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Optimization-based differential kinematic modeling exhibits a velocity-control strategy for dynamic posture determination in seated reaching movements

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Abstract

We proposed a velocity control strategy for dynamic posture determination that underlay an optimization-based differential inverse kinematics (ODIK) approach for modeling three-dimensional (3-D) seated reaching movements. In this modeling approach, a four-segment seven-DOF linkage is employed to represent the torso and right arm. Kinematic redundancy is resolved efficiently in the velocity domain via a weighted pseudoinverse. Weights assigned to individual DOF describe their relative movement contribution in response to an instantaneous postural change. Different schemes of posing constraints on the weighting parameters, by which various motion apportionment strategies are modeled, can be hypothesized and evaluated against empirical measurements. A numerical optimization procedure based on simulated annealing estimate the weighting parameter values such that the predicted movement best fits the measurement. We applied this approach to modeling 72 seated reaching movements of three distinctive types performed by six subjects. Results indicated that most of the movements were accurately modeled (time-averaged RMSE $< 5°$) with a simple time-invariant four-weight scheme which represents a time-constant, inter-joint motion apportionment strategy. Modeling error could be further reduced by using less constrained schemes, but notably only for the ones that were relatively poorly modeled with a time-invariant four-weight scheme. The fact that the current modeling approach was able to closely reproduce measured movements and do so in a computationally advantageous way lends support to the proposed velocity control strategy.

Keywords: Dynamic posture determination; Movement control; Kinematic redundancy; Optimization; Differential kinematics

1. Introduction

One problem often encountered in understanding as well as modeling human movement is that there are an infinite number of possibilities to determine a posture due to excessive degrees of freedom (DOF) possessed by the human body. This problem, often referred to as kinematic redundancy, is one integral component of a more general redundancy associated with motion trajectories, muscle activation patterns, and many other variables (Bernstein, 1967). Although it is widely appreciated that there is an underlying strategy adopted by human beings for resolving kinematic redundancy, quantitative representation of such an internal strategy is difficult.

A variety of optimization-based approaches have been proposed for human posture and movement modeling, wherein certain cost functions or performance criteria were hypothesized to represent a presumed optimal strategy (Crowninshield and Brand, 1981; Dysart and Woldstad, 1996; Hardt, 1978; Hatze, 1981; Park, 1973; Pedotti et al., 1978; Ryan, 1970; Yamaguchi, 1990). However, search for the most 'truthful' strategy representation or systematic validation of even a single criterion has been problematic, due mainly to the extreme computational complexity involved. For instance, the hypothesis that people tend to minimize the muscle stress during movement performance has been postulated with models incorporating musculoskeletal dynamics in conjunction

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with dynamic optimization to generate simulated human motions (Hatze, 1981; Pandy et al., 1992; Yamaguchi, 1990; Yamaguchi et al., 1995). As noted by Yamaguchi et al. (1995), the implementation of dynamic optimization, in particular dynamic programming, incurs an extreme computational demand for modeling even the simplest human movements. Therefore, empirical testing of muscle-stress-type cost functions for relatively complex, large-scale biomechanical systems is not practically feasible. Other strategies, such as minimum deviation from a 'neutral' configuration (Jung et al., 1994; Ryan, 1970), or optimal distribution of joint loading (Dysart and Woldstad, 1996) have also been postulated and formulated to allow the use of static optimization without incorporating the complex musculoskeletal dynamics. Modeling based on these optimal strategies provide some insight into the static posture selection process but not much into the dynamic posture determination or movement control. To test that a static posture selection strategy is used throughout a dynamic motion, individual static postures that are determined discretely would first have to be composed together as a sequence emulating a real motion. This composition, as attempted by Ryan (1970), is also computationally highly intensive: determination of every single static posture corresponds to a fairly sizable, often non-linear, optimization problem which has to be repeatedly resolved as many times as the number of frames comprised in a movement. Further, the applicability of sequential static motion emulation is challenged by the fact that there is a significant distinction between a static posture and an instantaneous posture sampled from a movement (Zhang and Chaffin, 1997).

In this article, we present a new optimization-based differential inverse kinematics (ODIK) approach for modeling moderately complex three-dimensional (3-D) seated reaching movements while representing the dynamic posture determination strategy as a velocity control strategy. We hypothesize that given a particular hand velocity, human beings adopt a strategy that distributes the joint angle velocities in some equitable manner, and that this apportionment is constant over an entire reaching movement. The approach models the apportionment by assigning each DOF a weighting parameter which quantifies the relative contribution of corresponding DOF to the instantaneous postural change. A computational advantage is offered by the linear mapping between the hand velocity and joint angular velocities, as the kinematic redundancy is efficiently resolved by a weighted pseudoinverse. We use an optimization procedure based on simulated annealing (SA) to estimate the weighting parameter values such that the prediction best fits the measurement. The proposed approach is illustrated by modeling of three distinctive types of right-handed seated reaching movements wherein a four-segment, seven-DOF biomechanical linkage is employed to represent the torso and right upper extremity. During the modeling process, we examine whether a hypothesized velocity-control, inter-joint motion apportionment strategy seems plausible, whether it changes over the course of a movement, whether it varies with type of movement, and between different subjects.

2. Methods

The biomechanical model created to describe the torso and right upper extremity postures during seated movements is a linkage representation composed of four rigid segments: torso, right clavicle, right upper arm, and right forearm (including hand). This linkage (Fig. 1) is constructed by allowing a total of seven degrees of freedom at the bottom of the spine, the sternum, right acromion, right elbow, and right wrist.

Joint or segment angles that measure the seven degrees of freedom incorporated in the linkage are defined here as Cardan angles (Andrews, 1995). Their names (see Fig. 1) therefore may not comply with clinical or anatomical conventions. Note that since the axial rotation of each link cannot be specified, torso axial rotation is modeled partly by a rotation of the clavicle with respect to the torso long axis. Similarly, in order to identify a possible change of forearm orientation caused by humeral rotation an extra DOF is modeled at the elbow which otherwise could be well represented by a 1-DOF revolute joint (Veeger and Yu, 1996). Of further note is that as the clavicle is only allowed to rotate about the torso long

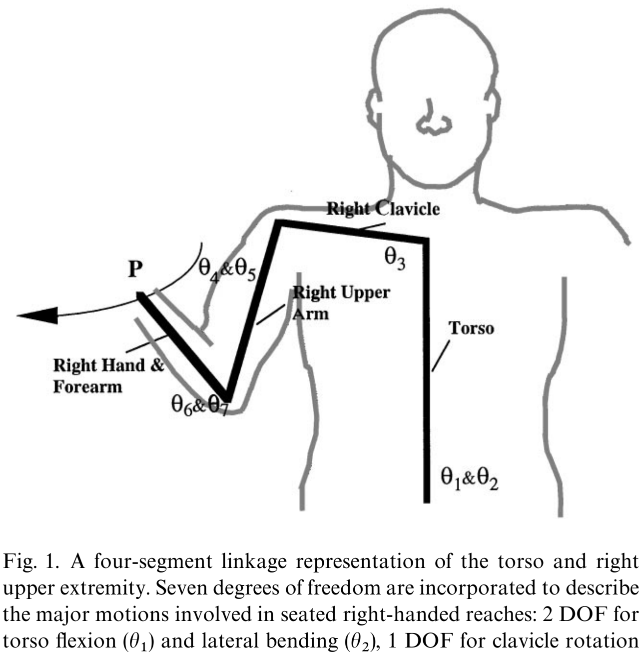

Fig. 1. A four-segment linkage representation of the torso and right upper extremity. Seven degrees of freedom are incorporated to describe the major motions involved in seated right-handed reaches: 2 DOF for torso flexion (θ_1) and lateral bending (θ_2), 1 DOF for clavicle rotation and partly for torso twisting (θ_3), 2 DOF for shoulder extension (θ_4) and abduction (θ_5), and 2 DOF for humeral rotation (θ_6) and elbow flexion (θ_7).

axis, the angle subtended by the clavicle and torso remains fixed.

The hand position with respect to the bottom of the torso link can be expressed in terms of the joint angles as variables, and link parameters including link lengths and link offsets which are assumed to be constants (Denavit and Hartenberg, 1955). Let $\mathbf{P} = [x\ y\ z]^{\mathrm{T}}$ and $\mathbf{\Theta} = [\theta_1\ \dots\ \theta_m]^{\mathrm{T}}$ represent the three-dimensional hand position and m joint angles, respectively ($m = 7$ for this specific modeling; T denotes transpose). The non-linear, complex relation between $\mathbf{P}$ and $\mathbf{\Theta}$ can be expressed in an abstract form as:

$$\mathbf{P} = [f_1(\theta_1, \dots, \theta_m)\quad f_2(\theta_1, \dots, \theta_m)\quad f_3(\theta_1, \dots, \theta_m)]^{\mathrm{T}} = f(\mathbf{\Theta}) \tag{1}$$

In fact, this type of kinematic relationship in the displacement domain is what a static posture prediction model usually deals with (e.g. Dysart and Woldstad, 1996; Ryan, 1970). With the aid of optimization, determination of Θ from an insufficient number of non-linear equations is possible but rather intricate, particularly when Θ is of sizable scale.

Differentiating Eq. (1) with respect to time results in a linear, differential kinematic relationship (Nakamura, 1991; Whitney, 1969) between the hand velocity and the joint angular velocities:

$$\dot{\mathbf{P}} = \dot{f}(\Theta) = \frac{\partial f}{\partial \Theta}\dot{\Theta} = \mathbf{J}(\Theta)\dot{\Theta}, \tag{2}$$

where $\mathbf{J}$ is the Jacobian, in this particular case, a $3 \times m$ matrix with each element

$$J_{i,j} = \frac{\partial f_i}{\partial \theta_j} \quad (i = 1, 2, 3; j = 1, \dots, m). \tag{3}$$

For redundant systems such as the one concerned in this work, the ordinary inverse of $\mathbf{J}$ is not defined. In other words, there are still an infinite number of $\dot{\Theta}$ sets that can provide the same $\dot{\mathbf{P}}$. However, a weighted pseudoinverse of $\mathbf{J}$ conveniently derives a solution as

$$\dot{\Theta} = \mathbf{W}^{-1}[\mathbf{J}\mathbf{W}^{-1}]^{\#}\ \dot{\mathbf{P}} \tag{4}$$

which minimizes the weighted Euclidean norm of angular velocity vector

$$\|\mathbf{W}\dot{\Theta}\|. \tag{5}$$

In Eq. (4), $\#$ symbolizes the pseudoinverse (Strang, 1976); $\mathbf{W}$ is a symmetric, positive-definite $m \times m$ weighting matrix that can be expressed as

$$\mathbf{W} = \mathrm{diag}(w_1\ \dots\ w_m), \tag{6}$$

where w_i $(i = 1, \dots, m)$ correspond to $\theta_i = (1, \dots, m)$, respectively. The implicit objective function (5) can be considered as a measure of the instantaneous weighted effort relating to kinetic energy (Whitney, 1969). Therefore, the weights (or weighting parameters) w_i collectively characterize how instantaneous effort is allocated among joint angles — a relatively smaller value of the weight signifies more participation of the corresponding angle whereas a greater value would tend to 'penalize' any change in the angle. In fact, as Eq. (4) indicates, given the hand motion trajectory or time history of $\dot{\mathbf{P}}$ and an initial posture, the weighting parameters in $\mathbf{W}$ are the only variables influencing the behavior of $\dot{\Theta}$ and Θ.

This formulation translates the problem into one of determining the weighting parameters which in turn define the distribution of motions. Analytical determination of the weighting parameter values for a measured movement using Eq. (4) is mathematically complex. A numerical method is proposed here to estimate the weighting parameter values such that the resulting or predicted movement profiles best approximate the measured ones. This numerical estimation presents an optimization problem that can be formulated as follows.

Since movement data are digitally acquired in discrete time frames at a certain sampling rate, it is more appropriate to express the kinematic variables in discrete forms. Let $\Theta[t]$ and $\dot{\Theta}[t]$, both vectors of length $m = 7$, represent respectively the joint angles and angular velocities at an instant of time t. Eq. (4) should now be rewritten as

$$\dot{\Theta}[t] = \frac{\Delta\Theta[t]}{\Delta t} = \frac{\Theta[t] - \Theta[t-1]}{\Delta t} = \mathbf{W}^{-1}[\mathbf{J}[t-1]\mathbf{W}^{-1}]^{\#}\ \frac{\Delta\mathbf{P}[t-1]}{\Delta t}, \tag{7}$$

where $\mathbf{J}[t-1]$ is in fact $\mathbf{J}(\Theta[t-1])$, a function of instantaneous angles at $t-1$ only. By eliminating the finite sampling time interval Δt at both sides, Eq. (7) becomes

$$\Theta[t] - \Theta[t-1] = \mathbf{W}^{-1}[\mathbf{J}[t-1]\mathbf{W}^{-1}]^{\#}\ \Delta\mathbf{P}[t-1]. \tag{8}$$

Using Eq. (8) recursively, $\mathbf{\Theta}[t]$ can be derived as

$$\Theta[t] = \Theta[1] + \sum_{k=1}^{t-1} \mathbf{W}^{-1}[\mathbf{J}[k]\mathbf{W}^{-1}]^{\#}\ \Delta\mathbf{P}[k]. \tag{9}$$

For a measured movement ($\Theta[t]$, $\mathbf{J}[t]$, and $\Delta\mathbf{P}[t]$ are known or derivable), $\mathbf{W}$ can be estimated by minimizing the time-averaged root mean square error (TaRMSE)

$$\mathrm{TaRMSE} = \frac{1}{\sqrt{mN}} \sum_{t=2}^{N} \left\| \mathbf{\Theta}[t] - \left(\mathbf{\Theta}[1] + \sum_{k=1}^{t-1} \mathbf{W}^{-1}[\mathbf{J}[k]\,\mathbf{W}^{-1}]^{\#}\ \Delta\mathbf{P}[k]\right) \right\|, \tag{10}$$

where N is the total number of time frames contained in a movement. This objective function is based on the Euclidean (L-2) norm of the difference between the predicted and measured angles. Once the weighting parameters are estimated through the above process, the following time-averaged absolute error (TaAE) may also be computed to more directly describe how the reproduced or predicted movement agrees with the measurement:

$$\mathrm{TaAE} = \frac{1}{m\mathrm{N}} \sum_{t=2}^{\mathrm{N}} \left| \mathbf{\Theta}[t] - (\mathbf{\Theta}[1] + \sum_{k=1}^{t-1} \mathbf{W}^{-1}[\mathbf{J}[k]\mathbf{W}^{-1}]^{\#}\, \Delta\mathbf{P}[k]) \right|. \tag{11}$$

Similar L1-norm-based measures for movement modeling accuracy were used by Ayoub et al. (1974) and Sepulveda et al. (1993).

The above numerical method affords the flexibility of formulating various hypotheses regarding $\mathbf{W}$ to simplify, as well as gain insight from, the modeling process. One such simplifying assumption is that the weighting parameters remain time-invariant (i.e. not a function of time). Another simplification would be to hypothesize that motions of a single segment are distributed equally amongst all of its degrees of freedom (e.g. torso flexion and lateral bending occur in concert). In other words, it represents an inter-joint motion apportionment strategy. Applying this simplification to the four distinct segment yields the weighting matrix

$$\mathbf{W} = \mathrm{diag}\,(w_1\ w_1\ w_2\ w_3\ w_3\ w_4\ w_4), \tag{12}$$

where w_1 is for the torso, w_2 for the clavicle, w_3 for the upper arm, and w_4 for the forearm. This is referred to as a four-weight scheme as opposed to a seven-weight scheme (see Eq. (6)). Tests of whether these assumed configurations or behavior of $\mathbf{W}$ result in a close match between model prediction and measurement are the basis for addressing the hypothesis and questions posed in the Introduction regarding dynamic posture control strategy.

The optimization problem of estimating weighting parameter values, as presented above, is extremely difficult to solve. It is highly non-linear while a general knowledge of the function surface is not available, thus making the conventional gradient-based non-linear optimization methods inapplicable. Simulated annealing (SA) provides a heuristic alternative of obtaining approximate solutions to difficult optimization problems without the need to compute the function gradient, nor much prior knowledge about its ‘terrain’ (Eglese, 1990; Kirkpatrick et al., 1983). Two facts make SA well suited for the specific problem considered here: (1) it is an organized ‘trial-and-error’ search method that requires only a point’s corresponding function value and no other information; and (2) a certain level of approximation is acceptable for estimating the weighting parameters.

Data for empirically testing the proposed modeling approach were acquired from an experiment in which six subjects performed three distinctive types of seated reaching movements (Fig. 2; see Zhang and Chaffin, 1997 for a full description). The experimental protocol was approved by the University of Michigan Human Subject Review Committee. A four-camera MacReflex™ motion analysis system was employed to measure the surface marker positions at a sampling frame rate of 25 Hz. The 3-D coordinates of surface markers for the wrist, elbow, and acromion were translated into those of the corresponding internal joint centers using a procedure developed by Nussbaum et al. (1996). Based on a linkage representation (Fig. 1), profiles of the seven joint angles as described previously were derived for 72 movement trials (12 movements per subject $\times 6$ subjects). Without any alteration (e.g. smoothing), these angle profiles along with those of hand trajectory served as the input for modeling.

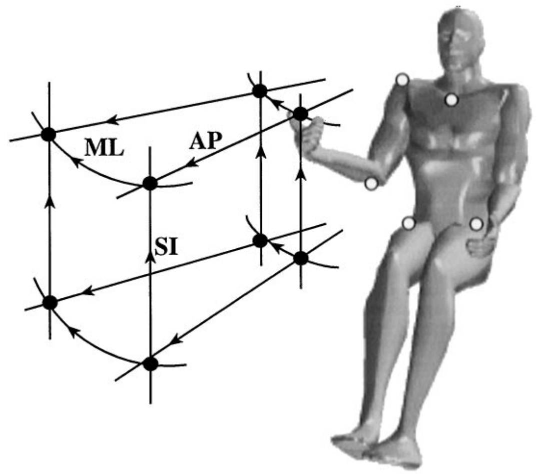

Fig. 2. An experiment in which six subjects performed three types of seated reaching movements, distinguished by the hand movement direction: anterior–posterior (AP), medial–lateral (ML), and superior–inferior (SI). Each type was performed at four hand path locations created by varying the height, distance to the body when resting, and asymmetry of the torso. Reflective spherical markers were placed over the subjects“ palpable body landmarks identifying the right wrist, right elbow, right acromion, sternum, right and left anterior superior iliac spine (ASIS). The two ASIS markers were utilized to approximately locate the bottom of spine assumed as the ASIS-bisection.

Each of the 72 movements was modeled using a four-weight scheme and a seven-weight scheme, both time-invariant. Additionally, in the interest of further examining the validity with time-invariant weights (and strategy which they represent), a four-weight modeling scheme that allowed the weights to vary over time was also attempted for some of the movements. This was conducted on a limited basis due to the computational complexity involved—the optimization-based fitting process was performed discretely for each time frame, minimizing the RMSE of instantaneous angular velocities (i.e. using Eq. (4) directly). The weight set previously estimated using a time-invariant scheme served as the initial value for this frame-to-frame search. A computer program that implements all the modeling procedures described above was developed using Mathematica®.

3. Results

Modeling based on time-invariant weighting schemes generally resulted in close representations of the measured movements (Table 1). With a simple time-invariant four-weight scheme, the majority of the 72 movements were accurately reproduced, as suggested by the overall TaRMSE mean of 4.26° and median of 2.99° (note that the corresponding TaAE values would be smaller). More specifically, modeling errors for 54 out of 72 movements were less than 5°, while greater errors (TaRMSE $> 5°$) were mostly associated with anterior-posterior (AP) movements. A seven-weight scheme relaxes the constraint imposed to a four-weight scheme, and potentially can better accommodate less coordinated postural behavior and thus improve the modeling accuracy. The improvement, however, was substantial only for those movements that were relatively poorly modeled by a four-weight scheme — the mean error decreased by 4.5° for the AP movements but only by 0.5–0.6° for both ML and SI movements (see Table 1). The overall modeling accuracy may be illustrated by graphically comparing the reproduced versus measured profiles of a movement that was relatively poorly represented using a four-weight scheme (Fig. 3).

Time-invariant weights designated to the four body segments demonstrated a certain level of consistency within each type of movement, while the motion apportionment among the segments varied considerably across different movement types (Fig. 4). Since the four-weight time-invariant scheme was shown to allow a close fit for most of the movements considered, emphasis of weighting parameter interpretation was placed on those resulting from a time-variant four-weight modeling scheme. As the weighting parameter values were being statistically summarized (Fig. 4), four out of 24 anterior–posterior (AP) movement trials were excluded for not having a close representation (TaRMSE $> 10°$). Note that the weighing parameters are normalized as weighting percentages — four parameters within each set are divided by their sum — to better visualize the apportionment among body segments. The trend exhibited by these weighting percentages appears to be intuitively consistent with the relative participation level of individual body segments. For instance, a constantly active role of the arm in completing reaching motions is reflected in the meager percentages (6%–13%) for both the forearm and upper arm throughout all the conditions.

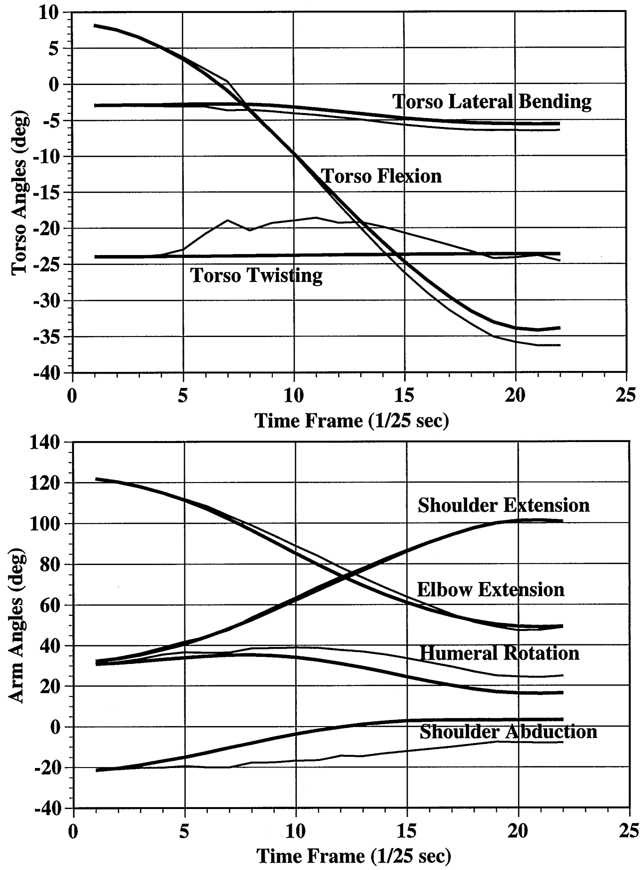

Fig. 3. An illustrative example of model-reproduced (thick) versus measured (thin) angular profiles for one particular movement trial with a 4.6° time-averaged root mean square error (TaRMSE, see expression (10)). The time-averaged absolute error (TaAE, see expression (11)) for this trial is 3.3°.

Table 1
A statistical summary of fitting errors (TaRMSE in degrees) that resulted from the modeling of three types of seated reaching movements ($n = 72$) using two weight schemes

Movement type	Four-weight scheme			Seven-weight scheme		
	Mean	S.D.	Median	Mean	S.D.	Median
Anterior–Posterior	7.97	4.04	7.20	3.47	1.83	3.11
Medial–Lateral	2.71	1.94	1.92	2.12	1.43	1.47
Superior–Inferior	2.11	0.93	1.81	1.69	0.76	1.64
Overall	4.26	3.72	2.99	2.43	1.58	1.94

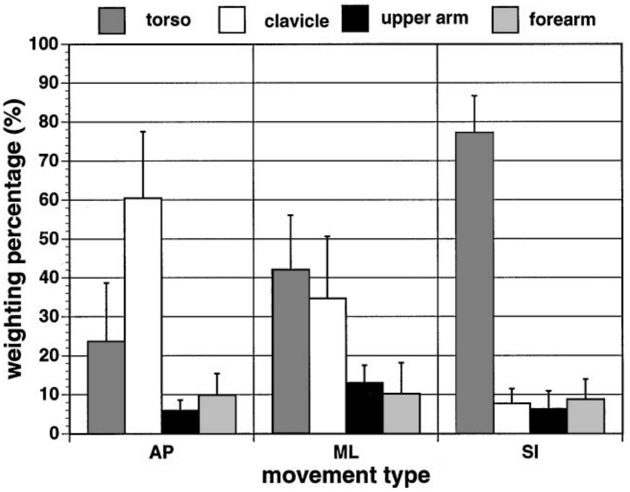

Fig. 4. Weighting parameter values, resulting from a time-invariant four-weight modeling scheme, normalized as percentages of the sum of individual sets. Bars represent the average across subjects for each type of movements. Whiskers indicate standard deviation.

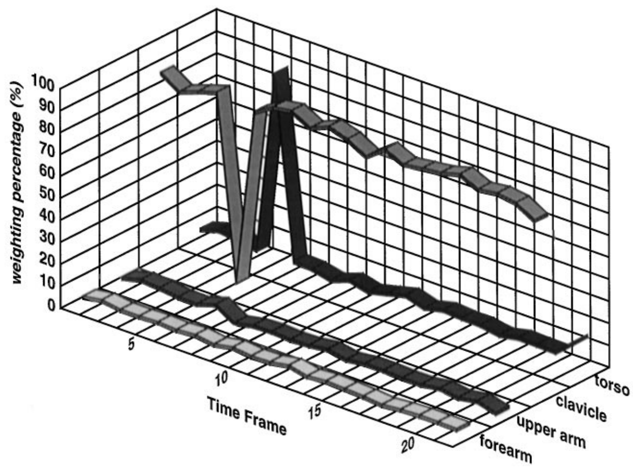

Fig. 5. Weighting percentages in time series obtained using a time-variant four-weight scheme for the same movement illustrated in Fig. 3. The resulting TaRMSE and TaAE were 3.9 and 2.84°, respectively. Except for one outlier time frame where the torso and clavicle appear to 'exchange' weights, the weighting parameter values remain fairly constant over time. This outlier may be attributed to the rather erratic pattern of clavicle motion (torso twisting) at the particular frame (see Fig. 3).

We found that, if a movement had been previously well fitted (TaRMSE $< 5°$) using a time-invariant modeling scheme, the weighting parameters did not vary much over time, and the better the previous accuracy the less the variation. A medium level variation over time can be qualitatively illustrated by plotting the weighting percentages as time series (Fig. 5) for the same movement trial presented earlier (Fig. 3). The modeling scheme that permitted the weights to vary over time reduced the fitting error but the reduction was also inversely correlated with the magnitude of previous error.

4. Discussion

Our optimization-based differential kinematics approach was developed to model moderately complex 3-D human movements with a reasonable computational demand, and to make inferences about the posture control strategy that underlies the movements being modeled. The latter depended on the success of the former. Although much effort has been invested in searching for one mysterious, presumably optimal strategy, there is little consensus regarding which optimal theory or criterion best explains human dynamic postural behavior (i.e. results in a best match in terms of kinematics). This is partly attributable to the prohibitive computational as well as empirical complexity of validating a particular theory using the existing optimization-based approaches. Motivated by the stipulation that multiple objectives may be involved in at least some movements, a generalized performance criterion has previously been formulated as a weighted linear combination of several subcriteria (Zajac and Winter, 1990). Strategic changes, therefore, are reflected by re-weighting or removing one or more subcriteria. However, such a generalized criterion has thus far served as a conceptual illustration, but has not been implemented or empirically tested. The modeling approach proposed in this work was an attempt to implement a more accommodating if not generalized representation scheme by 'parametrizing' the postural control strategy. This strategy indeed varied in different types of seated reaching movements considered in the empirical testing, as revealed by changes in the relative magnitudes of the parameters.

This modeling work, employing a velocity-domain method to resolve kinematic redundancy, argues for a velocity control strategy for dynamic posture determination. The primary support comes from the model's ability to closely represent measured movements. There are no data nor guidelines in the literature regarding what level of accuracy (i.e. closeness) would suffice for accepting assumed behavior or hypotheses based on a particular modeling process. Our empirical test demonstrated that the overall modeling accuracy achievable by the proposed approach was in the range of 2–4°. Such accuracy is comparable to the trial-to-trial repeatability of 2–5° (TaRMSE) we observed in a separate study (Chaffin et al., 1998) of a series of seated reaching movements similar to the ones modeled here. This suggests that on average a movement reproduced by the current model would emulate the actual movement as closely as if the movement were repeated by the same individual. A velocity control strategy is favored also in respect to the computational efficiency or simplicity. Once a set of 'tunable' weighting parameters is specified, there is no longer postural indeterminacy and the movement profiles are delivered via simple integration (see Eq. (9)) which is much faster than muscle stress control (which uses dynamic optimization) and displacement control (which uses sequential static optimization). We can see a great similarity between this scenario and what was proposed by Bernstein (1967) — the nervous system effectively eliminates redundancy by grouping multiple variables into functional units controlled by a single command. The choice of a strategy with computational advantage also appears to be consistent with the widely accepted general principle that motor control tends to avoid complex computations when multijoint movements are being executed (Gomi and Kawato, 1996).

It would be remiss not to discuss the limitations of the proposed modeling approach. One major limitation arises from its dependence on a specification of hand motion trajectory. This specification may in effect limit the applicability of our model. An improvement can be made through the implementation of a separate model which predicts the hand trajectory, provided the initial and terminal hand positions. For point-to-point discrete simple reaching motions, the issue of hand trajectory has been investigated quite extensively. One of the most robust results reported by several studies (Flash and Hogan, 1985; Morasso, 1981, 1983) is that the hand trajectory is essentially straight with a bell-shaped velocity profile. A model derived from a minimum jerk theory for predicting such hand motion trajectory is available and can readily be utilized (Flash and Hogan, 1985). Another limitation that could lead to worthwhile future investigation is that the scheme with time-variant weighting parameters has yet to be fully explored. This scheme, while losing some computational advantage, would improve the modeling accuracy, particularly for those cases that were not well accommodated by the time-invariant scheme. More important, it would help gain thorough insight into possible strategic change(s) during the course of a movement, or otherwise enhance the confidence level in using as well as interpreting the time-invariant representations.

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