

MATH 286 – WINTER 2008

Some more exercises for the final

Ex # 1. Classify ODEs #1 through #32 on page 131.

Ex # 2. Given $\alpha \in (0, 1)$, determine at least two different solutions of the initial-value problem

$$y' = y^\alpha, \quad y(0) = 0.$$

Ex # 3. Determine whether there is a non-zero solution of the initial-value problem

$$y' = y\sqrt{y}, \quad y(0) = 0.$$

Ex # 4. Let $y = \phi(x)$ be a solution of the initial-value problem

$$y'' + x^2y' + (\sin x)y = 0, \quad y(0) = b, \quad y'(0) = c.$$

Determine $\phi''(0)$, $\phi'''(0)$, $\phi^{(4)}(0)$.

Ex # 5. Let f be a piece-wise continuous periodic function with period T (i.e., $f(t + T) = f(t)$ for all $t \geq 0$ and some fixed positive T). Show that

$$\mathcal{L}\{f(t)\} = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt.$$

Ex # 6. Find the Laplace transform of the function

$$f(t) = \int_0^t (t - \xi)^2 \cos(2\xi) d\xi.$$

Ex # 7. Solve the initial-value problem

$$y'' + y = g(t); \quad y(0) = 0, \quad y'(0) = 0, \quad \text{where } g(t) = \begin{cases} t/2, & 0 \leq t < 6 \\ 3, & t \geq 6. \end{cases}$$