod has a length of 1.0 m and a resistance of 4.0 Ω. It slides with constant speed down a pair of frictionless vertical conducting rails that are joined at the bottom. Other than the rod, the rest of the circuit has zero resistance. A uniform magnetic field of magnitude 3.0 T is perpendicular to the plane formed by the rod and the rails as shown. Determine the speed of the rod.

\[ a) \ 0.38 \text{ m/s} \]
\[ b) \ 0.90 \text{ m/s} \]
\[ c) \ 2.6 \text{ m/s} \]
\[ d) \ 5.6 \text{ m/s} \]
\[ e) \ *8.7 \text{ m/s} \]

The EMF in the loop is $BdA/dt = B*L*v$. The current is $\text{EMF}/R$. The magnetic force on the wire is $iLB = \text{EMF}/R*L*B = B*L^2v/R = mg$ or $v = mg/R/B^2L^2$

$= 2.0*9.8*4.0 / 3.0^2*1.0^2 = 8.7 \text{ m/s}$
9: Charge is distributed uniformly on a plate which is 3.2 m wide and 4.5 m long. The electric field 3 cm above the center of the plate is 30 N/C. What is the electric field 6 cm above the plate?

- a) 120 N/C
- b) 60 N/C
- c) 30 N/C
- d) 15 N/C
- e) 7.5 N/C

This close to such a large plate, it looks like an infinite plane of charge. Such a plane has a constant electric field, so the answer is 30 N/C.

10: Eight identical spherical raindrops are equally charged, so that the potential at the surface of each sphere (taking the potential at infinity to be zero) is V. The drops coalesce, forming a single spherical raindrop. What is the electric potential at the surface of this new drop?

- a) V/8
- b) V/2
- c) 2V
- d) 4V
- e) 8V

The potential on the surface of a charged sphere is \( V = kQ/r \). If you put 8 drops together, you have 8 times the volume of water, which makes a sphere with 2x as big a radius. All the charge comes together, so you have 8x the charge and 2x the radius. This makes the potential at the surface 4x as large.
11: Two capacitors are identical except that one is filled with air and the other with wax. Both capacitors carry the same charge. The ratio of the electric fields inside the two capacitors $E_{air} / E_{wax}$ is:

a) Between 0 and 1
b) 0
c) 1
d) *Between 1 and infinity
e) Infinite

Wax is a dielectric (any solid will be a better dielectric than air). So the electric field in the capacitor is reduced, and the ratio of the two will be bigger than one.

12: An incandescent light bulb is operated by running a current through the filament. The power dissipated in the filament heats it until it glows. Which of the following best represents the relationship between voltage and current for this filament?

a) $V$ vs $I$
b) $I$ vs $V$
c) $V$ vs $I$
d) $I$ vs $V$
e) $V$ vs $I$

As you turn up the current, the filament heats. When you increase the temperature of a metal, its resistance rises. This rise in resistance means that the current increases more and more slowly as you increase the voltage (and hence the temperature).
13: Note the different types of electromagnetic radiation:

- X-rays
- radio waves
- gamma rays
- visible light
- infrared radiation
- ultraviolet radiation

Which list correctly ranks the electromagnetic waves in order of increasing frequency (from smallest to largest)?

a) 2, 5, 4, 6, 1, 3  
b) 3, 1, 6, 4, 5, 2  
c) 3, 6, 1, 4, 5, 2  
d) 2, 3, 4, 5, 6, 1  
e) 2, 5, 4, 1, 6, 3

14: Consider a cube of edge length L with one corner located at the origin. If this cube is placed in a region where the electric field is given by:

\[ \vec{E} = (a + bx)\hat{x} + c\hat{y} + (dz)\hat{z} \]

what is the total charge inside the cube?

a) (b-d)L^3\epsilon_0  
b) bdL^3\epsilon_0  
c) acL^3\epsilon_0  
d) *(b+d)L^3\epsilon_0  
e) (d-b)L^3\epsilon_0

Think about how much electric flux goes through each face of the cube:
- aL^2 + aL^2 + bL^3 on the x faces
- cL^2 + cL^2 on the y faces
- 0 + dL^3 on the z faces

Adding these up, you get (b+d)L^3 and the charge inside = (b+d)L^3\epsilon_0
15: Consider the map showing the electric potential around an electric quadrupole; two charges of +Q and two charges of -Q. The equipotential lines shown are spaced by 5 V. Solid lines represent equipotentials \( \geq 0 \) V, dashed lines are equipotentials \( \leq 0 \). The units of distance shown on the axes are in centimeters. Estimate the electric field at the point P.

a) 0 N/C  
b) 100 N/C up  
c) 50 N/C down  
d) *100 N/C down  
e) 50 N/C up

The potential changes from +5 at 55 cm to -5 at 45 cm, so \( E = \frac{dV}{dy} = -10 / 0.1m = -100 \text{ N/C} \)

16: Consider the circuit shown in the figure. Which of the following statements about the circuit is correct?

a) The current supplied by the battery long after the switch is closed is larger than the current supplied immediately after the switch is closed. 
b) *The current supplied by the battery long after the switch is closed is smaller than the current supplied immediately after the switch is closed. 
c) The current through the resistor \( R_2 \) is largest immediately after the switch is closed. 
d) The potential drop over the resistor \( R_1 \) is smallest immediately after the switch is closed. 
e) The current supplied by the battery long after the switch is closed is zero.

Initially, the capacitor will act like a wire and the battery will put out a current \( I = \frac{V}{R_1} \). Later, the capacitor will be a break in the circuit, and the battery will put out current \( I = \frac{V}{(R_1 + R_2)} \), which is less.
17: An electron travels with a velocity $5.2\times10^5$ m/s in the positive x direction. It experiences a total magnetic force $F_B = (3.4\times10^{-14}N)y - (6.7\times10^{-15}N)z$. What is the total magnetic field through which this electron travels?

a) $(-0.08T)y + (-0.41T)z$

b) $(0.08T)y + (-0.41T)z$

c) $(0.08T)y + (0.41T)z$

d) $(-0.08T)y + (0.41T)z$

e) *It is impossible to tell

This is somewhat tricky… Since the particle travels in the x-direction, it can experience a Lorentz force only in the y or z direction. Any x component of the magnetic field will not exert a force at all. The field listed as answer A could cause this force, but you cannot be sure that it is the total field. Any field with the same y and z components, but a different x component, would give the same force.

18: An incandescent light bulb radiates uniformly in all directions with a total average power of 100 W. What is the maximum value of the oscillating magnetic field at a distance of 0.50 m from the bulb?

a) $8.4\times10^{-7}$ T

b) *$5.2\times10^{-7}$ T

c) $3.1\times10^{-7}$ T

d) $1.6\times10^{-7}$ T

e) Zero

$$P = 100 \text{ W}$$

$$I = \frac{P}{4\pi r^2} = 31.8 \text{ W/m}^2 = \frac{cB^2}{2\mu_0}$$

$$B = \left(2\mu_0 I / c\right)^{1/2} = 5.2\times10^{-7} \text{ T}$$
19: A rectangular loop of wire is placed midway between two long straight parallel conductors as shown. The currents in the conductors are $i_1$ and $i_2$. If the current $i_1$ is constant, and $i_2$ is increasing, the induced current in the loop:

   a) is zero 
   b) *is clockwise 
   c) is counterclockwise 
   d) depends on $i_1 - i_2$ 
   e) depends on $i_1 / i_2$

If the current $i_1$ isn’t changing it has no effect. $i_2$ is increasing. This causes an increase in the magnetic field coming up through the loop. A current is induced which acts to oppose this change, in the clockwise direction.

20: A magnetic field exists between the plates of a capacitor:

   a) always 
   b) when the capacitor is fully charged 
   c) *while the capacitor is being discharged 
   d) when the charge on the capacitor is at a minimum 
   e) never

The displacement current due to the changing electric field produces a magnetic field whenever the capacitor is charging or discharging.
21: Here are Maxwell’s equations in the form we discussed:

\[ 1: \iiint E \cdot dA = \frac{q_{\text{enclosed}}}{\varepsilon_0} \]
\[ 2: \iiint B \cdot dA = 0 \]
\[ 3: \oint E \cdot ds = -\frac{d\Phi_B}{dt} \]
\[ 4: \oint B \cdot ds = \mu_0 i_{\text{enclosed}} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]

Which of these equations will need to be modified if physicists ever discover magnetic monopoles?

a) Only 1  

b) Only 2  

c) *Only 2 and 3  

d) Only 2, 3, and 4  

e) Only 3 and 4

Magnetic charges would add terms to both 2 and three, making 2 look like Gauss’ law, and 3 look like four, with both a Faraday’s law like term and an Ampere’s law like term for currents of magnetic charge.

22: A rectangular tank is filled to the top with an unknown liquid. An observer, whose eye is just level with the top of the tank, can see the lower right corner of the tank. What is the index of refraction of this liquid?

a) 1.75  

b) 1.67  

c) 1.50  

d) 1.33  

e) 1.25

Observer can see this corner

This one you didn’t have to answer. It has many possible answers.
23: At the orbit of the Earth, the intensity of the Sun’s radiation is 1370 W/m². Imagine you launch a satellite from the Earth, and after it leaves the atmosphere, you unfold a large solar sail. If this sail were a 100 m by 100 m square, had a mass of 13 kg, and were perfectly reflective, what would be its acceleration due to the radiation pressure of the sun?

a) 2.9x10⁻⁸ m/s²
b) *7.0x10⁻³ m/s²
c) 9.8x10⁻⁷ m/s²
d) 8.7x10⁻⁵ m/s²
e) 2.4x10⁻¹² m/s²

Because this reflects, the pressure exerted would be 2I/c = 9.2x10⁻⁶ N/m². The area is 10⁴ m², so the force is 9.2x10⁻² N. Since this weighs 13 kg, the acceleration you get is:

\[ A = \frac{F}{m} = 7.1x10^{-3} \text{ m/s}^2 \]

24: A parallel-plate capacitor, with air between the plates, is charged by a battery, after which the battery is disconnected. A slab of plastic dielectric is then slowly inserted between the plates. As it is being inserted:

a) a force repels the plastic out of the capacitor
b) *a force pulls the plastic into the capacitor
c) no net force acts on the plastic
d) a net charge acts on the plastic
e) the plastic makes the plates attract one another more strongly

While you are inserting the dielectric plate, it is pulled into the capacitor. This happens because polarization is induced in the dielectric.
25: A positively charged insulating rod is brought close to an object that is suspended by a string. If the object is attracted toward the rod we can conclude:

a) the object is positively charged
b) the object is negatively charged
c) the object is an insulator
d) the object is a conductor
e) *none of the above

The positively charged rod would attract an insulator, a conductor, or a negatively charged object. So you really can’t conclude that any one of these is true.