Comparative Vagueness*

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Draft of February 2019

Abstract

Despite vast literatures on linguistic vagueness, little attention has been given to comparatives. It is often assumed in linguistic semantics that explicit comparatives ‘x is ADJ-er than y’ cannot be vague. I show that certain adjectives exhibit vagueness phenomena such as sorites-sensitivity in the comparative form. Such phenomena cannot be reduced to cases of indiscriminability or fuzziness in relevant dimensions, standards, or measurement procedures. The comparative sorites arguments I provide raise a distinctive challenge for traditional degree-based and non-degree-based (delineation, partial-predicate, supervaluationist) semantics for gradation. In response I offer a degree-based semantics based on semiorders. The proposed framework captures vagueness phenomena with both positive (unmarked) and comparative forms. I close with discussion of possible further implications for typologies of gradable adjectives.

1 Comparative vagueness

Positive-form predications (‘is tall’, ‘is bald’) reign supreme in discussions of linguistic vagueness. Despite the vast literature, little attention has been given to phenomena with comparatives. It is often assumed in linguistic circles that explicit comparatives ‘x is ADJ-er than y’ using a comparative morpheme — are not vague;¹ for instance (see also Cooper 1995: 246; Kennedy 2007: 6, 2011: 74, 82–83, 93, 2013: 271; McNally 2011: 164n.10; van Rooij 2011a: 65–69):

*Thanks to Gunnar Björnsson and Nick Jones for discussion. Certain ideas in the paper draw on material from Silk 2016: chs. 6–7.

¹Unless otherwise noted, by ‘comparative’ I will mean ‘explicit comparative’ in this sense. I consider (so-called) “implicit comparatives,” which use the positive (unmarked) form, in §4.
“[T]he comparative form … is not vague… [A] core semantic difference between the positive [(unmarked)] and comparative forms … is that the latter lacks whatever semantic (or pragmatic) features give rise to the vagueness of the former, and simply expresses an asymmetric ordering relation.” (Kennedy 2013: 269–270)

“[A]djectives in the comparative are uniformly non-vague.” (Bochnak 2013: 56)

Such remarks are generally made in passing with an eye toward a limited range of “prototypical relative adjectives” (McNally 2011: 163) such as ‘tall’. Here is Kennedy:

“The comparative predicate taller than David … denotes a property that is true of an object just in case its height exceeds David’s height. This is a precise property, and is moreover fully objective, since whether it holds of an object or not is fully determined by facts about that object’s height.” (Kennedy 2013: 270)

Vagueness is treated as due to a fuzziness in standards of application — how many millimeters of height one must have to count as tall, how many cents one must have to count as rich, and so on. Hence, “Unsurprisingly, comparatives … do not give rise to the Sorites paradox, and do not have borderline cases” (McNally 2011: 164n.10).

But they do.

Suppose you are forced to decide between saving your dearest friend and saving some number of strangers. Plausibly we have some special obligations to those close to us, so that it is morally better for you to save your dear friend than to save two strangers. But there doesn’t seem to be any precise number of strangers that would tip the balance. Now consider:

(1) (P1) Your saving 2 strangers is not morally better than your saving your dear friend.

(P2) For all n, if your saving n strangers is not morally better than your saving your dear friend, then your saving n + 1 strangers is not morally better than your saving your dear friend.

(C) ∴ For all n > 2, your saving n strangers is not morally better than your saving your dear friend.

No one’s friends are that important.
Or suppose you like sugar in your coffee. Yet it’s not as if you care exactly how sweet it is. As far as your preferences go, one day’s sweetness is as good as any other (okay, at least up to a point, say $K$; there is, perhaps, such a thing as too sweet). Now consider (2) — letting $x_i$ be an ordinary cup of coffee, and $x_1 \ldots x_n \ldots x_K$ be a series of otherwise identical cups differing only in quantity of sugar, with $x_n$ being a (pre-$K$) cup with $n$ micrograms of sugar (cf. Luce 1956).

(2) (P1) $x_i$ is more preferable than $x_s$.
(P2') For all $n < K$, $x_n$ is as preferable as $x_{n+1}$.
(P3) For all $a$, $b$, $c$, if $a$ is more preferable than $b$, and $b$ is as preferable as $c$, then $a$ is more preferable than $c$. (PI-transitivity)
(C) \[ \therefore \] For all $n < K$, $x_i$ is more preferable than $x_n$.

Or in a perhaps more familiar form:

(3) (P1) $x_1$ is not more preferable than $x_s$.
(P2) For all $n$, if $x_n$ is not more preferable than $x_s$, then $x_{n+1}$ is not more preferable than $x_s$.
(C) \[ \therefore \] For all $n$, $x_n$ is not more preferable than $x_s$.

But not just any cuppa can be the best.

That is: The premises seem true — in (1) given the nature of morality, in (2)/(3) given the nature of one’s preferences; the arguments seem valid; yet the conclusions are false. There may be something wrong with sugar in one’s coffee, but not that thinking otherwise leads to paradox.

It is important to be clear about the force of comparative sorites cases such as (1)–(3). Unlike previous examples of comparative vagueness, (1)–(3) cannot be reduced to cases of indiscriminability between adjacent items in a sorites series, indeterminacy (uncertainty, indecision) in what dimensions are relevant, or unsettledness (indecision, imprecision) about measurement procedures (contrast Williamson 1994: 156; Endicott 2000: 43–45, 149–153; Keefe 2000: 13–14; Sassoon 2013: 76, 172–173). Fix on a particular dimension for preferability or moral value, and procedure for measuring/determining it, and the force of the comparative sorites remains. Many a monistic indirect consequentialist have countenanced

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2 Or consider Pat, who does, and read ‘preferable’ below as ‘preferable to Pat’. (Likewise for ‘morally good’ and ‘morally good according to Kant’ (or some such.).)

3 More on this shortly. See Hyde & Raffman 2018 for various formulations of the paradox.

4 Williamson and Keefe mention borderline cases arising from issues regarding measurement procedures and multidimensionality; Endicott and Sassoon appeal to multidimensionality (I am not aware of other precedents in the literature):
special obligations. Moreover the concern with denying the inductive premises isn’t simply that doing so would be unwarranted or in tension with a limited power of discrimination. As Wright (1987: 239–243) shows, indiscriminability between adjacent items is insufficient to generate the paradox.⁵ Only a maximally opinionated coffee maven could deny \((P_2')\) in (2). Discriminable though the adjacent cups might be, whether in quantity of sugar or quality of sweetness, one cup is as good as the next given one’s preferences.

Upshot: Linguistic vagueness can be associated not only with how ADJ something needs to be to count as ADJ, but with how ADJ things are. The latter sort of vagueness cannot be reduced to cases of indiscriminability or fuzziness in relevant dimensions, standards, or measurement procedures.

Comparative sorites arguments such as (1)–(3) raise a distinctive challenge for traditional semantics for gradation. To see this, let’s start by examining how (1)–(3) would be formalized in two standard degree-based and non-degree-based semantic frameworks.

“‘taller’…does not seem perfectly precise; stoops and curly scalps may produce borderline cases… A comparative such as ‘more intelligent’ is notably vague.” (Williamson 1994: 156)

“[T]hough there is a single dimension of height, people cannot always be exactly placed on it and assigned an exact height. For what exactly should count as the top of one’s head? Consequently there may also be borderline instances of ‘taller than’.

Comparatives associated with multi-dimensional predicates — for example ‘nicer than’ and ‘more intelligent than’ — are typically vague. They have borderline cases… This is particularly common when comparing people who are nice/intelligent in different ways.” (Keefe 2000: 13)

“[I]ncommensurability of dimensions yields vagueness. In our jobs example” — in which "job A [is] preferable to job B… just in respect of working hours, and less preferable… just in respect of pay,” and “there is no way to calibrate these two dimensions of working hours and pay… in the same units” — “the incommensurability in value of the two jobs means that we may face borderline cases for the application of ‘better.’” (Endicott 2000: 42, 44)

“Intuitively, the relations that hold between two entities may be unspecified. Consider, for example, comparatives such as tastier, cleverer, more normal or a more stereotypical tall person, whose interpretations — the set of dimensions, their relative importance and accordingly the ordering of the entities — are highly context-dependent.” (Sassoon 2013: 76)

It isn’t said how (if at all) such types of borderline cases might give rise to comparative sorites arguments or raise distinctive issues for the compositional semantics.

⁵Certain of the comparative sorites examples which I used in earlier work were problematic in failing to appreciate this point (Silk 2016: 198–199, 206). Thanks to Gunnar Björnsson for discussion.
2 Semantics for gradation

2.1 Degree-based semantics

A prominent approach is to treat gradable adjectives as associating individuals with degrees on a scale (e.g., Bartsch & Vennemann 1973, von Stechow 1984, Kennedy 1999, 2007, Heim 2001). For instance, ‘tall’ denotes a function tall from individuals to (positive) degrees of height, i.e. the individual’s maximal height; ‘hot’ denotes a function hot from individuals to (positive) degrees of temperature, i.e. the individual’s maximal temperature; and so on.⁶ Though many theories assume that degrees are isomorphic to real numbers and that scales are dense linearly ordered sets of degrees (e.g., Bartsch & Vennemann 1973, von Stechow 1984, Klein 1991, Fox & Menendez-Benito 2006, Sassoon 2010, 2013), at minimum it is required that the relation ≥ on the set of degrees D have at least the structure of a partial (if not total) order, i.e. that ≥ be a (possibly complete) reflexive, transitive, and antisymmetric relation on D (cf. Kennedy 1999, 2007, Barker 2002, Lassiter 2015, Morzycki 2015). Compositional details aside, the comparative (4) says that the (maximal) degree to which Alice is tall, tall(Alice), is greater than the (maximal) degree to which Bert is tall, tall(Bert), as reflected in (5).

(4) Alice is taller than Bert.

(5) (4) is true iff tall(Alice) > tall(Bert)

Recall the comparative sorites argument in (2). The above semantics renders the interpretation of (P3) as in (6), where pref (≠[preferable]) is a function from items to their degree of preferability, a representation of how preferable they are:⁷

(6) \[ \forall x \forall y \forall z \left( \left( (\text{pref}(x) > \text{pref}(y)) \land (\text{pref}(y) = \text{pref}(z)) \right) \rightarrow (\text{pref}(x) > \text{pref}(z)) \right) \]

(P3) is an instance of what is sometimes called PI-transitivity, which is a weakening of transitivity; i.e., if a relation ≥ satisfies transitivity, PI-transitivity in (7) is also satisfied, where > and ~ are the strict and non-strict parts, respectively, of ≥.

⁶To fix ideas I assume a Kennedy-style measure-function analysis (see also Bartsch & Vennemann 1973), treating gradable expressions as denoting functions from items to degrees (type \( \langle e, d \rangle \), or for broadly modal adjectives type \( \langle st, d \rangle \)). An alternative is to treat gradable expressions as denoting relations between items and degrees (for the individual case, type \( \langle d, et \rangle \) or \( \langle e, dt \rangle \); e.g. Cresswell 1977, von Stechow 1984, Heim 2001). For critical overviews see Kennedy 1999, Morzycki 2015. For simplicity I ignore complications from intensionality. I continue to focus only on vagueness phenomena associated with gradable adjectives.

(7) \textbf{PI-transitivity}  
\((X > Y \land Y \sim Z) \rightarrow X > Z\)

That is, \((P_3)\) is simply a weakening of transitivity. It follows from the transitivity of the relation \(\geq\) on the domain of degrees \(D\).

So: The premises \((P_1)\) are true (\textit{n. 2}). \((P_2')\) in (2) describes your non-obsessiveness about coffee sweetness; one cup is as good as the next given your preferences. \((P_3)\) is entailed by the general structure of scales, and thus holds with any adjective denotation (measure function). \((P_2)\) in (3) encodes these dual properties. \((P_2)\) in (1) likewise seems true given the nature of morality. Moreover the arguments seem valid. Yet the conclusions \((C)\) are false. Hence the paradox.

\subsection*{2.2 Delineation semantics}

Second, consider the other main approach to gradation in formal semantics: \textit{delineation semantics} ("partial predicate," "inherent vagueness" semantics). Delineation semantics treat gradable adjectives such as ‘tall’ and non-gradable adjectives such as ‘digital’ alike as ordinary predicates (type \(⟨e; t⟩\)), or for broadly modal adjectives type \(⟨st; t⟩\). What distinguishes gradable adjectives is (inter alia) their sensitivity to a contextually supplied comparison class (e.g., \textsc{Klein} 1980, \textsc{Doetjes et al.} 2009, \textsc{Burnett} 2012). In one context using ‘Alice is tall’ might express that Alice is tall for a basketball player; in another context it might express that she is tall for an American woman. Gradable adjectives are treated as denoting partial functions partitioning a comparison class \(CC\) into a positive extension, a negative extension, and an extension gap (the “borderline cases”):

\begin{equation}
[tall]^{CC} = \lambda x: \neg \text{gap}_{CC}(\text{tall})(x) \cdot x \text{ is tall in } CC
\end{equation}

Predicative uses are treated straightforwardly: ‘Alice is tall’ is true given \(CC\) iff Alice counts as tall in \(CC\). Delineation theories differ on how to analyze the meaning of the comparative given the comparison class. According to \textsc{Klein} 1980, a comparative ‘\(x\) is \(A\)D\(J\)-er than \(y\)’ is true iff there is a comparison class relative to which \(x\) counts as \(A\)D\(J\) but \(y\) does not (cf. \textsc{van Benthem} 1982, \textsc{von Stechow} 1984). On this line,

\*Some theories also invoke a parameter \(\delta\) setting relevant standards/thresholds for different adjectives (e.g., \textsc{Lewis} 1970, \textsc{Barker} 2002): ‘tall’ is treated as denoting those individuals in \(CC\) whose height is at least as great as the standard of tallness \(\delta_{\text{tall}}\) set (at least in part) by \(\delta\). For simplicity I ignore this potential layer of context-sensitivity. In delineation semantics, degrees may be invoked in the metalanguage (e.g. \textsc{Barker} 2002), but they aren’t included in the type system.
(4) ‘Alice is taller than Bert’ is true (given any comparison class) iff there is some comparison class $CC'$ such that Alice is tall in $CC'$ and Bert is not tall in $CC'$.

Degree-based and delineation-based approaches differ on issues regarding the morphology and internal compositional semantics, and the ontological status of degrees. Yet there are well-known logical correspondences between the two frameworks. In order to avoid problematic inconsistencies and entailments, delineation theories impose qualitative restrictions on comparisons among individuals across contexts (comparison classes). For instance, per Klein’s (1980) “consistency postulate,” if $x$ counts as tall in some $CC$ and $x$ has a greater height than $y$, then there can be no $CC'$ in which $x$ doesn’t count as tall but $y$ does (see also Fine 1975, Kennedy 1999, Fara 2000). Delineation theorists have proven that the relevant qualitative restrictions derive a preorder (reflexive, transitive, possibly total relation) $\succapprox_A$ “at least as ADJ as” over the set of individuals in the domain of ‘ADJ’ for any adjective ‘ADJ’ (Klein 1980, 1991, van Benthem 1982, van Rooij 2011a). The degree-theorist’s basic notions of degrees and scales may then be derived from these qualitative orderings $\succapprox_A$ (Cresswell 1977, Bale 2008, 2011, Lassiter 2011a, van Rooij 2011a): the set of degrees $D$ is the set of equivalence classes under $\succapprox_A$, and the relation $\succeq_A$ on $D$ is defined accordingly in terms of $\succapprox_A$, i.e. $[x]_A \succeq_A [y]_A := x \succapprox_A y$ (where $[a]_A$ is an equivalence class $\{b : b \succapprox_A a \land a \succapprox_A b\}$). Details of these derivations aside, what is important here are the results. The interpretation of any adjective is treated as relying on a preorder $\succapprox_A$ on the set of individuals related by the adjective. The transitivity of $\succapprox_A$ again validates (P3). Whether we go for Klein or for Kennedy, the paradox is off and running. The challenge posed by the comparative sorites — more on which presently — needn’t be hostage to debates between degree-based and delineation-based approaches regarding the basic vs. derived status of degrees, the role of degrees in object language and metalanguage, or the morphosyntax of predicative and comparative uses.

3 Upshots

3.1 Comparative vagueness and semantic competence

On the face of it, (1) and (3) would seem to be instances of the familiar sorites argument form, with ‘is not morally better than your saving your dear friend’ and ‘is not more preferable than $x_i$’ as the intuitively vague predicates. Compare (9)–(10), where $x_n$ is someone $4' + n$ nanometers tall:

(9) (P1) Someone who is $4'$ isn’t tall (for a pro basketball player).
If someone who isn't tall (for a pro basketball player) grows one nanometer, they still won't be tall (for a pro basketball player).

∴ No one is tall (for a pro basketball player).

(P1) \( x_0 \) is not tall.
(P2) For all \( n \), if \( x_n \) is not tall, then \( x_{n+1} \) is not tall.
(C) \( \therefore \) For all \( n \), \( x_n \) is not tall.

A comparative 'is ADJ-er than \( y \)' might not have precisely the same syntax or compositional semantics as a simple predicate of individuals or a positive-form use such as 'is tall'; but can we not still apply our favorite account of the sorites for 'is (not) tall' to 'is (not) more preferable than,' 'is (not) morally better than,' etc.? For instance, a standard move is to locate the problem in the inductive premise. Even if we can't point to any instance of (P2) with 'tall' in (10) that isn't true (cf. Soames 1999, Fara 2000),

(P2) For all \( n \), if \( x_n \) is not tall, then \( x_{n+1} \) is not tall.

perhaps we can know that it isn't true in any context (Fara 2000), or no matter what formally precise language we might be speaking (Lewis 1970; cf. Silk 2016), or no matter how the conversation might evolve (Shapiro 2006), or on any competent way of applying 'tall' (Kamp 1981, Raffman 2014). Why not say the same about the generalizations in (1)–(3)?

Recall our degree semantics from §2.1. The positive form is treated as relating a degree to a relevant threshold, or degree standard. To a first approximation, 'Alice is tall' is true iff the degree to which Alice is tall is at least as great as the relevant degree standard of tallness \( s_{\text{tall}} \), i.e. how tall one must be to count as tall:

(11) 'Alice is tall' is true iff \( \text{tall}(Alice) \geq s_{\text{tall}} \)

Details aside, the degree standard \( s \) provides a natural place for capturing felt vagueness in predicative uses. Suppose we fix on a particular procedure for measuring height and a relevant comparison class. Still we may not be willing commit to any specific degree of height as constituting how tall one must be to count as tall. Even if (12) — the predicted semantic content of the inductive premise (P2) in (10) — is

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*Details of the internal morphosyntax and compositional semantics which delivers these truth conditions won't be important for our purposes. For present purposes I treat the degree standard as a simple variable and the relation as \( \geq \) (see Kennedy 1999, 2007, Glanzberg 2009 for discussion). Many degree-based theories derive the positive form by combining the adjective with a null morpheme \( \text{pos} \) to yield a predicate of individuals (e.g., Cresswell 1977, von Stechow 1984, Kennedy 1999, 2007). I will abstract away from context-sensitivity from comparison classes.
false at any point of evaluation, speakers may find it compelling because (say) the falsifying instance determined by the degree standard is never where one is looking (Fara 2000); or because there is indeterminacy or uncertainty about which degree standard is determined in one’s concrete conversational situation (cf. Barker 2002); or because the speakers are undecided about what degree standard to accept for purposes of conversation (cf. Silk 2016); and so on.

(12) **IND-PRED**
\[\forall n [(\text{tall}(x_n) \not\geq s_{\text{tall}}) \rightarrow (\text{tall}(x_{n+1}) \not\geq s_{\text{tall}})]\]

The analogous surface forms of the intuitively vague expressions in (10) and (1)–(3) conceal an important difference between the positive-form and comparative-form inductive premises. Speakers’ assumptions in concrete discourses are typically compatible with a range of degree standards in such a way that makes IND-PRED with ‘tall’ seem plausible. Yet it is in principle possible to deny IND-PRED and settle on a precise standard of tallness. Treating the positive form ‘tall’ as semantically interpreted with respect to precise degree standards, or (possibly singleton) sets thereof, allows for this. Drawing the line here for what counts as tall may seem unwarranted but it isn’t incoherent; ontological vagueness notwithstanding, it needn’t do violence to the worldly facts or manifest a lack of semantic competence.

Not so with premises such as (P2) in (1)/(3) or (P2’) in (2). IND-COMP (13) is true given one’s preferences. Indeed even a supertaster could accept it; one simply doesn’t care exactly how sweet the coffee is. The non-arbitrariness of morality would seem to straight-up imply (14) (n. 2).

(13) **IND-COMP**
\[\forall n [\text{pref}(x_n) = \text{pref}(x_{n+1})]\]

(14) \[\forall n [(\text{morally-good(save-n)} \not\succ \text{morally-good(save-friend)})]
\[\rightarrow (\text{morally-good(save-n+1')} \not\succ \text{morally-good(save-friend)})]\]

Idealizations are commonplace in formal semantics. Yet it would distort our representation of speakers’ semantic competence — not to mention normative/evaluative facts — to treat measures of value, preferability, etc. as maximally “opinionated” in such a way as to necessarily falsify (13)–(14).

As may be expected, this worry needn’t be devastating. One might say: “Just as it would be ‘physiologically’ arbitrary to mark a particular counterinstance of IND-PRED (cutoff) with ‘tall’, so would it be (e.g.) ‘bouletically’ arbitrary to mark a particular counterinstance of (13) with ‘preferable’, ‘normatively’ arbitrary to mark a
particular counterinstance of (14) with ‘morally good’, etc. If using sorites-immune objects is legitimate in the formal semantics of the one, it is legitimate in the other.” Perhaps. (“One person’s modus ponens…”) Yet the apparent contrast remains: Though perhaps we might know that at the end of inquiry, no matter how the conversation might evolve, (12) with ‘tall’ is false, we would seem to know that (13)–(14) are true, given one’s preferences and the normative facts.\(^\text{10}\) It is a cost to a theory if it represents us as inevitably semantically incompetent with ‘preferable’, ‘morally good’, etc.

3.2 Going forward

Let’s recap. Vagueness phenomena with comparatives raise a distinctive challenge for formal semantics. Take (2). The argument seems valid, and we know that the base premise (P1) is true and the conclusion (C) false. The inductive premise (P2') is true given your lack of concern about precisely how sweet your coffee is. That leaves only (P3), which is validated by the general structure of standard semantics for gradation. This point is perhaps most evident in existing degree-based frameworks, which treat gradable expressions as associating items with degrees on a scale (§2.1); but it generalizes to delineation-based analyses that don’t invoke a notion of degrees (§2.2). It would be surprising if one could rebut fans of special obligations or sugar-taking coffee drinkers with facts about semantic scale structure.

Upshot: We need a semantic framework that allows for a certain kind of intrinsivity with gradable expressions.

Some theorists tie vagueness phenomena to features of particular types of expressions, often positive form relative gradable adjectives.\(^\text{11}\) A common worry is that seemingly similar phenomena also arise with expressions of other categories (‘heap’, ‘quietly’, etc.) (e.g. Stanley 2003: nn. 6, 8, Keeffe 2000: 14, Raffman 2014: 16–18, 22, 131–132). Consequently, some theories offer uniform accounts of linguistic vagueness, capturing vagueness phenomena via general features of the semantics or metasemantics. (Consider the general frameworks of supervaluations or many-valued/fuzzy logics, or the general claims of epistemicists about metasemantic complexity and imperfect knowledge of meanings.) Such apparatus could be applied to comparatives. For instance, one might posit systematic semantic association with

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\(^{10}\) Cf. “Each sharpening in the frame represents one way that some indeterminacies can turn out, consistent with the meaning of the predicates, the non-linguistic facts, and the externally fixed contextual factors” (Shapiro 2006: 76; also Raffman 2014). See n. 2.

\(^{11}\) E.g.: “the analysis of vagueness is linked to a particular semantic feature of the positive form (i.e. to a feature of the meaning of pos)” (Kennedy 2011: 83; cf. 74, 93, 2007: 6, 2013: 271, Fine 1975: 267, Cooper 1995: 246).
a discourse-level standard of precision or measure of granularity (cf. Lewis 1979, Krifka 2007, Sauerland & Stateva 2007, Morzycki 2015), and then restrict the evaluation of arguments to “admissible” contexts in which (inter alia) the adjective’s measure function isn’t less “opinionated” than the measures of relevant subvening properties (e.g., quantity of sugar, quality of sweetness). Or one might treat the formal semantics as supervaluating over numerical measure functions (Sassoon 2013) — or treat the formal semantics as interpreted with respect to a particular numerical measure function and treat token uses as adjusting a set of live measure functions (cf. Lewis 1975, Barker 2002, Lassiter 2011b) — where the various measure functions provide different counterinstances of the inductive premises.

There are reasons to be cautious about pursuing “global” strategies such as these. A growing body of linguistic work has stressed the importance of distinguishing sources of apparent vagueness phenomena, whether by distinguishing kinds of vagueness (Sauerland & Stateva 2011) or by distinguishing vagueness from imprecision or “loose talk” (Lasersohn 1999, Kennedy 2007, Morzycki 2015). A more specific concern comes from our remarks from § 3.1. Knowing the facts about usage and the extra-linguistic circumstances (think: epistemicism, contextualism), and settling on (meta)normative issues about what determines preferability, moral value, etc. (think: precise contexts), is insufficient to undermine the intuitive force of the comparative sorites. In degree-semantic terms, the problem isn’t that our measures are insufficiently precise, or that we are aren’t settled on what measure function to associate with the expressions (or that we don’t know what dimensions are relevant for determining preferability, moral value, etc., or how to measure things with respect to these dimensions); it is that we need the measures to allow for certain intransitivities.

This conclusion is, again, not inevitable. Yet given the prominence of capturing at least some instances of linguistic vagueness in terms of features of semantic gradability, I would like to put global approaches to the comparative sorites aside. The next section explores a more local approach that revises the semantics of gradation. I leave investigation of apparent vagueness phenomena with non-gradable expressions for elsewhere.

4 Degrees and scale structure: Semiorders in a degree semantics

To fix ideas let’s assume a Kennedy-style degree-based semantic framework. § 2.1 assumed a traditional degree semantics on which degrees are conceived as points on a scale, and scales \( (D, \geq) \) are treated as imposing a relation \( \geq \) with at least the structure of a partial order on the domain of degrees \( D \). (Analogously, in delin-
cation theories (§2.2) the meanings of adjectives are treated as relying on a relation ≿ with at least the structure of a partial preorder on a set of individuals.) Natural places for revising the semantics are the representation of degrees and the representation of scales. For instance, one might treat degrees as sets of points, perhaps intervals, and treat adjectives as associating individuals with such (perhaps fuzzy) sets (cf. Kennedy 2001, Schwarzschild & Wilkinson 2002, Solt 2014; though see Silk 2016: 186–187). In the remainder of the paper, however, I would like to examine one way of pursuing the latter option and reconsidering the assumptions about scale structure. In particular, I suggest that we treat the domain of degrees as coming with a semiorder. Adjective denotations may still be treated as associating items with degrees, conceived as points on a scale; yet a scale is now a structure \((D, >)\), where \(>\) is a semiorder on the set of degrees \(D\).

Formally, a semiorder \(>\) is an interval order that satisfies semitransitivity ((15)). Equivalently, \(>\) is a semiorder iff there is a real-valued function \(f\) such that \(x > y\) iff \(f(x) > f(y) + \epsilon\), for some fixed positive number \(\epsilon\) (Luce 1956, Scott & Suppes 1958, Fishburn 1985, van Rooij 2011b).

(15)  
Irreflexivity: \(\forall x \ x \not> x\)
Interval-order: \(\forall x, y, z, w: (x > y \land z > w) \rightarrow (x > w \lor z > y)\)
Semitransitivity: \(\forall x, y, z, w: (x > y \land y > z) \rightarrow (x > w \lor w > z)\)

From \(>\) a relation \(~\) can be defined, where \(x ~ y\) iff \(x \not> y \land y \not> x\). Crucially, although \(~\) is reflexive and symmetric, it needn’t be transitive (unlike the non-strict part of a partial or total order). Semiorders have been used fruitfully in measurement theory and choice theory for modeling intransitive indifferences (Luce 1956, Scott & Suppes 1958, Fishburn 1985, van Rooij 2011c). The broader research on semiorders provides an independently motivated resource to incorporate into the semantics of gradation.

One way of interpreting the above formalism is to understand \(f\) as mapping degrees of ADJ-ness to measures of a property on which ADJ-ness may (possibly trivially) supervene — e.g., as mapping degrees of preferability to measures of sweetness. The value \(\epsilon\) can be understood as representing a threshold of distinguishability with respect to the property associated with the adjective, now written \(\epsilon_A\) (hereafter distinguishability threshold). Intuitively, the greater the value of \(\epsilon_A\), the less distinguishing in matters of ADJ-ness. Schematic truth conditions are as follows, where \(\text{adj}\) is the measure function denoted by ‘ADJ’ (nn. 7, 9):

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From the above, it is evident that the use of semiorders in the semantics of adjectives provides a richer and more nuanced approach to understanding the meaning of adjectives, particularly in the context of degrees and scales. This approach not only addresses the issue of indistinguishability but also allows for a more flexible and realistic model of how adjectives relate to properties in the world.
\[ a \text{ is ADJ-er than } b \] is true
\[ \text{iff } \text{adj}(a) >_A \text{adj}(b) \]
\[ \text{iff } f_A(\text{adj}(a)) > f_A(\text{adj}(b)) + \epsilon_A \]

\[ a \text{ is as ADJ as } b \] is true
\[ \text{iff } \text{adj}(a) \sim_A \text{adj}(b) \]
\[ \text{iff } |f_A(\text{adj}(a)) - f_A(\text{adj}(b))| \leq \epsilon_A \]

Take ‘preferable’. If quantity of sugar was the sole property relevant for preferability, \( \epsilon_P \) would represent a level of sugar sufficing to distinguish cups with respect to how preferable they are; anything less than \( \epsilon_P \) and any differences in sugar fail to render one more preferable than the other. For degrees of preferability \( x \) and \( y \), \( x >_P y \) says that \( x \)'s amount of sugar is relevantly more than \( y \)'s, i.e. greater than \( \epsilon_P \); and \( x \sim_P y \) says that the difference in sugar between \( x \) and \( y \) is less than \( \epsilon_P \).

Several remarks on interpreting the formalism are in order. It is important in semantics for gradation not to read too much into the ‘measure’ in ‘measure function’, or into the appeal to “degrees” in the formal semantics. I use ‘measure function’ broadly, not only for adjectives associated with measurement procedures or numerical units of measurement (e.g. height in inches, with ‘tall’), but for any mapping which would determine an order on objects (contrast Sassoon 2010, 2013, Crespo 2015, Kamp & Sassoon 2016). Talking of the adjectives’ “measure functions” doesn’t presuppose that properties of preferability, value, etc. are quantifiable. What is important about degrees for our purposes is simply that they represent assessments of how preferable, tall, etc. things are, and thus that they can be associated with qualitative orderings on the items in the adjectives’ domains. Nothing of metaphysical significance is presupposed in our talk of things having “degrees” of preferability, value, etc.

Likewise, one shouldn’t be misled by the use of numerical values with \( f \) and \( \epsilon \) in the definition of semiorders. A relation is a semiorder only if there is such a function and number that satisfy the stated condition. As noted above, degrees aren’t treated as isomorphic to numbers, and properties of ADJ-ness needn’t be quantifiable. The numerical threshold \( \epsilon \) is used in representing a relation between items that fail to count as relevantly distinguishable. Talk of the “difference” between \( f_A(\text{adj}(a)) \) and \( f_A(\text{adj}(b)) \) being “less than” or “equal to” \( \epsilon_A \) is compatible with (inter alia) the items \( a \) and \( b \) being identical, as ADJ as one another, imperceptibly different in ADJ-ness, or even incomparable.

Finally, the notion of “(non-)distinguishability” is to be understood as specific to matters of the property described by the adjective in the context. \( \text{adj}(a) >_A \text{adj}(b) \) says that \( a \) and \( b \) are relevantly distinguishable in ADJ-ness — e.g., that the difference
in sugar or sweetness between \( a \) and \( b \) suffices to distinguish them in how preferable they are, and render \( b \) more preferable than \( a \). Being discriminable in some respect doesn’t imply being “distinguishable,” in the present sense of being related by \( \succ \) in a context. On the flip side, saying ‘\( \text{adj}(a) \sim \text{adj}(b) \)’ — that \( a \)'s and \( b \)'s degrees fail to exceed the “threshold of distinguishability” \( \epsilon_a \) — doesn’t imply that \( a \) and \( b \) are indiscriminable, either in general or in properties relevant to determining how ADJ they are. Indiscriminability is one basis for being related by \( \sim \) (i.e., for it neither being the case that \( x \succ y \) nor that \( y \succ x \)), but it isn’t the only one. As noted in §§1, 3.1, a supertaster might be able to discriminate between adjacent coffee cups \( x_n, x_{n+1} \) in their quantity of sugar or quality of sweetness. Saying that the items’ degrees are related by \( \sim_p \) is to say that any such difference fails to constitute a relevant distinction in preferability. The act of saving 2 strangers at the expense of your dear friend may be discriminable in, say, utility from the act of saving 3 strangers at the expense of your dear friend. Such a difference may even constitute a discriminable difference in moral value.

Let’s apply the above semantics to our comparative sorites arguments from §1. Consider (2) with “preferable”’; the truth conditions of (P2′) and (P3) are as follows:

\[
\begin{align*}
(18) \quad (\text{P2'} \text{ (in (2)) is true}) \\
& \text{iff } \forall n \left[ \text{pref}(x_n) \sim_p \text{pref}(x_{n+1}) \right] \\
& \text{iff } \left| f_p(\text{pref}(x_n)) - f_p(\text{pref}(x_{n+1})) \right| \leq \epsilon_p \\
(19) \quad (\text{P3} \text{ (in (2)) is true}) \\
& \text{iff } \forall a \forall b \forall c \left[ \left( (\text{pref}(a) \succ_p \text{pref}(b)) \land (\text{pref}(b) \sim_p \text{pref}(c)) \right) \\
& \land (\text{pref}(a) \succ_p \text{pref}(c)) \right] \\
& \rightarrow (\text{pref}(a) \succ_p \text{pref}(c)) 
\end{align*}
\]

The semantics in (18) captures the truth of (P2′). Every pair of adjacent cups is related by \( \sim_p \); for any \( x_n, x_{n+1} \), the difference in how preferable they are falls below the threshold \( \epsilon_p \). Discriminable though they might be, one cup is as good as the next, given your preferences. However, PI-transitivity (P3) is violated. The counterinstance for (19) occurs at the cup \( x_i \) such that \( f_p(\text{pref}(x_i)) - f_p(\text{pref}(x_{i+1})) = \epsilon_p \) (where \( x_i \) is, again, an ordinary sweetened cup of coffee): \( x_i \) is more preferable than \( x_i \), since \( f_p(\text{pref}(x_i)) > f_p(\text{pref}(x_{i+1})) + \epsilon_p \); yet it’s not the case that \( x_i \) is more preferable than \( x_{i+1} \), since \( f_p(\text{pref}(x_i)) = f_p(\text{pref}(x_{i+1})) + \epsilon_p \). \( x_{i+1} \) still fails to differ from \( x_i \) (say, in sugar or sweetness) in such a way as to render it less preferable. So, (i) we can accept that \( \text{pref}(x_i) \succ_p \text{pref}(x_{i+1}) \), i.e. that \( x_i \) is more preferable than \( x_{i+1} \); and (ii) we can allow that \( \text{pref}(x_i) \sim_p \text{pref}(x_{i+1}) \land \ldots \text{pref}(x_{i-1}) \sim_p \text{pref}(x_i) \), i.e. that adjacent cups aren’t distinguished in preferability; and yet, due to the intransitivity of \( \sim_p \),
(iii) we can maintain that \( \exists x; \text{pref}(x_i) \not\succeq_p \text{pref}(x_j) \), i.e. that there is a cup \( x_j \) which \( x_i \) isn’t preferable to.

The semantics also avoids satisfying the premises (P2) in (1)/(3). The truth conditions of (P2) in (1) are as follows:

\[
\text{(20) \quad (P2) (in (1)) is true if } \forall n \left[ \left( \text{morally-good(\text{save-n})} \not\succeq \text{morally-good(\text{save-friend})} \right) \rightarrow \left( \text{morally-good(\text{save-n+1})} \not\succeq \text{morally-good(\text{save-friend})} \right) \right] \]

The counterinstance is at the act \( \text{save-i} \) such that \( f_A(\text{morally-good(\text{save-friend})}) - f_A(\text{morally-good(\text{save-i+1})}) = \epsilon_M \). The falsity of the inductive premise is compatible with it being the case that \( \text{morally-good(\text{save-n})} \sim_M \text{morally-good(\text{save-n+1})} \), for any \( n \), i.e. that the difference — indeed moral difference — between the acts of saving \( n \) strangers over your friend and saving \( n+1 \) strangers over your friend is insufficient to relevantly morally distinguish them in the context.

In these ways, the proposed semiorder-based degree semantics avoids validating PI-transitivity or satisfying the inductive premises in sorites arguments such as (1)–(3) (or (10) for that matter); and it does so without needing to represent measures of preferability, value, etc. as maximally discriminating and opinionated (§3.1). As Farah (2000) emphasizes, predicting that the inductive premise is not true doesn’t suffice for an account of the sorites (cf. e.g. Wright 2003: 87, 97–98, Raffman 2014: 122). If the inductive premise isn’t true in any context, why do we find it so plausible? What should we say about the seemingly predicted “sharp boundary” between (e.g.) cups that aren’t more preferable than \( x_i \) and cups that are? This isn’t the place to hazard a general theory of the semantics, epistemology, and psychology of vagueness; however, several directions for approaching such questions in the above framework might be as follows.

First, the intransitivity of the defined non-distinguishability relation \( \sim \) locates a place for explaining some of the sorites’ intuitive appeal. Though the semantics doesn’t verify (P2) in (1)/(3) or PI-transitivity (P3) in (2), it verifies claims expressing how adjacent items in the series are relevantly non-distinguishable in preferability, moral value, etc. We may find (P2)/(P3) compelling because of failing to distinguish them from the true claim that \( \text{adj}(x_1) \sim_A \text{adj}(x_2) \land \cdots \land \text{adj}(x_{n-1}) \sim_A \text{adj}(x_n) \).

Second, the distinguishability threshold \( \epsilon_A \) locates a place for importing ideas from broader theories of vagueness (epistemicism, contextualism, supervaluationism, etc.). For instance, epistemicists claim that facts about competent use determine precise extensions for intuitively vague expressions (Sorensen 1988, Williamson 1992).

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12See Silk 2016: chs. 6–7 for one sort of broadly contextualist approach.
1994, Barker 2002). In terms of the present semantics, an epistemicist might treat a specific value of \( \epsilon_A \) as determined by the world of evaluation. Apparent fuzziness in the distinguishability threshold could be diagnosed as uncertainty about what formally precise language (in the sense of Lewis 1975) is being spoken.

Alternatively, on a broadly contextualist line, the distinguishability threshold may be treated as a contextual parameter \( \epsilon_c = (\epsilon_{A_1}, \epsilon_{A_2}, \ldots) \), with different contexts determining different levels of distinguishability.\(^{13}\) For the maximally opinionated among us, context may supply a value of \( \epsilon_{A_c} = 0 \); no difference in properties relevant to determining how ADJ things are goes undetected or uncared-for in matters of ADJ-ness. For the rest of us, context supplies \( \epsilon_{A_c} > 0 \) and the comparative sorites is off and running. Further, even if the compositional semantics takes as given a particular value for \( \epsilon_c \), there may be indeterminacies in what value is determined in concrete discourses (cf. Silk 2016). In token uses there may be a range of live representations of context and values for \( \epsilon_c \) compatible with the interlocutors’ interests (Fara 2000), psychological states or verbal dispositions (Raffman 1994, 1996), or discourse moves (Kamp 1981, Soames 1999, Shapiro 2006, Silk 2016). We may not be able to point to any instance of (P2)/(P2’)) we reject, or any instance of the sharp boundaries claim which we accept.

To my knowledge, the only precedent in linguistic semantics for invoking semi-orders is the delineation-based semantics for “implicit comparatives” (Kennedy 2011, Bochnak 2013) from van Rooij 2011a. Implicit comparatives are sentences ‘\( x \text{ is ADJ compared to } y \)’ in which a comparison is made using the positive form, in contrast to explicit comparatives ‘\( x \text{ is ADJ-er than } y \)’ (as considered in this paper), which use a comparative morpheme. Implicit comparatives contrast with explicit comparatives in seeming to imply that there is a significant difference between the items, as reflected in (21).

\(^{13}\)‘Broadly contextualist’ in the sense of covering different combinations of indexicalism/non-indexicalism and contextualism/relativism (MacFarlane 2009) – i.e. different views on whether a particular value for the parameter \( \epsilon_c \) figures in calculations of compositional semantic value, and whether a particular value is determined by the context of utterance or a “context of assessment” in the definition of truth-in-a-context. I take it that many accounts in the vagueness literature classified as “contextualism about vagueness” (see below in the main text) are neutral on such matters of the compositional semantics and postsemantics (cf. Åkerman 2009: 47–53, Kölbl 2010: 324–325, Kennedy 2013: 270n.19, Silk 2016: §§6.1, 6.3.3). For alternative contextualist and relativist approaches, see e.g. Silk 2016 and Richard 2008, Lassiter 2011b, respectively.
Though Alice’s height is greater than Bert’s height, as required by (21a), it isn’t significantly greater, as intuitively required by (21b). van Rooij (2011a) appeals to semiorders to capture this “significantly ADJ-er than” relation in the interpretation of implicit comparatives. Explicit comparatives are analyzed via weak orders, as usual (§2.2), and they are denied to be vague. By contrast, the semantics in this section invokes semiorders in the scale structure for all uses, and allows for vagueness with both positive and comparative forms. Note that the proposed semantics is compatible with contrasts between implicit and explicit comparatives such as in (21). The semiorder on the set of degrees represents a relation of relevant distinguishability in matters of ADJ-ness. The truth of (21a) simply requires that Alice’s height be distinguishably greater than Bert’s — which it is. The distinguishable difference required by an explicit comparative needn’t be “significant” in such a way that verifies a corresponding implicit comparative such as (21b).

Let’s recap. This section has begun to develop a semiorder-based semantic framework for gradation. Although semiorders have been widely used in measurement theory and choice theory, they have received little currency in linguistic semantics. Previous appeals to semiorders in treatments of vagueness have focused on predicative uses and the positive form (Luce 1956, Halpern 2008, van Rooij 2011a,b,c). The proposal in this section invokes semiorders in the basic structure of semantics for gradation. In degree-based terms, the semantics treats the relation on the set of degrees as a semiorder, rather than, say, a partial or total order. The proposed semiorder-based semantics allows for vagueness phenomena with both positive and comparative forms, and captures the relevant intransitivities in comparative sorites cases such as (1)–(3).

§1 began by noting how consideration of adjectives such as ‘tall’ has led various theorists to assume that the comparative form cannot be vague. Vagueness phenomena arises from fuzziness in how many nanometers of height one must have to

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14Interestingly, Fara’s (2000) semantics for the positive form utilizes a relation of “being significantly greater than,” which is the interpretation typically given to semiorders in preference theory and measurement theory. Ackerman (1994: 132–135) raises the connection between the sorites and the paradox of intransitive preferences but doesn’t develop it further.
count as tall, how many cents one must have to count as rich, and so on. Yet com-
parative sorites arguments such as (1)–(3) illustrate that linguistic vagueness cannot
be wholly traceable to features specific to the positive form. Vagueness phenomena
such as sorites-sensitivity can be associated not only with how ADJ something needs
to be to count as ADJ, but with how ADJ things are (Silk 2015). The latter sort of
vagueness cannot be assimilated to indiscriminability or fuzziness in measurement
procedures or relevant dimensions.

Many questions remain. The account in this section allows for vagueness with
comparatives and avoids diagnosing linguistic vagueness in terms of features specific
to positive-form relative gradable adjectives (pace Cooper 1995, Kennedy 2007,
2011, 2013, McNally 2011, van Rooij 2011a, Bochnak 2013). Yet there remains a
potential concern that vagueness phenomena are still addressed piecemeal, in terms
of the semantics of gradation. Whether we should prefer a more unified account
of apparent vagueness phenomena in natural language remains to be seen (§3.2).
Extensions to higher-order vagueness and expressions of other categories (e.g. ‘def-
initely’), interactions between positive predications and (implicit and explicit) com-
paratives, and implications for scalar semantics more broadly call for careful in-
vestigation. Interactions in the compositional semantics with phenomena such as
context-sensitivity (Silk 2015, 2016), granularity (Sauerland & Stateva 2007),
multidimensionality (Sassoon 2013) may afford interesting directions for future
research.

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