Physics 240 Fall 2005: Exam #1 Solutions

Please print your name: _____________________________________________________

Please list your discussion section number: ________________________________

Please list your discussion instructor: ______________________________________

Form #1

Instructions

1. Fill in your name above
2. This will be a 1.5 hour, closed book exam. The exam includes 20 questions.
3. You may use a calculator, please do not share calculators
4. You may use one 3”x5” note card with notes and equations you think may be useful. You can write on both sides of the card if you like.
5. You will be asked to show your University student ID card when you turn in your exam.

ID checked by: _________________________________________________________

Table of constants:
\( \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 \)
\( k = 1/(4\pi\varepsilon_0) = 8.99 \times 10^9 \text{ Nm}^2/\text{C} \)
\( q_{\text{electron}} = -1.6 \times 10^{-19} \text{ C} \)
\( q_{\text{proton}} = 1.6 \times 10^{-19} \text{ C} \)
\( m_{\text{electron}} = 9.1 \times 10^{-31} \text{ kg} \)
\( m_{\text{proton}} = 1.67 \times 10^{-27} \text{ kg} \)
\( G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \)
1: Imagine four identical metal spheres; A, B, C, and D. Sphere A has an initial charge of +12Q, sphere B of -14Q, and sphere C of +16Q. The initially neutral sphere D is touched to sphere A, then they are pulled apart. Sphere D is then touched to sphere B, and they are pulled apart. Finally sphere D is touched to sphere C, and they are pulled apart. What is the final charge on sphere D?

a) 8Q  

b) *6Q  

c) 3.5Q  

d) -6Q  

e) 0

The idea here is that each time you touch the two identical spheres the divide their charge up equally. So the first time, A and D divide up +12Q, leaving D with +6Q. Then D and B divide up -8Q (-14 + 6), leaving D with -4Q. Then D and C divide up +12Q, leaving D with 6Q.

2: Consider an infinitely long cylinder with radius R. The cylinder has a uniform charge density \( \rho \) (measured in C/m\(^3\)). What is the magnitude of the electric field at a point \( r \) which is inside the cylinder?

a) \( \frac{\rho \pi r}{2\varepsilon_0} \)

b) *\( \frac{\rho r}{2\varepsilon_0} \)

c) \( \frac{\rho}{\varepsilon_0} \)

d) \( \frac{\rho \pi r^2}{2\varepsilon_0} \)

e) \( \frac{2\rho \pi r}{\varepsilon_0} \)

Here you can use Gauss’ law:

\[
\iiint E \cdot d\mathbf{A} = \frac{q_{\text{inside}}}{\varepsilon_0}
\]

Choose a surface which is a cylinder of radius \( r \). The electric field points out from the center, and so the electric flux is just

\[
\iiint E \cdot d\mathbf{A} = E(2\pi rL) = \frac{q_{\text{inside}}}{\varepsilon_0}
\]

The charge inside is the charge density times the volume, so

\[
E(2\pi rL) = \frac{q_{\text{inside}}}{\varepsilon_0} = \frac{\pi r^2 L \rho}{\varepsilon_0}
\]

\[
E = \frac{r \rho}{2\varepsilon_0}
\]
3: Because charge can flow freely in them, conductors exhibit a variety of special properties. Which of the following statements about conductors in electrostatics is false?

a) *The electric potential everywhere inside a conductor must be zero
b) The total electric charge on a conductor may be positive or negative
c) The electric field inside a conductor is everywhere zero
d) Any excess positive or negative charge inside a conductor must be distributed on its surface
e) The electric field just outside the conductor is directed perpendicular to its surface

4: The electric field inside a parallel plate capacitor is $5 \times 10^3$ N/C. A proton starts out moving perpendicular to this field with a velocity of $3 \times 10^4$ m/s. What is the magnitude of the proton’s velocity after $10^{-7}$ seconds? The charge on the proton is $1.6 \times 10^{-19}$C and its mass is $1.67 \times 10^{-27}$kg.

a) $3.0 \times 10^4$ m/s
b) $6.2 \times 10^6$ m/s
c) *$5.7 \times 10^4$ m/s
d) $4.8 \times 10^4$ m/s
e) $1.5 \times 10^2$ m/s

The motion of the proton in the y direction remains unchanged. In the x direction it is accelerated by the field:

$v_x = a_x \Delta t$

$a_x = F_x / m = qE / m = 4.8 \times 10^{11}$ m/s$^2$

so at the end of this time

$v_x = 4.8 \times 10^{11} \times 10^{-7} = 4.8 \times 10^4$ m/s

and the total velocity is:

$v_{tot} = \sqrt{(v_x^2 + v_y^2)} = 5.7 \times 10^4$ m/s
5: An electric dipole is made of a positive charge \( q \) and a negative charge \( -q \) separated by a distance \( d \). This dipole is placed near a positive point charge \( Q \) in the position shown in the diagram, so that each of the dipole charges is a distance \( D \) from the point charge. What is the magnitude of the net force on the dipole?

The positive charge will be pushed away, and the negative charge attracted. You can see from the diagram that the \( y \) components of these forces cancel, and that the two \( x \) components add together.

\[
F_x^{+q} = -\frac{kqQ}{D^2} \sin(\theta) = -\frac{kqQ}{D^2} \frac{d}{2D}
\]

Where I have noted that \( \sin(\theta) = (d/2)/D \). Since the negative charge experiences an identical force the total is:

\[
F_x = -\frac{kqQ}{D^2} \frac{d}{D} = \frac{kqQd}{D^3}
\]

Then you want the magnitude of this.

6: Consider two point charges, \( A \) and \( B \), which have charges \( +Q \) and \( +3Q \) respectively. One has mass \( M \), the other has mass \( 2M \). The two charges are placed a distance \( R \) apart and released so that they fly off in opposite directions. What is the velocity of mass \( B \) when the two charges are finally very far apart?

Momentum and energy are conserved here. So \( PE_i = KE_f \) and \( P_i = P_f \)

\[
\frac{3kQ^2}{R} = \frac{1}{2} MV_A^2 + \frac{1}{2} (2M) V_B^2
\]

\[
MV_A = 2MV_B \quad \text{or} \quad V_A = 2V_B
\]

\[
\frac{3kQ^2}{R} = 2MV_B^2 + MV_B^2 = 3MV_B^2
\]

\[
V_B = \sqrt{\frac{kQ^2}{MR}}
\]
7: What is the total electrical potential energy of the arrangement of charges shown below? You may take the potential energy when the charges are infinitely far apart to be zero.

\[ \text{Here you just have to add up the potential energies of all the pairs:} \]
\[ \frac{-2kQ^2}{R} + \frac{2kQ^2}{2R} + \frac{4kQ^2}{3R} + \frac{-4kQ^2}{R} + \frac{-8kQ^2}{2R} + \frac{8kQ^2}{R} = \frac{kQ^2}{3R} \]

a) \( \frac{12kQ^2}{r} \)

b) \( \frac{-4kQ^2}{r} \)

c) \( * \frac{kQ^2}{3r} \)

\[ \text{You can understand this from Gauss’ law. If you draw a Gaussian surface around this region, it will have more electric flux coming into it than going out. Since the net electric flux will be negative, the total charge enclosed must be negative. There must be more negative than positive charge in this region.} \]

d) \( \frac{9kQ^2}{4r} \)

e) \( \frac{kQ^2}{r} \)

8: Suppose the electric field in some region is constant in direction, but decreases in strength along the direction the field points. What can you conclude about the distribution of charge in that region?

a) There must be some positive charge in that region
b) There must be no charge in that region
c) There must be equal amounts of positive and negative charge in that region
d) There must be more positive than negative charge in that region
e) *There must be more negative than positive charge in that region
9: On the Wimshurst machine we saw in class, sparks would connect the two brass balls when the electric field between the two balls was everywhere larger than the minimum field required to “break down” the air. If you take this breakdown field to be \(6 \times 10^6\) N/C, the distance between the balls to be 15 cm, and treat the balls as oppositely charged point charges, what is the minimum charge required to create this large a field everywhere between the two terminals?

a) \(1.9 \times 10^{-6}\) C
b) \(5.2 \times 10^{-2}\) C
c) \(1.8 \times 10^{-9}\) C
d) \(5.5 \times 10^{-5}\) C
e) \(2.6 \times 10^{-4}\) C

You need to make sure the electric field is large enough everywhere between these balls. Since you treat these as point charges, the field will be minimum directly between them. So you need to have:

\[
\frac{kQ}{\left(\frac{d}{2}\right)^2} + \frac{kQ}{\left(\frac{d}{2}\right)^2} = \frac{8kQ}{d^2} \quad \text{or} \quad Q = \frac{d^2E}{8k} = 1.9 \times 10^{-6}\ C
\]

10: A spacecraft propels itself using an ion motor which ejects protons out backwards. If the electric potential of the spacecraft is +40,000 V (with the potential at infinity defined as 0 V), and \(5 \times 10^9\) protons per second are ejected, what is the “thrust” of the motor? The thrust is defined as the total force exerted on a rocket by the fuel expelled out the back. It may help you to recall the impulse-momentum theorem: \(\vec{F}\Delta t = \Delta \vec{p}\).

a) \(1.9 \times 10^{-5}\) N
b) \(230\) N
c) \(2.3 \times 10^{-11}\) N
d) \(0.34\) N
e) \(8.1 \times 10^{-4}\) N

Each proton which is fired out converts it’s electric potential energy into kinetic energy. From this, you can find the final velocity of the protons:

\[qV = 6.4 \times 10^{-15} J = \frac{1}{2} m_p v_f^2 \quad \text{or} \quad v_f = 2.8 \times 10^6\ m/s\]

Once you know this, you know how much the momentum of each proton changes, and from this you can find the force:

\[F = \frac{\Delta p}{\Delta t} = 5 \times 10^9 \frac{\text{protons}}{s} \times 4.7 \times 10^{-21} \frac{kgm}{s \times \text{proton}} = 2.3 \times 10^{-11}\ N\]
11: The electric potential in a certain region can be written:

\[ V(x, y, z) = 4.0x^2 - 5.0y + 2.0z^3 + 24.6 \text{ (in volts)} \]

What is the associated electric field at the point \( x=2, y=4, z=2 \)?

a) \( \vec{E} = -16\hat{x} + 20\hat{y} - 18\hat{z} \) (in N/C)
b) \( \vec{E} = 8\hat{x} + 5\hat{y} - 12\hat{z} \) (in N/C)
c) \( \vec{E} = -16\hat{x} + 20\hat{y} - 18\hat{z} \) (in N/C)
d) * \( \vec{E} = -8x\hat{x} + 5\hat{y} - 6z^2\hat{z} \) (in N/C)
e) \( \vec{E} = 18\hat{x} - 5\hat{y} + 12\hat{z} \) (in N/C)

\[ \vec{E} = -\vec{\nabla}V = -\frac{\partial V}{\partial x}\hat{x} - \frac{\partial V}{\partial y}\hat{y} - \frac{\partial V}{\partial z}\hat{z} \]
\[ \vec{E} = -8x\hat{x} + 5\hat{y} - 6z^2\hat{z} \]

Now obviously when I wrote this I meant to plug the numbers for \( x, y, \) and \( z \) into the equation for answer d. But I didn’t, and none of the discussion instructors noticed this either. Sorry if that caused unnecessary confusion.

12: You could use two pennies to make a parallel plate capacitor. Given what you know about pennies, how far apart would you have to place the pennies to create a capacitor with a capacitance of \( 3 \times 10^{-9} \text{ Farads} \)?

a) \( 9 \times 10^{-12} \text{ m} \)
b) \( 3 \times 10^{-15} \text{ m} \)
c) \( 2 \times 10^{-18} \text{ m} \)
d) \( 4 \times 10^{-4} \text{ m} \)
e) *\( 9 \times 10^{-7} \text{ m} \)

A penny is about 1cm in radius. This makes its area about \( \pi r^2 = 3.1 \times 10^{-4} \text{ m}^2 \). The capacitance is \( C = \varepsilon_0 A / d \), or \( d = \varepsilon_0 A / C = 9 \times 10^{-7} \text{ m} \)
13: Two conducting spheres are nested together. The smaller one has radius \( a \), and it sits inside the second which has radius \( b \). What is the capacitance of this pair of nested spheres?

a) \( \frac{ab}{k(b-a)} \)

b) \( \frac{b}{k} \)

c) \( abk \)

d) \( \frac{b-a}{abk} \)

e) \( \frac{a}{bk} \)

The capacitance is determined from \( C = \frac{Q}{V} \). So we need to know the potential difference between the inner and outer shells. This is:

\[
V = kQ/a - kQ/b = kQ(b-a)/ab
\]

So the capacitance is:

\[
C = \frac{Q}{V} = \frac{ab}{k(b-a)}
\]

14: Two opposite electric charges \(+Q_1\) and \(-Q_2\) are separated by a distance \( b \). There is a Coulomb force between them. If I insert an electrically neutral metal cube between these two charges, which of the following statements is true?

a) *The electrostatic force on charge \( Q_1 \) increases

b) The electrostatic force on charge \( Q_1 \) decreases

c) The electrostatic force on charge \( Q_1 \) remains the same

d) The induced separation of charge in the metal sphere will create an internal electric field in the cube

e) There will be no electrostatic force on charge \( Q_1 \) in the presence of a metal

The force between the two charges increases. This is because the intervening metal cube becomes polarized, with negative charge lining the left side and positive charge lining the right. This adds to the electrical force pulling \( Q_1 \) to the right.
15: The net electric flux through the surface of a donkey was initially 150 \( \text{Nm}^2/\text{C} \). After eating a potato, the net flux is -250 \( \text{Nm}^2/\text{C} \). What was the charge on the potato?

a) \( 6.2 \times 10^{-12} \text{ C} \)
b) \( -4.7 \times 10^{-15} \text{ C} \)
c) \( *-3.5 \times 10^{-9} \text{ C} \)
d) \( 9.0 \times 10^{-13} \text{ C} \)
e) \( -4.9 \times 10^{-14} \text{ C} \)

Gauss’ law tells us that if the net flux through a surface (the Donkey) changes, the charge inside must change. Here, the flux change is:

\[
\Delta \text{Flux} = -250 \text{Nm}^2/\text{C} - 150 \text{Nm}^2/\text{C} = -400 \text{Nm}^2/\text{C}
\]

This change in flux comes about because of the change in charge

\[
\Delta \text{Flux} = \Delta q / \varepsilon_0
\]

Or

\[
\Delta q = \varepsilon_0 \Delta \text{Flux} = -3.5 \times 10^{-9} \text{ C}
\]

16: A small charged sphere hangs in a lab from the side of a functionally infinite charged plate as shown in the picture below. If the positive surface charge density on the plate is \( \sigma \), the charge on the ball is \( q \), and the angle the string makes with the vertical is \( \theta \), what is the mass of the ball?

a) \( 2\sigma q / \tan(\theta)g\varepsilon_0 \)
b) \( 2\tan(\theta)\varepsilon_0 / \sigma qg \)
c) \( *\sigma q / 2\tan(\theta)g\varepsilon_0 \)
d) \( 2\sigma g / \tan(\theta)q\varepsilon_0 \)
e) \( \sigma q\tan(\theta) / 2g\varepsilon_0 \)

Here you are summing forces:

\[
\Sigma F_x = T \sin(\theta) - qE = T \sin(\theta) - q\sigma / 2\varepsilon_0 = 0 \quad \text{or} \quad T = q\sigma / 2\varepsilon_0 \sin(\theta)
\]

\[
\Sigma F_y = T \cos(\theta) - mg = 0 \quad \text{or} \quad m = T \cos(\theta) / g
\]

Combining these gives:

\[
M = q\sigma \cos(\theta) / 2g\varepsilon_0 \sin(\theta) = q\sigma / 2g\varepsilon_0 \tan(\theta)
\]
17: The drawing below shows an electrically neutral conductor with a small cavity in it. In the cavity there is a point charge $+Q$. Taking the potential at infinity to be zero, which of the following curves could represent the electric potential as a function of position along the solid line drawn through the figure?

![Diagram of electric potential curves]

B has potential falling off linearly outside the conductor, C has it dropping to zero, D and E both have the potential changing in the conductor. Only A is possible.

18: Consider two conducting spheres, sphere A with radius R and sphere B with radius $5R$, charged so that the electric potential at their surfaces is the same. Which of the following statements about the charge they hold ($Q_A$ and $Q_B$) and the electric field near their surfaces ($E_A$ and $E_B$) is correct? You can assume the spheres are isolated, and do not affect one another.

a) $Q_A = 5Q_B$ and $E_A = 25E_B$
b) $Q_A = Q_B/5$ and $E_A = E_B/5$
c) $Q_A = Q_B/5$ and $E_A = 5E_B$
d) $Q_A = 5Q_B$ and $E_A = 5E_B$
e) $Q_A = Q_B$ and $E_A = 5E_B$

$V = kQ/R$
So the large sphere has to have more charge, five times more, to maintain the same potential. Given this $E = kQ/R^2$
The larger one has 5 times the charge, but $R^2$ is 25 times as big, so its electric field is $1/5$ as large.
19: What is the equivalent capacitance of the network of identical capacitors shown in the picture if the capacitance of each capacitor is C?

a) 6C  
b) 0.5C  
c) 2.5C  
d) C/3  
e) 3C

Each of the two pairs of series capacitors becomes C/2. They are then in parallel and become just C. Now you have three capacitors which are identical in series, and their equivalent capacitance is C/3.

20: You are handed an object. How do you tell if it is negatively charged? Give the most inclusive answer possible.

a) See if it attracts a neutral conductor or a neutral insulator  
b) See if it attracts a positively charged conductor  
c) See if it attracts a positively charged conductor or insulator  
d) *See if it repels a negatively charged insulator  
e) Any of the above

The only way to be sure and object is negatively charged is to see if it repels a negatively charged object.