

MATH 286 – WINTER 2008
HOMEWORK SET 10

Due Friday 04/11 by 2PM

What you need to know:

- How to multiply matrices
- $M_{m \times n}(\mathbb{K}) = \{\text{space of matrices with } m \text{ rows and } n \text{ columns with coefficients in } \mathbb{K}\}$
- \mathbb{O} denotes the zero matrix
- Sections 7.1, 7.2, 7.3

Ex # 1. Suppose that $A, B, C, D \in M_{m \times n}(\mathbb{R})$ are the following matrices:

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}.$$

Determine whether the following matrix expressions are defined, and, when they are, determine the size of and compute the resulting matrix.

- i) $A(BC)$
- ii) DC
- iii) D^2
- iv) $(AB)C - D$

Ex # 2. Suppose that $A \in M_{m \times n}(\mathbb{R})$ is given by

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}.$$

Compute $p(A)$ where $p(x)$ is the polynomial $x^3 - 2x + 4$.

Ex # 3. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{m \times n}(\mathbb{R})$, with $\det A \neq 0$. Using the definition of inverse, show that

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Ex # 4. Find matrices $A(t), \vec{x}(t), \vec{b}(t)$ such that the given system of ODEs can be expressed as $\vec{x}' = A\vec{x} + \vec{b}$.

$$\begin{cases} 4tx_1 - 3x_3 + x_4 - t^2 + 1 = x_2' \\ x_2 + x_3' + e^{2t} - 5x_1 - 3 = 8x_4 \\ 9x_3 + 4 - x_4 - 5t^2x_2 + 2x_1' = \sin t \\ 3x_2 + 3e^t \cos t - x_3 + 7e^t x_4 = 2x_4'. \end{cases}$$

Ex # 5. Let F be the map from \mathbb{C} to $M_{2 \times 2}(\mathbb{R})$ defined by

$$F(a + ib) = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}.$$

- i) Show that F is one-to-one.
- ii) Find $F(1 - 2i), F(1), F(i), F(a)$ for $a \in \mathbb{R}$.
- iii) Show that $(F(i))^2 = F(-1)$.
- iv) Show that $F(zw) = F(z)F(w)$ for any $z, w \in \mathbb{C}$.
- v) The *norm* of a complex number $z = a + ib$ is defined by $\sqrt{a^2 + b^2}$, and is denoted by $\|z\|$. Show that $\|z\|^2 = \det F(z)$ for any $z \in \mathbb{C}$.

Ex # 6. Let $A = \begin{bmatrix} a & 1 \\ b & 2 \end{bmatrix} \in M_{2 \times 2}(\mathbb{R})$ where a is equal to the number corresponding to the first letter of your first name (e.g., mine would be 1 ...). Find b and a matrix $B \neq \mathbb{O}$ such that $AB = \mathbb{O}$.

Ex # 7. Suppose that the vectors $\vec{x}_1, \dots, \vec{x}_k$ are in \mathbb{R}^n , where $n < k$. Show that they are linearly dependent.

Ex # 8. Let

$$\vec{x}(t) = \begin{bmatrix} e^t \\ te^t \end{bmatrix}, \quad \vec{y}(t) = \begin{bmatrix} 1 \\ t \end{bmatrix}.$$

Show that $\vec{x}(t)$ and $\vec{y}(t)$ are linearly dependent for each $t \in [0, 1]$, but that they are linearly independent functions on $[0, 1]$.