Physics 240 Fall 2005: Exam #2 Solutions

Please print your name: _____________________________________________________

Please list your discussion section number: _________________________________

Please list your discussion instructor: _______________________________________

Form #1

Instructions

1. Fill in your name above
2. This will be a 1.5 hour, closed book exam. The exam includes 20 questions.
3. You may use a calculator, please do not share calculators
4. You may use two 3”x5” note cards (one from the first exam, plus one new one) with notes and equations you think may be useful. You can write on both sides of the card if you like.
5. You will be asked to show your University student ID card when you turn in your exam.

ID checked by: ________________________________

Table of constants:
ε₀ = 8.85 x 10^{-12} \text{ C}^2/\text{Nm}^2
k = 1/(4\pi\varepsilon₀)
q_{\text{electron}} = -1.6 \times 10^{-19} \text{ C}
q_{\text{proton}} = 1.6 \times 10^{-19} \text{ C}
m_{\text{electron}} = 9.1 \times 10^{-31} \text{ kg}
m_{\text{proton}} = 1.67 \times 10^{-27} \text{ kg}
\mu₀ = 4\pi \times 10^{-7} \text{ Tm/A}
G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2
1: An electron moving with a velocity of 525 m/s in the x direction enters a region filled with a constant magnetic field. The magnitude of this field is 0.5 Tesla, and it points into the page. This constant field region is 3 m long and 1.5 m wide. The electron enters 20 cm from the top of the region. What is the magnitude of the electron’s velocity when it leaves the region?

a) 367 m/s
b) 692 m/s
c) 560 m/s
d) 920 m/s
e) *525 m/s

Because the magnetic force is always perpendicular to the direction of motion, it can never do work on the particle, and its kinetic energy (and hence speed) never changes. So it leaves the region with the speed it entered with.

2: Two long, straight wires are perpendicular to the plane of the paper as shown in the drawing. Each wire carries a current of magnitude I. The currents are directed out of the paper toward you. Which one of the following expressions correctly gives the magnitude of the total magnetic field at the origin of the x, y coordinate system?

$$a) \frac{\mu_0 I}{2d}$$
$$b) \frac{\mu_0 I}{\sqrt{2}d}$$
$$c) \frac{\mu_0 I}{2\pi d}$$
$$d) \frac{\mu_0 I}{\pi d}$$
$$e) \frac{\mu_0 I}{\sqrt{2}\pi d}$$

Each of the wires will create a magnetic field with magnitude:
$$B = \frac{\mu_0 I}{2\pi d}$$

The wire on the x axis will produce this field in the $-y$ direction, the wire on the y axis will produce this field in the $+x$ direction. To get the total magnitude, I have to find the magnitude of the vector with these two components:
$$B_{total} = \sqrt{(B_x^2 + B_y^2)} = \sqrt{((\mu_0 I/2\pi d)^2 + (\mu_0 I/2\pi d)^2)} = \sqrt{2*\mu_0 I/2\pi d}$$
$$B_{total} = \frac{\mu_0 I}{\sqrt{2}\pi d}$$
3: Two loops carry equal currents $I$ in opposite directions. The loops are held parallel, in the positions shown in the figure, and are then released. Which one of the following statements correctly describes the subsequent behavior of the loops? You may ignore any possible effects of gravity.

a) The current in the bottom loop is gradually reversed.

b) The loops remain in the positions shown.

c) The top loop moves to the right; the bottom loop moves to the left.

d) *The loops repel each other.

e) The loops attract each other.

There are two ways to think of this. The loops are parallel wires with current going in opposite directions. Such pairs repel one another. Or you can use the fact that each is a magnetic dipole, and they are placed so that their south poles face one another, so they repel.

4: If you stack up 12 pennies, you get a cylindrical column of copper as tall as it is wide. Given that the resistivity of copper is $1.7 \times 10^{-8}$ $\Omega\cdot m$, estimate the resistance of this stack measured from top to bottom. (Even though the raised features on the surface of the pennies would make them touch in only a few spots, you may treat this as a solid column of copper for your calculation.)

a) *$1 \times 10^{-6}$ $\Omega$

b) $3 \times 10^{-2}$ $\Omega$

c) $7 \times 10^{-11}$ $\Omega$

d) $0.3$ $\Omega$

e) $4 \times 10^{-15}$ $\Omega$

Remembering your estimates of the penny size from the first exam, you might guess $D_{\text{penny}} = 2$ cm (really it’s about 1.8 cm). This would give:

$$R = \rho \frac{L}{A} = \rho \frac{D_{\text{penny}}}{\pi (D_{\text{penny}}/2)^2} = 4\rho / \pi D_{\text{penny}} = 1.2 \times 10^{-6}$ $\Omega$$
5: Three light bulbs are connected in a circuit as shown. Each of the bulbs has a resistance of 30 Ω, and the battery supplies a voltage of 30 V. How much power does light B consume?

a) *30W
b) 60W
c) 10W
d) 20W
e) 50W

\[ P = \frac{V^2}{R} = \frac{(30 \text{ V})^2}{(30 \text{ Ω})} = 30 \text{W} \]

6: Consider the circuit shown in the figure. Which of the following statements about the circuit is correct?

a) The current supplied by the battery long after the switch is closed is larger than the current supplied immediately after the switch is closed.
b) *The current supplied by the battery long after the switch is closed approaches a finite current which is smaller than the current supplied immediately after the switch is closed.
c) The current through the resistor R_2 is largest immediately after the switch is closed.
d) The potential drop over the resistor R_1 is smallest immediately after the switch is closed.
e) The current supplied by the battery long after the switch is closed is zero.

A & B: When the switch is initially closed current both flows through the resistors and charges the capacitor, so it is large. Later, once the capacitor is charged, it only flows through the resistors. So it starts large and become smaller (but not zero!).
C & D: Initially the potential difference across R_2 (and C) is zero, so no current flows through it. Later, as the capacitor becomes charged, current begins to flow through R_2. The potential drop across R_1 starts out equal to the battery voltage V. This potential drop is reduced as charge builds up on the capacitor.
7: Consider a flat rectangular conductor connected to a battery as shown. The conductor is embedded in a magnetic field coming out of the page. Electrons are the charge carriers in this conductor. Which of the following statements is true?

a) The electrons which flow feel a magnetic force into the page
b) The electrons which flow feel a magnetic force up the page
c) The electrons which flow feel no magnetic force
d) The electric potential on the left side of the conductor is higher than the electric potential on the right side of the conductor
e) *The electric potential on the left side of the conductor is lower than the electric potential on the right side of the conductor

Consider each statement in turn:
A) The magnetic field is out of the page. The electrons move up the page, so the force they feel is to the left.
B) See A…
C) See A…
D) If the electrons move to the left (as in A), the potential on the left side will be low (positive charge would move there), not high.
E) When the electrons move to the left, potential on the left side is lower than on the right. So this is correct.

8: A current loop carrying a counterclockwise current I through a single turn of wire has sides of length L_x and L_y as shown in the figure. The loop lies flat in the plane of the paper. A magnetic field \((B_x x_{\text{unit}} + B_y y_{\text{unit}})\) lies in the plane of the page. What is the vector which best describes the magnetic torque on this current loop? Here the boldface \(x_{\text{unit}}\) and \(y_{\text{unit}}\) represent the unit vectors in the x and y directions.

a) \(IL_x L_y (B_x^2 + B_y^2)^{1/2}\)
b) \(-IL_x L_y (B_x x_{\text{unit}} + B_y y_{\text{unit}})\)
c) \(-IL_x B_x y_{\text{unit}} + IL_y B_y x_{\text{unit}}\)
d) \(*IL_x L_y B_x y_{\text{unit}} - B_y x_{\text{unit}}\)
e) \(IL_x B_x y_{\text{unit}} - IL_y B_y x_{\text{unit}}\)

To do this use the expression for the torque as is \(\boldsymbol{\tau} = \mu \times \mathbf{B}\). Here the magnetic moment of the loop is given by: \(I\mathbf{A} = IL_x L_y\). Given the current direction shown, the right hand rule tells us that this magnetic moment is in the z direction. Now calculate the torque:

\[
\boldsymbol{\tau} = \mu \times \mathbf{B} = iL_x L_y (B_x x + B_y y) = iL_x L_y (B_x x + B_y z + B_z x y) = iL_x L_y (B_y - B_z x)
\]
9: A parallel plate capacitor with capacitance \( C \) is connected to a battery until the potential between the plates is \( V \) and it holds a total charge \( Q \). The battery is then disconnected and the capacitor is left charged. The gap between the plates is then filled with a dielectric material with dielectric constant \( \kappa \). Which of the following statements is true?

a) The energy stored in the capacitor increases  
   b) *The energy stored in the capacitor decreases*  
   c) The charge stored on the capacitor increases  
   d) The charge stored on the capacitor decreases  
   e) The electric field inside the dielectric is zero

The charge on the capacitor remains the same, so this is about what happens to the energy. The energy stored in the capacitor is \( Q^2/2C \). When we put in the dielectric, \( C_{\text{initial}} \) changes to \( \kappa C_{\text{initial}} \), so the energy stored is reduced by a factor of \( 1/\kappa \).

10: Consider the circuit shown in the figure. If the circuit has been connected for a long time, what is the charge on the capacitor \( C_1 \)?

a) *0*  
   b) \( V/C_1 \)  
   c) \( V/(C_1 + C_2) \)  
   d) \( V - V/C_2 \)  
   e) \( V(C_1+C_2)/C_1C_2 \)

After the circuit has been connected for a long time the capacitor \( C_2 \) is fully charged, and no current runs through the circuit. That is, the potential rise across the battery is entirely balanced by the potential fall across \( C_2 \). Since there is no current, there is no potential drop across \( R_1 \), and hence no potential across \( C_1 \), and hence no charge on it.
11: Consider the set of light bulbs, powered by a battery, shown in the figure. Rank the bulbs in order of brightness. If two bulbs are equally bright, it is written as $B_i = B_j$.

a) $*B_1 > B_4 > B_2 = B_3 > B_5 = B_6$
b) $B_1 > B_2 = B_4 > B_4 > B_5 = B_6$
c) $B_4 > B_1 > B_2 = B_6 > B_2 = B_3$
d) $B_1 > B_5 = B_6 > B_4 > B_2 = B_3$
e) $B_4 > B_1 > B_2 = B_3 > B_5 = B_6$

All the current goes through $B_1$. Then it splits and goes evenly through $B_2$ and $B_3$, each taking half. Then it splits and sends $2/3$ through $B_4$ and $1/3$ through both $B_5$ and $B_6$. This gives brightnesses which are:

$B_1 > B_4 > B_2 = B_3 > B_5 = B_6$

12: Consider the circuit shown at the right. It includes two identical batteries, each producing potential $V$, and each with internal resistance $r$. These are connected in parallel across the external resistance $R$. What is the current $i_R$ through the external resistor $R$?

a) $V / (R+r)$
b) $*V / (R+r/2)$
c) $V / (R+2r)$
d) $V / (2R+r)$
e) $V / (2R+r/2)$

\[
\begin{align*}
V - i_3R - i_1r &= 0 \\
V - i_2R - i_3r &= 0 \\
i_1 + i_2 &= 2i_1 = i_3 \\
V - i_3R - i_3r/2 &= 0 \text{ or } i_3 = V / (R+r/2) \text{ and } i_1 = i_2 = V/(2R + r)
\end{align*}
\]
13: What is the equivalent resistance across the terminals of the battery due to all the resistors?

a) 22.0 Ω  
b) *2.3 Ω  
c) 14.5 Ω  
d) 18.1 Ω  
e) 24.6 Ω

The left hand side reduces to a 7 Ω resistor in series with a 3 Ω resistor. The right hand side reduces to a 1.5 Ω resistor in series with a 1.5 Ω resistor. Putting these together, I have a 10 Ω resistor in parallel with a 3 Ω resistor, which has an equivalent resistance of 2.3 Ω. This is the equivalent resistance of the whole network.

14: An electron with initial velocity \( v = 4 \times 10^5 \text{ x}_{\text{unit}} \text{ m/s} \) (where the \( \text{x}_{\text{unit}} \) indicates the unit vector in the x-direction) enters a region with uniform electric and magnetic fields. The electric field in this region is given by \( E = 300 \text{ z}_{\text{unit}} \text{ N/C} \). Which of these magnetic fields has a magnitude and direction which will allow the electron to pass undeflected through this region?

a) \( 3 \times 10^{-3} \text{ T y}_{\text{unit}} \)  
b) \( -4.2 \times 10^{-6} \text{ T z}_{\text{unit}} \)  
c) \( *-7.5 \times 10^{-4} \text{ T y}_{\text{unit}} \)  
d) \( 2.4 \times 10^{-5} \text{ T x}_{\text{unit}} \)  
e) \( -4.2 \times 10^{-6} \text{ T y}_{\text{unit}} \)

The electric force the electron feels is given by \( q_eE \), while the magnetic force is \( q_ev_xB \). We want these to balance, so \( v_xB = E = 300 \text{ z}_{\text{unit}} \text{ N/C} \).

Since the velocity is in the \( \text{x}_{\text{unit}} \) direction, and we want \( v_xB \) to be in the \(-z_{\text{unit}} \) direction. That means we need to have \( B \) in the \(-y_{\text{unit}} \) direction. The magnitude would be:
\[
B = \frac{300 \text{ N/C}}{4 \times 10^5 \text{ m/s}} = 7.5 \times 10^{-4} \text{ T}
\]
15: Consider the circuit shown in the figure. All the capacitors start uncharged. When the switch is first closed, what is the current through the 120 Ω resistor?

a) 0 A  
b) 0.67A  
c) 3.0 A  
d) 0.33A  
e) *0.5 A

When the switch is first closed, each capacitor acts like a wire, so no current passes through the 60 Ω resistor, and all of it passes through the 120 Ω resistor. This makes the current:

\[ I = \frac{V}{R} = \frac{60 \text{ V}}{120 \text{ Ω}} = 0.5 \text{ A} \]

16: A power plant produces 100 A of current and sends it through some power lines to a consumer. When a voltmeter is connected to the power lines at the power station, the potential between the lines is 600 V. When a voltmeter is connected to the power lines at the consumers end, the potential observed is 580 V. How much power is being wasted in the transmission lines?

a) 1000W  
b) *2000W  
c) 58,000W  
d) 59,000W  
e) 60,000W

Power = IV, so the power coming out of the station is 600 V * 100 A = 60,000 W, and the power going into the consumers house is 580 V * 100 A = 58,000 W. The power lost is the difference, or 2,000 W
17: An electron and a proton travel with the same speed in a region of constant magnetic field. The velocity of each is in the plane of the paper, while the magnetic field is into the paper. The mass of an electron is about 1800 times less than that of a proton. The charges of the two are equal, but opposite, with the electron negative and the proton positive. Which of the following statements is correct?

a) The electron travels in a clockwise circle about 1800x larger than the counterclockwise circle in which the proton travels
b) *The proton takes about 1800x as long as the electron to orbit around once
c) The proton travels in a clockwise circle while the electron travels in a counterclockwise circle which is 1800x smaller
d) The electron takes 1800x as long as the proton to orbit around once
e) The electron and the proton take exactly the same amount of time to orbit around once

The gyroradius is $r = \frac{mv}{qB}$, and the period of gyration is $\Gamma = \frac{2\pi m}{qB}$. The electron will travel in a circle which is clockwise while the proton goes counterclockwise. The protons circle will be 1800x larger than the electrons, and the proton will take 1800x as long to orbit.

18: Four long straight wires carry equal currents into and out of the page in the manner shown in the figure. What is the direction in which the magnetic force on wire A points?

a) Northwest
b) Southeast
c) Northeast
d) *Southwest
e) The total force is zero by symmetry

Parallel wires attract while antiparallel wires repel. The magnitude of this force is proportional to 1/distance. See the forces drawn on the figure…
19: A magnetic dipole starts at rest in a region of non-uniform magnetic field. The field lines are shown in the figure. The dipole is a square current loop lying perpendicular to the page, so that current in the edge on the left is coming out of the page, while current in the edge on the right is going into the page. Which of the following combinations correctly describes the net force and net torque which this loop feels at this moment?

<table>
<thead>
<tr>
<th>Force direction</th>
<th>Torque direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) To the right</td>
<td>Clockwise</td>
</tr>
<tr>
<td>b) *Down the page</td>
<td>Clockwise</td>
</tr>
<tr>
<td>c) No force</td>
<td>Clockwise</td>
</tr>
<tr>
<td>d) To the left</td>
<td>Counterclockwise</td>
</tr>
<tr>
<td>e) Up the page</td>
<td>There is no torque</td>
</tr>
</tbody>
</table>

The wire on the right feels a downward force which is relatively large (the field lines are close there), while the wire on the left feels an upward force which is smaller (because the field lines are farther apart there). The two side wires experience no force. So the net torque is clockwise, and the net force is down the page.

20: You buy a light bulb labeled “75 W”. This label means that:

a) No matter voltage you connect it to, the power will be 75 W
b) The bulb was filled with 75 W in the factory
c) The actual power dissipated will be much greater than 75 W since most of the power appears as heat
d) The bulb is expected to burn out after you use up its 75 W
e) *None of the above

All of these answers are wrong.