Physics 240 Fall 2003: Exam #2 Solutions

Please print your name:___________________________________________________

Please list your discussion section number:_________________________________

Please list your discussion instructor:_______________________________________

Form #1

Instructions

1. Fill in your name above
2. This will be a 1.5 hour, closed book exam. The exam includes 20 questions.
3. You may use a calculator, please do not share calculators
4. You may use two 3”x5” note cards (one from the first exam, plus one new one) with
   notes and equations you think may be useful. You can write on both sides of the card
   if you like.
5. You will be asked to show your University student ID card when you turn in your
   exam.

ID checked by:__________________________________________________________

Table of constants:
\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 \]
\[ k = 1/(4\pi\varepsilon_0) \]
\[ q_{\text{electron}} = -1.6 \times 10^{-19} \text{ C} \]
\[ q_{\text{proton}} = 1.6 \times 10^{-19} \text{ C} \]
\[ m_{\text{electron}} = 9.1 \times 10^{-31} \text{ kg} \]
\[ m_{\text{proton}} = 1.67 \times 10^{-27} \text{ kg} \]
\[ \mu_0 = 4\pi \times 10^{-7} \text{ Tm/A} \]
\[ G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \]
1: Which of the following is equivalent to the quantity 1000 Ohms * 20 microFarads?

A) 20000 Joules/second  
B) 0.02 kilogram*meters/second  
C) 0.0002 volts  
D) *0.02 seconds  
E) 20000 Newtons/Coulomb

This is an RC time:

\[ 1000 \Omega \times 20 \mu F = 0.02 \text{ seconds} \]

2: What is the equivalent resistance of the resistor network seen by the battery shown in the figure?

A) 5R/2  
B) *5R/3  
C) 2R  
D) 11R/5  
E) R/2

This problem has one feature you have to be careful about. The resistor coming down on the diagonal is actually shorted out. That is, it has a wire connected from one end to the other. This means the potential difference across this resistor will be zero. No current will flow through it and there will be no voltage drop across it. It has no effect on the circuit. You can see this most easily in the redrawn circuit below.

Working on this redrawn circuit, we have resistors of R and 2R in parallel, which has an equivalent resistance of 2R/3. This combination is in series with the resistor R, so the total resistance is 5R/3.
3: Three identical light bulbs are connected in a circuit as shown. Each of the bulbs has a resistance of 30Ω, and the battery supplies a voltage of 30V. How much power does light B consume?

A) *30W  
B) 60W  
C) 10W  
D) 20W  
E) 50W

Each bulb sees 30V across it, so the power through each is just: 
\[ P = \frac{V^2}{R} = \frac{30^2}{30} = 30W \]

4: A mass spectrometer accelerates a singly charged ion to a velocity of 4.5x10^5 m/s before injecting it into region with a constant 0.400 Tesla field. A ‘singly charged ion’ has a charge equal in magnitude to the electron charge. If it bends to the left in a circular path with a radius of 25 cm, what is the mass of the ion?

A) 2.6x10^{-27} kg  
B) 9.1x10^{-31} kg  
C) *3.6x10^{-26} kg  
D) 8.3x10^{-25} kg  
E) 3.3x10^{-24} kg

The centripetal force for circular motion in a mass spectrometer is provided by the magnetic force:
\[ \frac{mv^2}{r} = qvB \quad \text{or} \quad m = \frac{qvB}{v} \]
\[ m = \frac{(0.25m*1.6x10^{-19}C*0.4T)}{4.5x10^5 \text{ m/s}} \]
\[ m = 3.6x10^{-26} \text{ kg} \]
5: Five ions, labeled A-E, all enter a region of uniform magnetic field with the same velocity. **Which ion lands at position 3?**

<table>
<thead>
<tr>
<th>Ion</th>
<th>Mass</th>
<th>Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2 units</td>
<td>+e</td>
</tr>
<tr>
<td>B</td>
<td>4 units</td>
<td>+e</td>
</tr>
<tr>
<td>C</td>
<td>6 units</td>
<td>+e</td>
</tr>
<tr>
<td>D</td>
<td>2 units</td>
<td>-e</td>
</tr>
<tr>
<td>E</td>
<td>4 units</td>
<td>-e</td>
</tr>
</tbody>
</table>

There are three positively charged particles. They curve to the left. They all have the same positive charge and same velocity. So the one which curves most quickly, and lands at position three, must be the one with the smallest mass. It is easiest to turn in a circle. The answer is A.

6: In Oersted’s laboratory a compass needle is aligned with a constant magnetic field. This field has a magnitude of $6.0 \times 10^{-6}$ T, and points in the $y_{\text{unit}}$ direction. A wire, extending up and down in the $z_{\text{unit}}$ direction, passes through the table 30 cm from the compass, as shown in the figure. What current must he pass through this wire to make the compass point $45^\circ$ to the right of the $y_{\text{unit}}$ direction? You may treat the wire as infinite.

A) $9.0$ A in the $-z_{\text{unit}}$ direction
B) $6.3$ A in the $-z_{\text{unit}}$ direction
C) $6.3$ A in the $z_{\text{unit}}$ direction
D) $12.6$ A in the $-z_{\text{unit}}$ direction
E) $8.4$ A in the $-z_{\text{unit}}$ direction

In this problem you use the compass needle to tell you the direction of the total magnetic field. Since the needle turns by $45^\circ$, you know that the $x$ component of the magnetic field, which is created by current in the wire, must be equal in magnitude to the $y$ component which is imposed initially. So you have:

$$\mu_0 I / 2\pi = 6.0 \times 10^{-6} \text{ T}$$

or

$$I = 2\pi * 6.0 \times 10^{-6} \text{ T} / \mu_0 = 9.0 \text{ Amps}$$

It must be going into the page (the $-z_{\text{unit}}$ direction) to have the right effect.
7: A circular loop with radius \( r \) lies flat in the \( xy \) plane. If I look down the \( z \) axis I see a current \( i \) circulating in the counterclockwise direction in the loop. The loop is immersed in a magnetic field \( \mathbf{B} = (B_x \hat{\mathbf{x}} \text{unit} + B_y \hat{\mathbf{y}} \text{unit} + B_z \hat{\mathbf{z}} \text{unit}) \). What is the \textbf{magnitude} of the magnetic torque felt by the loop?

A) \( i \pi r^2 (B_x^2 + B_y^2 + B_z^2)^{1/2} \)

B) \( i \pi r^2 (B_x^2 + B_y^2 + B_z^2)^{3/2} \)

C) \( *i \pi r^2 (B_y^2 + B_z^2)^{1/2} \)

D) \( i \pi r^2 (B_x^2 - B_y^2)^{1/2} \)

For this problem we rely on the definition of magnetic torque:
\[
\tau = \mu \times \mathbf{B}
\]
where the magnetic moment of the current loop is given by \( iA \), with a direction from the right hand rule. So the magnetic moment here is:
\[
\mu = i \pi r^2 \hat{\mathbf{z}} \text{unit}
\]
Taking the cross product of this with the magnetic field given provides:
\[
\tau = i \pi r^2 \hat{\mathbf{z}} \text{unit} \times (B_x \hat{\mathbf{x}} \text{unit} + B_y \hat{\mathbf{y}} \text{unit} + B_z \hat{\mathbf{z}} \text{unit})
= i \pi r^2 (B_y \hat{\mathbf{y}} \text{unit} - B_z \hat{\mathbf{z}} \text{unit})
\]
The magnitude of this vector is then the square root of the sum of the squares of its components:
\[
|\tau| = i \pi r^2 (B_x^2 + B_y^2)^{1/2}
\]

8: A capacitor is charged by a battery until the charge on the capacitor is \( Q \). It is then disconnected from the battery, leaving the charge in place. A dielectric material with dielectric constant \( K \) is now slowly inserted into the capacitor. How does the energy stored in this capacitor change in this process?

A) The energy stored increases by a factor \( K \)

B) The energy stored increases by a factor \( K^2 \)

C) \( *\)The energy stored decreases by a factor \( 1/K \)

D) The energy stored decreases by a factor \( 1/K^2 \)

E) The energy stored remains unchanged

Once the battery is disconnected, it’s the charge on the capacitor that remains fixed. When the dielectric is put in place, the effective capacitance changes to \( C_{\text{eff}} = \kappa C \). How does this change the energy stored? This stored energy is given by \( Q^2/C \), and since \( C \) becomes larger while \( Q \) is fixed, the stored energy of the capacitor goes down by a factor of \( K \).

Why does the stored energy go down? The dielectric reduces the electric field inside the capacitor, making the positive and negative charges pull on each other less strongly. Thus there is less energy to release stored in their separation.
9: A three way light bulb has two filaments with resistances 50Ω and 100Ω connected in series. The resistors are connected to three terminals as indicated in the figure. The light switch determines which two of the three terminals are connected to a potential difference of 120V at any given time. Which two terminals should you connect to make the light bulb shine most brightly?

A) A and C  
B) B and C  
C) C and B  
D) *A and B  
E) A and C

Since the voltage is fixed at 120V, you want to connect this across the smallest possible resistance to get the largest power out \((P = V^2 / R)\). Hooking it across A and B will do this.

10: What is the equivalent capacitance between points A and B for the group of capacitors shown in the figure?

A) 14.1µF  
B) 9.5µF  
C) *21.0µF  
D) 26.3µF  
E) 12.4µF

The equivalent capacitance here is:
\[ C_{eq} = 15\mu F + \left( \frac{1}{1/8.2\mu F + 1/22\mu F} \right) = 21\mu F \]
11: You find a 20V battery on the shelf and want to determine its internal resistance. To find out, you connect a 200Ω resistor across the terminals of the battery and measure a current of 65mA. What is the internal resistance of the battery?

A) 12Ω  
B) 28Ω  
C) 36Ω  
D) *108Ω  
E) 8Ω  

The current is given by:

\[ I = \frac{V}{R_{\text{ext}} + R_{\text{int}}} \]

Or

\[ R_{\text{ext}} + R_{\text{int}} = \frac{V}{I} \]

Or

\[ R_{\text{int}} = \frac{V}{I} - R_{\text{ext}} \]

\[ = \frac{20}{0.065} - 200 = 108 \Omega \]

12: An ion with a positive charge of 4.8x10^{-19} C and a mass of 6.6x10^{-25} kg has a velocity of 3x10^6 m/s. This particle enters a magnetic field, pointing out of the page, with magnitude 3.5x10^{-3}T. The particle travels in a semicircle, eventually leaving the region with the magnetic field. What is the kinetic energy of the particle when it is at the point shown, halfway through this semicircle?

A) *3.0x10^{-12} J  
B) 6.8x10^{-11} J  
C) 4.4x10^{-11} J  
D) 1.7x10^{-11} J  
E) 5.5x10^{-11} J  

The kinetic energy of the particle will be unchanged by this field, which can do no work on the particle. So when it’s halfway through, it has the same KE it started with:  
\[ KE = \frac{1}{2}mv^2 = 0.5 \times 6.6 \times 10^{-25} \times (3 \times 10^6)^2 = 3.0 \times 10^{-12} J \]
13: Consider a flat rectangular conductor connected to a battery as shown. The conductor is embedded in a magnetic field coming out of the page. The electric potential on the conductor is larger on the left than on the right. Which of the following statements is true?

A) The charges which flow feel a magnetic force into the page  
B) The charges which flow feel a magnetic force to the right  
C) The charges which flow in this conductor must be negative  
D) The electric field in the conductor points to the left  
E) *The charges which flow in this conductor must be positive

Consider each statement in turn:
A) Since the field comes out of the page, and the charges move up (if negative) or down (if positive), the magnetic force must be to the left.
B) See A....
C) The electric potential is larger on the left, so we can’t have negative charges piling up on the left, where they would be forced by the magnetic force if they were moving. So this is false.
D) Larger potential on the left means the field points right.
E) Positive charges flowing would be pushed left, making the potential there higher, so this is correct.

14: A high voltage power line carries a current of 110A at a location where the Earth’s magnetic field has a magnitude of 6x10^{-4} T and points north and dips down 32° below the horizontal (toward the ground). Calculate the magnitude of the magnetic force on a 250 meter segment of wire in which current flows east.

A) 8.7 N  
B) *16.5 N  
C) 14.0 N  
D) 4.3 N  
E) 0.87 N

Here we just use: 
\[ F_B = iL \times B \]

The angle between L and B is 90°, even though the field is dipping down. So the magnetic force is just: 
\[ F_B = 110A \times 250m \times 6x10^{-4} = 16.5N \]
15: We might make a simple model for the hydrogen atom in which the electron orbits the proton in a circle just like the Moon orbits the Earth. This orbiting electron travels with speed $2.2 \times 10^6$ m/s in a circle with radius $5.3 \times 10^{-11}$ m. To us this orbiting electron looks like a little current loop. In this model, what is the magnetic moment we would associate with this current loop?

A) $5.2 \times 10^{-23}$ Am$^2$
B) $4.2 \times 10^{-21}$ Am$^2$
C) $5.4 \times 10^{-23}$ Am$^2$
D) $4.0 \times 10^{-26}$ Am$^2$
E) *$9.4 \times 10^{-24}$ Am$^2$

Current is $I = dq/dt$. Here the charge is the electron charge and the time is the time it takes to orbit once:

\[
I = q_e / \left(2\pi r / v \right)
\]

\[
= 1.6 \times 10^{-19} \text{C} / \left(2\pi \times 5.3 \times 10^{-11} \text{m} / 2.2 \times 10^6 \text{m/s} \right)
\]

\[
= 1.1 \times 10^{-3} \text{A}
\]

Given this, the magnetic moment is:

\[
\mu = iA = 1.1 \times 10^{-3} \text{A} * \pi * (5.3 \times 10^{-11})^2 = 9.4 \times 10^{-24} \text{Am}^2
\]

16: Four long parallel conducting wires, arranged in a square, carry the same current $I$ in a direction perpendicular to the page. In which of the following arrangements is the magnitude of the magnetic field at the center of the square largest?

In every case in which wires on opposite corners conduct in the same direction there is a cancellation of magnetic field from this pair. So in cases A, B, and D, there is zero total field at the center. In case C there is a non-zero field, created by the lower left and upper right wires. But in case E, there are two pairs (upper left and lower right, then lower left and upper right) which contribute to the field. Hence the total field at the center in E is larger.
17: A rectangular loop of area $A$ carries a current $I_{\text{loop}}$ in the direction shown in the figure, forming a magnetic dipole with dipole moment $\mu = I_{\text{loop}}A$. This loop is placed next to a long straight wire which carries a current $I_{\text{wire}}$. Which of the following statements correctly describes what happens?

A) The loop feels a torque pulling side A into the page and pushing side B out of the page.
B) The loop feels a torque pushing side A out of the page and pulling side B into the page.
C) The loop feels no net force or torque.
D) *The loop feels a net force pulling it toward the wire.*
E) The loop feels a net force pushing it away from the wire.

![Diagram of a loop and a wire]

The field from the long wire points into the page in the region of the loop. Side A feels a force toward the long wire, side B feels a force away. The forces on the other two sides cancel. But the force on A is larger than on B, because the field from the long wire is falling off with distance from it.

18: Consider the set of light bulbs, powered by a battery, shown in the figure. With the switch open, only bulbs A and B are lit. When the switch closes, how does the circuit change?

A) Bulbs A and B get brighter
B) Bulb A gets brighter, B remains the same
C) Bulb A remains the same, B gets brighter
D) Bulb A and B get fainter
E) *Bulb A gets brighter, B gets fainter*

When the switch is closed the net resistance of the circuit goes down. This means the total current increases. Since all the current goes through A, we know it must get brighter. But now only half the total current goes through B, so it gets dimmer.
19: Which of the following statements about voltmeters and ammeters is correct?

A) To function properly, a voltmeter must have very small internal resistance
B) To function properly, a voltmeter must be placed in series with the circuit element it is meant to measure
C) To function properly, an ammeter must have a very large internal resistance
D) *To function properly, an ammeter must be placed in series with the circuit element it is meant to measure
E) To function properly, a voltmeter should be placed in parallel with the ammeter it is being used with

20: A cylinder of material 0.5 m long and 8 cm in diameter is connected across the terminals of a 9 V battery and a current of $3.5 \times 10^{-3}$ A is seen to flow. What is the resistivity of this material?

A) $3.9 \times 10^{-4}$ Ωm
B) 227 Ωm
C) *26 Ωm
D) $9.4 \times 10^{-2}$ Ωm
E) $3.1 \times 10^{-3}$ Ωm

What’s the resistance of the cylinder?

$$V = IR \quad \text{or} \quad R = \frac{V}{I}$$

$$= \frac{9V}{3.5 \times 10^{-3}} = 2.6 \times 10^{3} \Omega$$

This resistance comes from resistivity and geometry:

$$R = \frac{\rho(L/A)}{L} \quad \text{or} \quad \rho = \frac{RA}{L}$$

$$\rho = 2.6 \times 10^{3} \Omega \times (\pi \times 0.04^{2}) / 0.5 = 26 \Omega m$$