\( \beta = 10 \log_{10} \left( \frac{I}{I_0} \right) \)

\[ I = 10^{-6} \text{ W/m}^2 \quad I_0 = 10^{-12} \text{ W/m}^2 \]

\[ \frac{I}{I_0} = \frac{10^{-6}}{10^{-12}} = 10^6 \]

\[ \beta = 10 \log_{10} (10^6) = 10 \cdot 6 = 60 \text{ dB} \]

\( \beta = 10 \log_{10} \left( \frac{I}{I_0} \right) \)

\[ I = \frac{\text{Power}}{\text{Area}} = \frac{P}{4\pi R^2} \]

\[ I_1 = \frac{P}{4\pi R_1^2} \quad I_2 = \frac{P}{4\pi R_2^2} \]

\[ \frac{I_2}{I_1} = \frac{\frac{P}{4\pi R_2^2}}{\frac{P}{4\pi R_1^2}} = \frac{R_1^2}{R_2^2} \]

\[ \frac{I_2}{I_1} = \frac{\frac{R_1^2}{(3R_1)^2}}{\frac{1}{9}} = \frac{1}{9} I_2 = \frac{I_1}{9} = 10^{-6} \text{ W/m}^2 \]

\[ I_2 = 1.11 \times 10^{-7} \text{ W/m}^2 \]

\[ \beta_2 = 10 \log_{10} \left( \frac{1.11 \times 10^{-7} \text{ W/m}^2}{10^{-12} \text{ W/m}^2} \right) = 50.45 \text{ dB} \]
\[ I = \frac{40 \, \text{W}}{4\pi (6\Omega)^2} = 0.0884 \, \text{W/m}^2 \]  

\[(3)\]

\[ \beta = 5 \, \text{dB} \]

\[ \beta = 10 \log_{10} \frac{I}{I_0} \]

\[ \frac{P}{4\pi R^2} \]

\[ I = \left[ 10^{-12} \, \text{W/m}^2 \right] 10^{5/10} = 3.162 \times 10^{-12} \, \text{W/m}^2 \]

\[ I = \frac{P}{4\pi R^2} \quad \text{R}_0 \text{ does not change} \Rightarrow (R_2 = R_1) \]

\[ \frac{I_2}{I_1} = \frac{P_2}{4\pi R^2} \cdot \frac{4\pi R_1^2}{P_1} = \frac{P_2}{P_1} \Rightarrow \]

\[ I_2 = \frac{P_2}{P_1} I_1 = \frac{50 \, \text{W}}{5 \, \text{W}} \left( 3.162 \times 10^{-12} \, \text{W/m}^2 \right) = 3.162 \times 10^{-11} \, \text{W/m}^2 \]

\[ \beta_2 = 10 \log_{10} \left[ \frac{3.162 \times 10^{-11} \, \text{W/m}^2}{10^{-12} \, \text{W/m}^2} \right] = 14.996 \, \text{dB} = 15 \, \text{dB} \quad \text{(A)} \]
(4) ALTERNATIVE SOLUTION

\[(B_2 - B_1) = 10 \log_{10} \left[ \frac{I_2}{I_1} \right] \]

\[
B_2 = B_1 + 10 \log_{10} \left[ \frac{I_2}{I_1} \right]
\]

\[
\frac{I_2}{I_1} = \frac{P_2}{P_1} \quad (R \text{ is constant}) = \frac{50 \text{W}}{5 \text{W}} = 10
\]

\[
B_2 = 5 \text{dB} + 10 \log_{10} 10
\]

\[
B_2 = 5 \text{dB} + 10 \text{dB}
\]

\[
B_2 = 15 \text{dB} \quad (A)
\]
(5) \[ I = 10^{-12} \text{ W/m}^2 \]

\[ R = 1.0 \text{ m} \Rightarrow P = 4\pi R^2 I = 4\pi (1\text{ m})^2 \times 10^{-12} \text{ W/m}^2 \]

\[ P = \frac{1.2566 \times 10^{-11}}{\text{J/s}} \]

\[ E = P\Delta t = \frac{1.2566 \times 10^{-11}}{\text{J/s}} \times 100\text{s} \]

\[ E = 1.2566 \times 10^{-9} \text{ J} \]

(6) \[ N_{\text{mosq.}} \times P = 10 \text{ W} \]

\[ P = 1.2566 \times 10^{-9} \text{ W} \]

\[ = N_{\text{mosq.}} = \frac{10 \text{ W}}{1.2566 \times 10^{-9} \text{ W}} = 7.957 \times 10^{9} \text{ mosquitoes} \]

(7) \[ I = I_0 = \frac{P_{\text{swarm}}}{4\pi R^2} \]

\[ R = 10 \text{ meters} \]

\[ P_{\text{swarm}} = N_{\text{swarm}} \times P_{\text{mosquito}} \]

\[ I_0 = \frac{N_{\text{swarm}} \times P_{\text{mosquito}}}{4\pi R^2} \]

\[ N_{\text{swarm}} = \frac{(4\pi)(10 \text{ m}^2)10 \text{ W/m}^2}{(4\pi)(1 \text{ m})^2} = 100 \]

\[ P_{\text{mosquito}} = 4\pi (1\text{ m})^2 \times I_0 \]
\[
\frac{I_2}{I_1} = 10 \quad \frac{(B_2 - B_1)}{10}
\]

\[
(B_2 - B_1) = 10 \log_{10} \left( \frac{I_2}{I_1} \right)
\]

\[
B_1 = 20 \text{ dB} \quad R_1 = 30 \text{ m} \\
R_2 = 3 \text{ m}
\]

\[
\Rightarrow \quad \frac{I_2}{I_1} = \frac{(30 \text{ m})^2}{(3 \text{ m})^2} = 100
\]

\[
\Rightarrow B_2 = B_1 + 10 \log_{10} \left( \frac{I_2}{I_1} \right)
\]

\[
B_2 = 20 \text{ dB} + 10 \log_{10} (100)
\]

\[
B_2 = 20 \text{ dB} + 10 \times 2 \text{ dB} = 40 \text{ dB} \quad \text{(B)}
\]
What \( f \) to "SEE" an insect which is 1 cm (10\(^{-2}\) m) in size?

"SEE" \( \equiv \lambda < 1 \text{ cm} \)
\[ \lambda < 10^{-2} \text{ m} \]

\[ v = f \lambda \quad \text{CALCULATE} \quad v \Rightarrow \lambda = 10^{-2} \text{ m} \]

\[ f = \frac{v}{\lambda} = \frac{343 \text{ m/s}}{10^{-2} \text{ m}} = 343 \times 10^{2} \text{ Hz} \]
\[ = 3.43 \times 10^{4} \text{ Hz} \]
\[ = 34,300 \text{ Hz} \]

\[ \Rightarrow f > 34.3 \text{ kHz (B)} \]
\( f_0 = \frac{f_s \left[ \frac{1 + \frac{V_b}{V_s}}{1 + \frac{V_s}{V_b}} \right]}{1 + \frac{V_b}{V_s}} \)

- \( V_b \): observer's speed
- \( V_s \): source's speed
- \( V \): sound speed

\[ \begin{align*}
V_{\text{bat}} &\rightarrow V_{\text{bat}} = V_{\text{insect}} \\
&= 10 \text{ m/s EAST}
\end{align*} \]

There is zero relative speed between BAT and INSECT, so the frequency that the insect receives is the same as the source frequency. The reflected signal the BAT receives is also \( f_{\text{source}} \) with no shift in frequency.

Bat detects original frequency \( = 30 \text{ kHz} \) (B)

"NO RELATIVE SPEED <=> NO DOPPLER SHIFT!"
(5) \[ V_{\text{bat}} = 15 \text{ m/s} \]

Obstacle which is stationary.

I) What is frequency observed at obstacle?

\[ f_0 = f_s \left[ \frac{1 \pm \frac{V_0}{V_{\text{sound}}}}{1 \pm \frac{V_s}{V_{\text{sound}}}} \right] \]

\[ V_0 = 0 \quad (\text{Obstacle is stationary}) \]

Source is approaching observer ->

Use top symbol ->

\[ f_0 = f_s \left[ \frac{1}{1 - \frac{V_s}{V}} \right] \]

\[ V_s = V_{\text{bat}} \]

II) Now consider the reflected sound from the obstacle to be the new source and apply Doppler formula to approaching obstacle, the bat!
\[ f_s' = f_s \left[ \frac{1}{1 - U_{bat}/v} \right] \]

\[ f_0 = f_{\text{detected by bat}} = f_s' \left[ \frac{1 + U_0/v}{1 + U_s/v} \right] \]

Observer (Bat) is approaching \( \Rightarrow \) Top symbol

Wall is stationary \( \Rightarrow v_s = 0 \)

\[ f_{\text{detected by bat}} = f_s' \left[ 1 + U_{bat}/v \right] \]

\[ f_{\text{detected by bat}} = f_s \left[ \frac{1}{1 - U_{bat}/v} \right] \left[ 1 + U_{bat}/v \right] \]

Original "squeak" frequency of Bat

\[ f_{\text{detected by bat}} = f_s \left[ \frac{1 + U_{bat}/v}{1 - U_{bat}/v} \right] \]

\[ f_{\text{detected}} = 50 \text{ kHz} \left[ \frac{1 + 15m/5/343 m/s}{1 - 15m/5/343 m/s} \right] = 54.573 \text{ kHz} \]

\[ \approx 55 \text{ kHz (D)} \]

\[ \text{N.B.: } f_{\text{detected}} > f_{\text{bat}} \Rightarrow \text{D is only possible correct answer!} \]