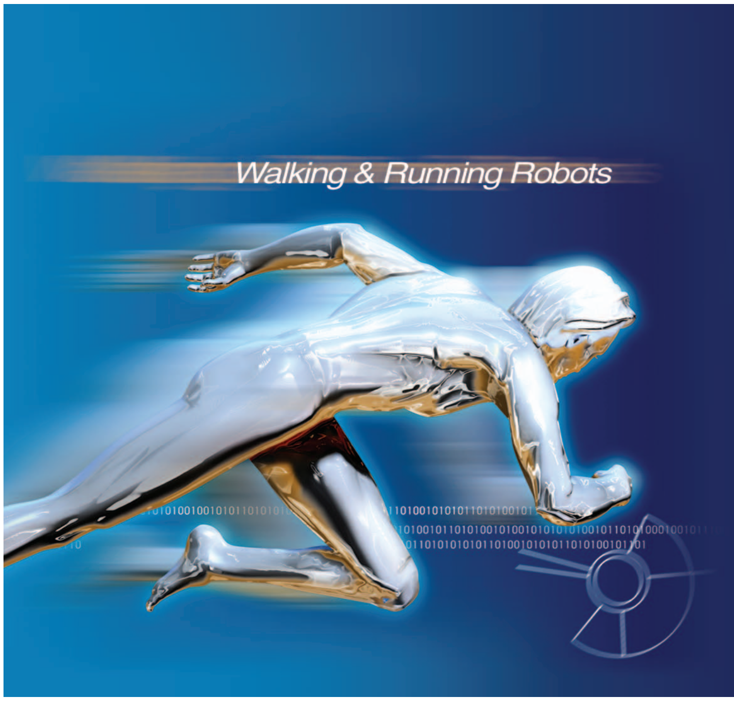

Choosing Your Steps Carefully

Trade-Offs Between Economy and Versatility in Dynamic Walking Bipedal Robots

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State-of-the-art bipedal robots can perform a great variety of tasks, demonstrating impressive versatility. The best present-day robots can walk, stand, turn, and climb stairs (e.g., Honda ASIMO [1]). Once the challenge for versatility has been met, other requirements become important in making bipeds practical. Chief among the requirements for most methods for locomotion is energy economy. Autonomous robots may be designed with intent to perform primary tasks such as sensing or manufacturing, and so secondary goals such as economic transportation may seem unimportant. However, improved energy economy allows a machine greater range, greater capacity to carry loads or perform primary tasks, and greater independence. As robots gain in capabilities, energy economy will gradually grow as a feature, much as it has for mobile devices such as notebook computers, cellular phones, and personal music players. Here we examine the means by which high economy can be achieved in dynamic walking robots—machines designed expressly to harness the natural dynamics of the legs during walking (see Figure 1). We then examine the issues in achieving high economy while also combining the versatility already demonstrated in other robots.

Energy economy may be defined in a variety of ways, but the most objective measure is the energetic cost of transport (COT). Defined as the energy consumed to move a unit weight a unit distance, COT allows for objective comparison between many forms of locomotion, with energy derived from a variety of sources. When calculated in SI units, COT is a dimensionless quantity (e.g., $\mathrm{J} \cdot \mathrm{N}^{-1} \cdot \mathrm{m}^{-1}$). Here we base energetic COT on actual energy consumed, defined as that extracted from readily-available sources such as gasoline, electricity, food, and air. Normalization for body weight and distance traveled, the product of which is work, accounts for the expectation that more work is required to move a heavier machine. A separate and also useful quantity is the mechanical COT, where mechanical work rather than energy expenditure is used in the numerator. Mechanical COT is helpful for examining inefficiencies such as friction effects, but energetic COT also includes the conversion of energy to work, which may also be performed with a variety of inefficiencies. Another advantage of energetic COT is that energy expenditure is easy to quantify, whereas the mechanical work performed is not always readily available for walking robots.

Compared to other vehicles and to animals, it is apparent that robots span a wide range of economies (see Figure 2). For example, the modern Honda ASIMO has COT = 3.2 [2], about ten times that of a walking human (COT = 0.3; e.g., [3]). It is understandable that many robots are not economical, because economy has not commonly been a design consideration. At the low end of the range of robot economies is the Cornell Efficient Biped [2], with COT = 0.2. This is a considerable improvement over ASIMO, but it is still more than double the COT of a bicyclist or a Toyota Prius. The Cornell machine is also a specialist, capable of walking economically and little else. It performs none of the other practical tasks of ASIMO, with its sole purpose being to illustrate the possible economy of a walking robot. It does not, however, demonstrate what further gains may be possible. For example, it is unclear whether walking robots must perform far less economically than wheeled machines such as the Segway i2 (COT = 0.08; Segway Inc., Bedford, New Hampshire) or the GM EV-1 (COT = 0.04; General Motors Corp., Detroit, Michigan).

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To determine the potential gains in bipedal energy economy, we must examine the principles of locomotion. It is instructive to compare these principles with those of wheeled transport. Wheels are the preferred mode for most transportation needs, so much so that a vast infrastructure has been constructed to provide fuel and pavement. Many mobile robot tasks may be performed as well or better with wheels rather than legs. We shall also compare two different paradigms for bipedal walking. The first is the conventional zero moment point (ZMP) approach (e.g., [4]), which is the basis for most versatile bipeds including ASIMO. The second is the principle of dynamic walking, originally developed by McGeer [5] and now applied in several other robots and even demonstrated to be harnessed by humans.

A robot designed solely for walking economy may accomplish no other tasks of note. As demonstrated by the Cornell Efficient Biped [2], economical machines may have design constraints that disallow other useful tasks such as standing. We will therefore also consider the design of a hypothetical robot that is both economical and versatile. As a benchmark we will adopt the guidelines of the W Prize (see announcement on page 13). The W Prize will be awarded to the machine that walks at least 1 m/s with COT = 0.10 and surmounts a series of obstacles including tight turns, a short flight of stairs, and stepping stones.

We begin our examination with a review of the general types of energy losses that are experienced in locomotion. We will then compare wheeled and legged locomotion in terms of those losses. This comparison will include ZMP and dynamic walking approaches, as well as an examination of the strategies by which humans maximize their economy. Finally, we will consider the design and technical challenges of a hypothetical bipedal robot that is both economical and versatile enough to compete for the W Prize.

Energy Losses

Energetic economy is essentially a study of energetic losses. In steady locomotion, the goal is to perform no net work on the machine. The only positive work required is, therefore, that needed to offset negative work. We shall broadly and somewhat loosely classify negative work into the categories of energy conversion, friction, and inertia/braking. Energy conversion includes any transformation of energy from one domain to another, as well as transformations within a single domain. Transformation between domains includes the conversion of chemical energy (fuel) into mechanical work, as is performed by an internal combustion engine. Humans also convert food energy into mechanical work through the respiration cycle. Motors produce mechanical work by converting electrical energy, and this conversion also entails a loss, albeit a relatively low loss compared to the others. Energy conversion also includes within-domain transfer such as that performed by gears, ball screws, levers, and transformers, all of which can be subject to energy loss.

The second category of friction includes unavoidable energy losses associated with motion. Items that fall into this category include joint articulations, air resistance, and rolling resistance. Many of these only count as friction in a general sense; for example, rolling resistance is associated with deformation of tire material and not necessarily due to relative sliding motion. However, these friction-like losses are of concern for most major forms of locomotion. Energy conversion within the mechanical domain, such as gears and other transmissions, usually dissipate energy through friction, and so could also be categorized as such.

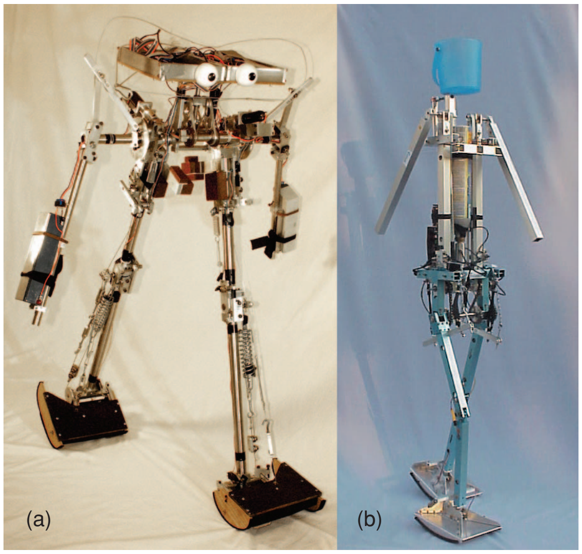

Figure 1. *Dynamic walking robots. (a) Cornell Efficient Biped and (b) Delft University Denise are two machines that rely on the leg dynamics to produce passively stable walking gaits. Both robots walk without use of feedback control of positions, relying primarily on event-triggered control for powering. The Cornell robot [2] pushes off at the ankles, and Denise [15] pumps the hips to power the gait and offset energetic losses due to collisions. Dynamic walking robots can produce economical, human-like gaits but also tend to have poor versatility.*

The final category to be considered includes inertial and braking losses. Inertial losses are those associated with acceleration of a mass. Energy is usually dissipated when a mass is decelerated by negative work, and energy is usually expended when a mass is accelerated by positive work. In some cases, positive energy may be expended even to perform negative work. For steady locomotion, every deceleration requires an acceleration, and so inertial losses may be reduced through the avoidance of decelerations. Braking is not typically associated with inertial losses, but it is in our consideration because the negative work of braking must be offset by an equal amount of positive work for steady locomotion. In that sense, braking and deceleration both result in negative work and have the same consequences. Note that braking is often performed using friction but is classified as an inertial loss because braking is an elected action performed for some reason, whereas the friction category only includes unavoidable consequences of motion.

Comparison of Modes of Transport

Although the primary consideration here is legged locomotion, it is instructive to also consider the advantages of wheeled locomotion. This helps to clarify the benefits to be gained from legged locomotion. We shall also consider two main categories of legged locomotion, the ZMP principle and the principle of dynamic walking. These will be compared in terms of both economy and versatility, as well as control. We shall see that each mode of transport has qualities that favor economy or versatility, and that neither form of legged locomotion currently provides both. Finally, we will compare these mechanistic forms of locomotion with humans.

Wheeled Locomotion

Wheeled vehicles and especially automobiles are economical compared to most alternatives. A vehicle with perfectly round wheels can theoretically travel on flat ground, supporting their body weight, at zero energetic cost, because no mechanical work except to overcome friction need be performed to maintain constant speed rolling [see Figure 3(a)]. In practice, mechanical energy losses are quite significant even at constant speed, and mechanical work is also necessary if only to accelerate a vehicle from standstill. We take as an example the typical modern automobile. The most substantial energy loss occurs in conversion of fuel to mechanical work, with an internal combustion engine efficiency of about 32%. With pumping and friction losses within the engine, the net efficiency of work delivered to the drive train is about 23%. The further losses are due to overhead costs such as accessories and maintaining idle, and due to friction, primarily in the form of rolling resistance and aerodynamic drag. These combined effects account for an energetic cost of transport of about 0.17 in a modern economy car.

A wheeled robot can improve significantly on the automobile. Reliance on an electrical motor immediately boosts efficiency, because fuel energy has already been converted elsewhere. In our definition of COT, this conversion is excluded from vehicle economy. Conversion of battery power to mechanical work can occur at efficiencies of 85 to 95%, depending on the quality of the motor. A robot's transmission can be comparable to an automobile's or can even be dispensed with via direct drive. When rolling on a smooth

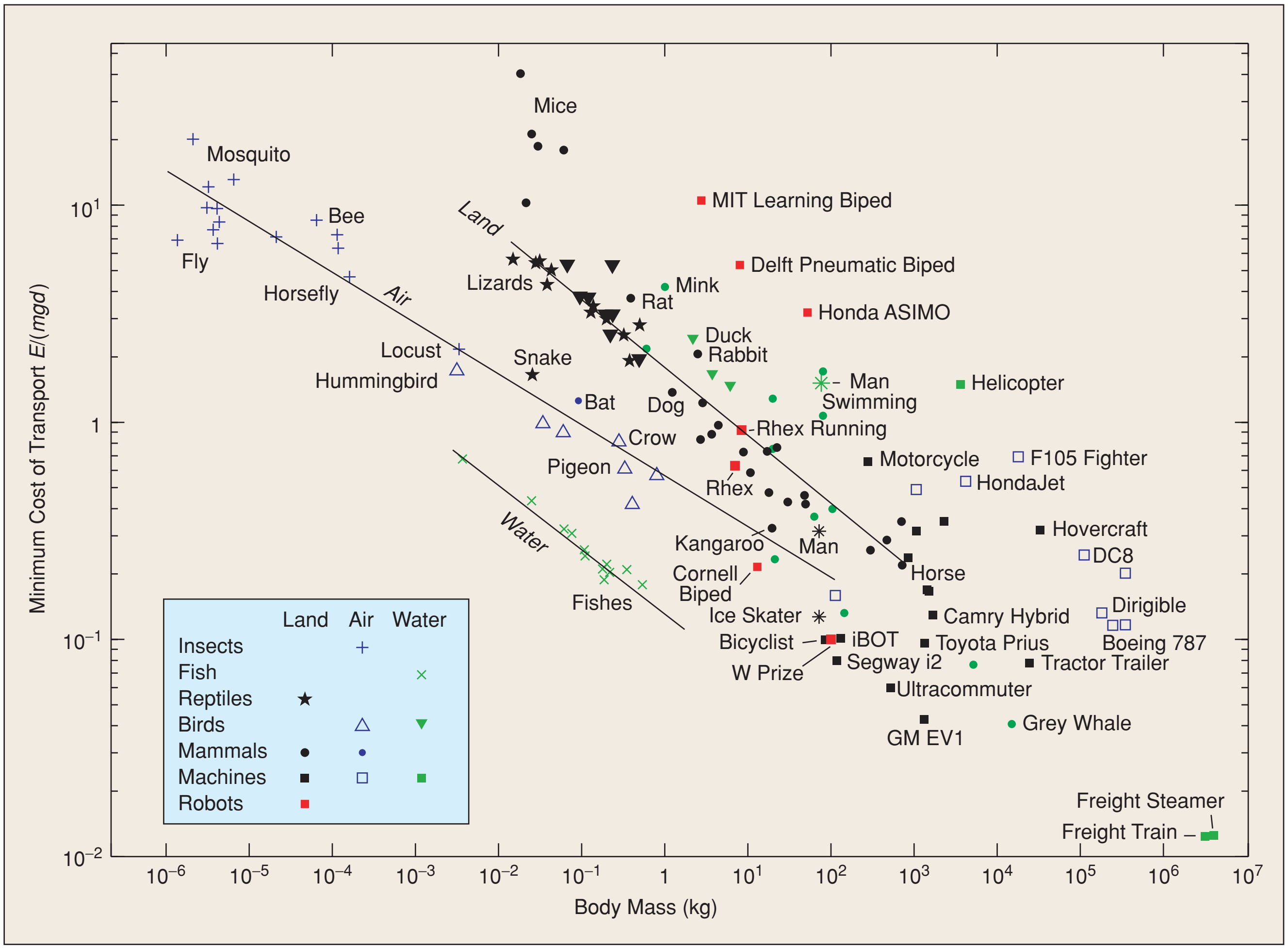

Figure 2. *Comparison of minimum cost of transport as function of body mass for a variety of robots, animals, and vehicles. Cost of transport is defined as energy expended, per weight and distance traveled, yielding an objective, dimensionless quantity summarizing overall economy. Both animals and machines tend to have reduced cost of transport as a function of body mass. Data points are from Tucker [26], with a variety of new and updated values.*

floor at slow speed, both tire and aerodynamic losses can be very small. Because most mobile robots are not designed for economy, they do not provide suitable benchmarks. In their stead, the GM EV-1 electric vehicle, with an energetic cost of transport of 0.043, indicates the economy possible in a wheeled device with good performance and considerable versatility.

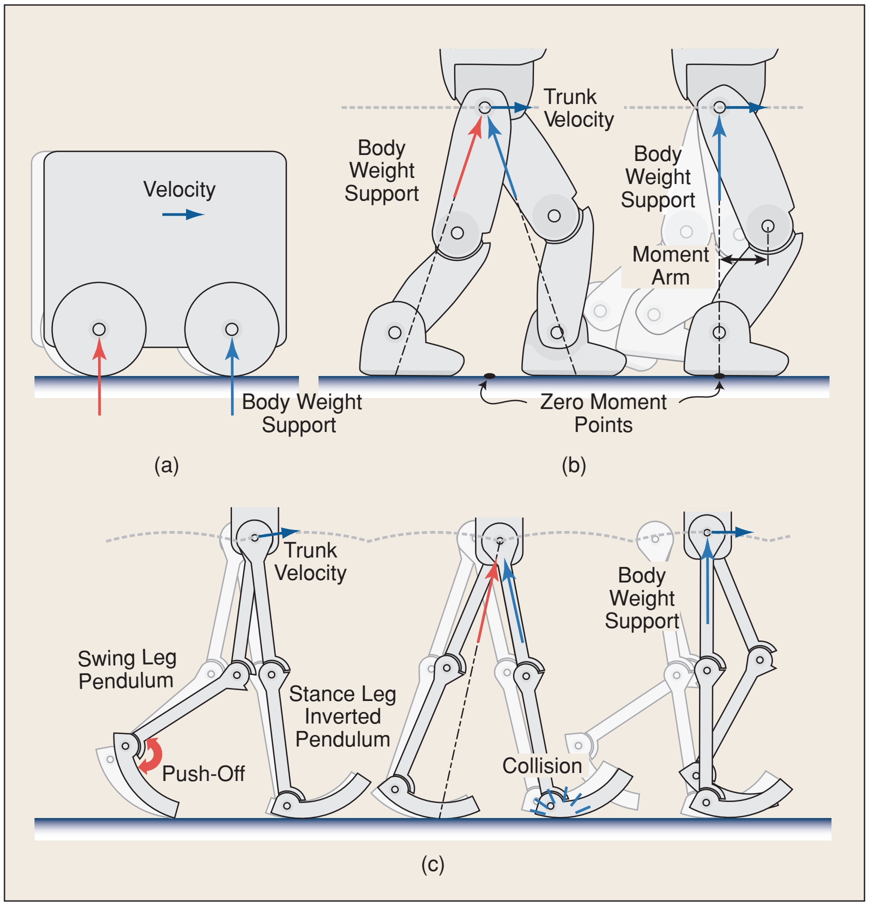

Figure 3. *Three different modes of transport. (a) Wheeled locomotion. (b) ZMP locomotion. (c) Dynamic walking.*

ZMP Locomotion

The most versatile bipedal robots to date employ some form of ZMP [see Figure 3(b)] control [4]. ZMP refers to the location within the base of support about which the ground contact forces exert no moment in the lateral and fore-aft directions. By controlling this location, the robot may induce forward motion while maintaining dynamic balance. ZMP control places few constraints on the motion of specific joints, allowing for considerable freedom to the designer. However, controllability of the ZMP requires avoidance of kinematic singularities such as full extension of the knee and maintenance of a flat (and preferably large) foot contact surface. These features enable robots not only to walk but also to perform complex maneuvers such as turning, climbing stairs, and kicking balls. Some robots demonstrate impressive maneuvers despite not explicitly employing ZMP control, instead following ad-hoc trajectories that emerge from a designer's experience and intuition. These machines nevertheless embody the spirit of ZMP control, and their architecture relies on similar features. Whether the ZMP is controlled explicitly or implicitly, the associated platforms demonstrate considerable flexibility and maneuverability.

The same architecture, however, suffers in terms of economy. Even if the robot's center of mass is kept on a level path at constant forward speed, considerable energy must be expended. One expense is simply to support body weight. The knees of both legs must be flexed through much of a step in order to accommodate a level path for the center of mass, and to avoid the kinematic singularity of full knee extension. Extensor torques must then be produced about both knees, with a substantial energetic cost even in a static weight-supporting configuration. Adding motion to this weight support requires positive work performed by the trailing leg and braking negative work by the leading leg, because the trailing leg extends and the leading leg bends while both legs produce extensor torque. The active production of both positive and negative work requires energy, all without accelerating the robot itself. Some energy could theoretically be recovered from the negative work through regenerative braking, but the efficiency of such regeneration is limited to no more than 30% in most practical applications. Although some advantage can be gained from regeneration, it is more advantageous simply to minimize the amount of simultaneous positive and negative work that is to be performed. Forward motion of the center of mass of a ZMP robot is in any case highly uneconomical compared to a wheeled vehicle, where braking is unnecessary for maintaining speed.

The mechanical requirements of supporting weight on bent legs also induce ancillary costs. The torque needed to support body weight typically entails either large or heavily-geared motors, so that inertial or frictional losses may be high when producing the reciprocal motion of the legs. Inertial losses occur even when the robot's body moves at constant speed, because the back-and-forth motion of the legs requires continual acceleration and deceleration. The forward motion of the legs, required to prepare for each step, induces a frictional loss that is not incurred in wheeled vehicles, where the continual rotational motion requires no such resetting of configuration, and therefore no unnecessary work against friction.

The feedback control performed in the ZMP paradigm may also entail an energetic cost. ZMP robots often employ positional control of the joints, where energy may be

We shall see that each mode of transport has qualities that favor economy or versatility, and that neither form of legged locomotion currently provides both.

expended simply to follow commanded positions regardless of workload. The commanded positions are enforced through negative feedback, where the feedback gain produces corrective torques that act against tracking error. Large gains contribute to low tracking error, but with a trade-off in energetic cost. High torques may be needed to enforce accurate tracking, especially if the legs do not naturally follow the commanded motions. In addition, large control transients may result from a variety of factors, such as random disturbances, unmodeled deterministic disturbances, sensor noise, quantization and sampling effects, and frictional and inertial loads. The actual energetic cost of high-gain position control is difficult to predict, but it generally increases with the magnitude of feedback gains and the desired accuracy of position tracking. Position control for walking contributes to the energetic differences with wheeled vehicles, which do not generally control for wheel position. ZMP control, and indeed other methods that also use position control, might gain considerable energetic benefits by reducing position control gains or even eliminating position control altogether.

These various economical factors are offset by considerable advantages in versatility. ZMP robots are highly adaptable for different maneuvers. This is because ZMP control is a general strategy for maintaining stability, for which locomotion is a special case where the advancement of the ZMP is specified. As long as the ZMP is stabilized, many other tasks may be performed. The state-of-the-art in this regard is the Honda ASIMO, which is capable of turning, climbing stairs, carrying a load, and pushing a cart. These capabilities also make ZMP robots suitable for tasks and environments that may be unsuitable for wheeled mobile robots. It is the focus on these tasks that has made energy a secondary priority. With proper optimization, many of the energetic costs detailed above can be reduced considerably. However, even at the theoretical minimum energetic cost, this method of locomotion is far less economical than wheeled locomotion.

Dynamic Walking

Another approach is to rely on the passive dynamics of the legs to produce walking [see Figure 3(c)]. This approach eschews position control and focuses on production of a cyclic gait. An early demonstration of dynamic walking was by Miura and Shimoyama [6], whose robot walked by allowing the stance leg to behave as an inverted pendulum, and controlling the beginning and ending conditions to produce a steady gait. Raibert [7] similarly demonstrated running gaits that resulted from controlled spring action of the legs. McGeer [5] took dynamic walking to a fully passive extreme, showing that walking could in fact be produced with no control whatsoever, by descending a gentle slope under gravity power, with the legs moved freely as pendulums. The leg dynamics both produced a periodic motion and exhibited passively stability, with the only energetic loss incurred during the collision of the swing leg with ground. McGeer [5] also explained how power could be added through judicious actuation. Others [2] have subsequently produced actively powered robots that walk on level ground, though with a gait that is still governed primarily by passive dynamics. Here we define the term *dynamic walking* to refer specifically to machines designed to harness leg dynamics, using control more to shape and tune these dynamics than to impose prescribed kinematic motions. One powering strategy for dynamic walking is to produce torques about the hips, either forcing one leg against the other or against the torso. The passive dynamics allow considerable freedom in the torque program, so long as the hips perform net positive work [5]. In fact, one means of achieving stable powered walking is simply to lean the torso forward, and to support that lean against hip torque produced by the alternating stance legs. Another strategy for powered walking is to push off with the trailing leg's ankle [8]. Again, positive work must be performed, and with proper timing considerable energy savings may be realized. A feature shared by both strategies is that the other joints may be left unactuated, allowing for minimal frictional losses. The swinging of the legs also involves no inertial losses, because the reciprocal motion is produced passively. Also, energy need not be expended to support body weight, because the stance knee may be kept fully extended with a passive mechanical stop.

The only fundamental energetic loss associated with dynamic walking is inertial, in the collision of leg with ground. We may illustrate these losses with a simple model, in which the robot's mass is lumped almost entirely into a point at the pelvis. The mass of the legs may similarly be lumped into points at the feet, taken to be much less massive than the pelvis. Such a model retains the pendulum-like properties of the legs, yet simplifies the computation of energy losses considerably. Each leg prescribes a circular arc for the pelvis, so that the velocity of the center of mass is directed forward and downward at the end of one arc, and forward and upward at the beginning of the next. Termed the step-to-step transition [3], the change in velocity entails a collisional loss of energy and may be interpreted as the projection of the precollision velocity onto the admissible post-collision velocities. The energy loss per step is proportional to the square of speed and the square of the angle between the legs. With respect to the center of mass, hip torque performs positive work during the subsequent step, restoring the energy lost due to the collision. The mass distribution of an actual machine such as the Delft Pneumatic Biped [2] is different from the simple model's but not sufficiently to alter the fundamental energetics, which remain dominated by collisional losses.

The model also illustrates the advantages of the alternative powering strategy using ankle push-off. Rather than performing positive work throughout the step, push-off is ideally performed impulsively by the trailing leg immediately preceding the impact of the leading leg with ground [8]. This concentrated force pushes upward on the center of mass, thereby reducing the velocity of the impact that follows. Under ideal conditions, this reduces the energetic loss to one-fourth that of the hip-powered case. The timing of the impulse is particularly important; pushing off immediately after the leading leg impact has no advantage over hip powering. In practice, it is difficult to produce a large push-off force impulsively and just prior to collision. For example, the Cornell Efficient Biped appears to perform some of its push-off work during and after the collision, but enough before collision to contribute to the machine's low cost of transport. There are also practical limitations in the impulsiveness of the collision that can be sustained by a walking machine, which may benefit from some active or passive cushioning of impact.

Practical walking machines encounter some minor energetic losses not accounted for in the simple model. Realistic mass distribution yields collision losses that are somewhat higher than, yet fundamentally very similar to, those of the simple model. In addition, most dynamic walking machines (e.g., Cornell Efficient Biped, Delft Pneumatic Biped) achieve ground clearance of the swing leg with knees that flex and extend passively during the swing phase. The alternative to knees is to actively shorten the leg for ground clearance [5]. The knees can support body weight at no cost through a mechanical stop that prevents knee hyperextension. Passive dynamics cause the swing knee to be bent at mid-stance and to reach full extension before heel strike [9]. During the stance phase, the stance leg remains extended against its knee stop as long as the ground reaction force passes in front of the knee. This is ensured by offsetting the feet to point forward from the ankles. Knees result in a slightly increased energetic cost, due to the collision of the swing leg hitting its stop. But the forward-pointing foot alters the collision geometry significantly, increasing the heel strike collision cost much more than the knee collision. Kneed walking machines are typically less economical than their straight-legged counterparts.

Two design features can improve the economy of dynamic walking machines. The first is the addition of a curved foot bottom, which reduces the vertical component of collision velocity (see [10]). The radius of curvature and length of foot can significantly influence energy losses. For example, a radius of 0.3 leg length (as employed by McGeer [5]) can reduce collisional losses by about one-half compared to a pointed foot [10]. A radius equal to leg length will theoretically reduce losses to zero, but with the disadvantage of requiring very large and unwieldy feet that will hamper maneuverability. In fact, matching foot length to the curvature for a human-like step length, a radius of 0.3 leg length yields a human-like foot length [11], suitable for climbing stairs and negotiating an environment designed for humans.

The second feature for improving economy is a mechanical spring acting torsionally between the legs at the pelvis [8], with the axis of torque passing laterally through the hips. Such a spring will speed and accentuate the pendulum motion of the legs, allowing for faster and shorter steps for the same amount of energy input. For a given walking speed, as the spring theoretically approaches infinite stiffness, the steps become shorter and shorter, and energetic losses approach zero. Springs may alternatively be applied between each leg and the robot torso, in which case they can also contribute to stabilization of the torso, lessening the burden on an active balancing system. Passive hip springs of either type have yet to be applied to a physical walking machine. However, active springs have been implemented in the pneumatically driven robots of the Delft group [12]–[14]. In these cases, McKibben actuators produced not only work but also stiffness about the hips and knees, simultaneously powering the gait (as described above) and stabilizing and speeding the leg motion. One machine, Denise [15], also used passive springs at the ankles to accentuate the advantages of curved foot bottoms. These features were implemented for purposes of powering and stability but could be used to improve economy.

In terms of overall energetics, dynamic walking compares poorly with wheeled vehicles. Wheeled vehicles do not encounter significant collisions when moving on smooth ground. The most analogous cost to the collisions of walking is in fact rolling resistance, which may be thought of as a continuous series of collisions performed by a continuous and infinite set of legs, spaced infinitely closely together. However, the close spacing means that the center of mass (COM) undergoes minimal redirection, so that the energy lost in tire deformation is far lower than that in walking collisions. As steps become shorter and faster in walking, the collision costs also decrease. However, the ability to move the legs quickly, even if performed passively with springs, is likely to reach a practical limit that will make the energetic cost of dynamic walking considerably higher than most wheeled alternatives.

Dynamic walking can nonetheless be energetically far superior to ZMP walking. The mechanical knee stop in a dynamic walking machine allows body weight to be supported at no energetic cost [see Figure 3(c)]. Although heel strike collisions remain costly, they are far less so than the simultaneous positive and negative work of smooth ZMP translation. A dynamic walking machine has negligible friction in the unactuated pin joints, compared to the substantial friction of the geared motors in most ZMP robots. Dynamic walking machines also require minimal control, in most cases using no positional control. Also, no energy need be expended to move the legs back and forth, as is accomplished through passive pendular motion alone. The control and active movement of the legs contribute to a much higher energetic cost of locomotion in ZMP robots.

Many of these advantages may be attributed to underactuation in dynamic walking machines. In an actuated joint, control is performed with a direct energetic cost that increases with the amount of control effort. An actuated joint also requires a control infrastructure, in the form of motor and transmission, that exacts significant indirect costs due to inertia and friction. Underactuation is hardly a desirable feature in robots, but it benefits dynamic walking robots that have it. Actuation, while necessary for many functions, conversely tends to eliminate passive dynamics, requiring active work to produce a motion that might otherwise occur naturally. This is not to say that actuation must have this disadvantage; it is merely a disadvantage of most present-day drive systems. Alternative actuation schemes, yet to be developed, may well co-exist with passive dynamics.

The remarkable economy of dynamic walking comes, however, with a serious trade-off in poor versatility. Most dynamic walking machines can perform no other tasks, mainly due to underpowered or unactuated joints, and due to poor static stability. The Cornell Efficient Biped, for example, cannot even stand upright because of curved foot bottoms and unactuated hips. The focus in dynamic walking machines to date has been on the production of stable or economical locomotion. The simplicity of such machines puts them far behind the capabilities of ZMP robots in realms other than walking. Moreover, the reliance on limbs that swing freely makes it difficult to actuate joints without adversely affecting the pendulum-like dynamics that depend on low friction. Just as every unactuated degree of freedom yields an energetic advantage, every actuated degree of freedom yields a versatility advantage.

The energetics of the three forms of locomotion considered here may be summarized as follows (see Figure 3). In a wheeled machine, the entire weight of the body is supported passively, with no energetic cost. No mechanical work is needed to locomote beyond that needed to overcome friction. These factors contribute to an overall low cost of transport. A ZMP robot, in contrast, requires active support of body weight through much of each step. This is demonstrated by the moment arm of the center of gravity about the knee joint, where substantial extensor torque is needed. This active production of torque to support weight comes at an energetic cost, along with that for the horizontal motion of the center of mass. During each double support phase, one leg must extend actively, while the other must shorten actively, sustaining a braking loss. One leg therefore performs positive work, and the other negative work, both with a positive energetic cost. Finally, we consider a dynamic walking machine. During each single support phase, body weight is supported passively by the straight leg, with a mechanical stop keeping the leg from hyperextending. Motion of the center of mass along a pendular arc can also be accomplished passively, with no mechanical work. The single support phase therefore requires no energetic cost. However, the collision of the leading leg with ground will dissipate mechanical energy, which must then be restored through positive work. This work exacts an energetic cost, with no other work required to maintain steady walking. Dynamic walking is less economical than wheeled transport but has substantial advantages over ZMP walking.

Human Walking

We next examine how humans accomplish economical walking while remaining capable of complex maneuvers. It is interesting to note that humans convert chemical energy to mechanical work using a respiratory cycle that differs significantly in operation from internal combustion engines but is remarkably similar in efficiency. Humans convert glucose into the fuel used by muscle and other cells, adenosine triphosphate (ATP), at an efficiency of about 50%. Muscle then converts ATP into work with an efficiency of about 50%, so that muscle's total efficiency is about 25%, comparable to the 23% delivered to a vehicle drive train by an engine. Muscle can also actively perform negative work, with an efficiency of about −120%, meaning that some positive energy must be expended to perform negative work.

Humans rely heavily on passive limb dynamics and power their walking gait predominantly with ankle push-off (see Figure 4). Instead of pushing off impulsively just before leading-leg heel strike, humans push off with finite force over a finite and nonzero duration. An impulsive push-off, while theoretically desirable for minimum work, requires impractical force amplitudes. Humans begin push-off prior to heel strike of the leading leg, continuing positive work about the ankle even after heel strike and through most of the double support period [3]. The leading leg also collides with the ground over finite time, with the body COM undergoing a U-shaped displacement. The work performed during this displacement is higher than the theoretical minimum for impulsive push-off and collision. The duration of collision is apparently determined by practical limitations in impulsive force production, such as pain and damage potentially sustained by high impacts.

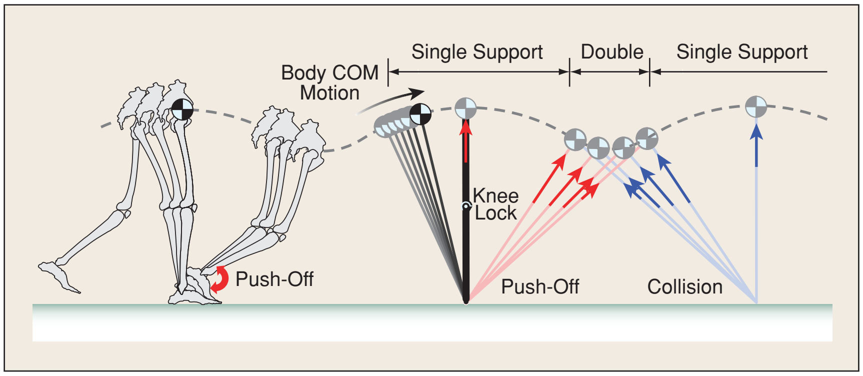

Figure 4. *Human dynamic walking.*

The collision appears to be performed both actively by muscle and passively through soft tissue deformation. Some of the collision energy also appears to be returned elastically, as the COM rebounds after the collision. However, this rebound may be damped, so that it remains unclear whether substantial collision energy is returned in a useful manner. Nevertheless, the result is that energy dissipation may actually occur over an extended duration, starting with heel strike and ending at mid-stance (when the stance leg is approximately halfway through its pendular arc). Similarly, humans also appear to extend the duration of push-off while distributing the positive work across multiple muscles. Following midstance, the Achilles tendon stores energy elastically, performing negative work on the body COM. This energy supplements the positive work performed by the calf muscles during push-off, so that the total push-off need not be performed by those muscles alone. The combined energetic cost of the positive and negative work associated with step-to-step transitions is about 60 to 70% of the net cost of walking.

The human hip muscles perform relatively little positive work, but part of their function is to function as actively-tunable springs [16]. The hip muscles are activated at the forward and backward extremes of leg swing, so that sufficient force is produced to speed reversal of motion. However, relatively little work appears to be performed actively on the leg, with elastic tendons accounting for much of the reversal. The hip muscles therefore speed leg swing much as mechanical springs, but at a metabolic energetic cost to produce the requisite muscle force. It might appear disadvantageous to actively produce muscle force when the legs could swing more slowly at no energetic cost whatsoever. However, as discussed above, sprung hips can reduce collision costs for the same walking speed. Hip action is therefore energetically favorable overall, insofar as the savings in collisions trade off against the costs of actuating the hip muscles.

Humans also enjoy some of the advantages of rigid curved feet, despite the fact that human feet are neither rigid nor curved. The geometry, actuation, and flexibility of the foot are quite complex but actively engaged in such a way as to mechanically resemble a curved foot. In terms of the forward progression of the center of pressure of ground reaction forces, human feet resemble curvatures with radius of 0.3 leg length [5], [11]. This effective radius of curvature appears to be an optimum match with leg geometry, yielding energetically favorable walking [10].

Humans also produce short bursts of high power by using muscles strategically [17]. For each foot, ankle push-off only occurs over approximately 15% of a stride (the time for one leg to repeat its motion). However, the calf muscles are active for almost double that time, allowing the Achilles tendon to store energy prior to push-off. The high peak power can then be produced with muscles that are relatively small. A similar scheme is employed in the Cornell Efficient Biped, which uses a small motor, active during most of a step, to store push-off energy in a spring. The energy is then released suddenly for push-off with a latch mechanism. If the same push-off were to be produced directly, a larger and heavier motor would be required.

The features of dynamic walking are remarkably well-suited to the drive capabilities of human muscles. Muscles can only act in tension, producing high forces at low speeds. Humans produce the appropriate shortening speed through a combination of musculoskeletal geometry and muscle architecture. Geometry refers to the placement of the muscle across one or more degrees of freedom, each with position-dependent moment arms. Muscle architecture refers to the parallel and series alignment of muscle fibers and their respective pennation angle (i.e., alignment with the tendon's pulling direction). Unilateral force production also means that muscles are back-drivable, although with the disadvantage that reversal of direction requires an opposing muscle. This disadvantage is somewhat mitigated by the fact that only half the muscles need produce high forces to resist gravity, with the opposing muscles (usually flexors) producing less force and weighing less. The tendons also contribute to economical movement by acting like springs, storing and returning mechanical energy during locomotion. Tendons also allow for proximal placement of massive muscle tissue, thereby keeping the distal extremities relatively light. These capabilities certainly match the needs of locomotion very well, to the extent that we would expect different actuators to optimally produce a different gait, or to suffer a significant energetic penalty in attempting a poorly-matched animal gait.

Zero moment point refers to the location within the base of support about which the ground contact forces exert no moment in the lateral and fore-aft directions.

Design of an Economical and Versatile Bipedal Robot

Let us consider the design of a hypothetical bipedal robot that combines a low cost of transport with the ability to execute a variety of complex maneuvers. This effectively requires the economy of a dynamic walking machine with the sophisticated capabilities of a ZMP robot. To achieve the advantages of both in a single machine will require careful design. We first consider the mechanical degrees of freedom necessary to be versatile, constrained by the advantages of minimal actuation for walking economy. We then consider new technologies that may contribute to economical actuation, through new actuators and devices, and through new control schemes.

Mechanical Degrees of Freedom

The prospective robot that can walk economically and yet execute complex maneuvers must possess many degrees of freedom. It is typical for ZMP robots to have many degrees of freedom; for example, the Honda ASIMO possesses 34. The requirement for energy economy, however, favors restraint in this regard. Even though engineering capabilities make many joints possible, each degree of freedom and its associated actuation presents a mass penalty. Extra mass in the torso is not a major concern, because the energetic cost of transport normalizes for body mass. However, mass in the legs is costly in terms of both collisions and leg swinging. It is therefore instructive to consider the minimum necessary

We define **dynamic walking** *to refer specifically to machines designed to harness leg dynamics, using control more to shape and tune these dynamics than to impose prescribed kinematic motions.*

number of joints that will accomplish each of the desired functional specifications, along with the corresponding actuation requirements.

Walking minimally requires only a few degrees of freedom. A straight-legged biped walking only in the sagittal plane (the plane bisecting the body into left and right halves) only requires one degree of freedom for each leg to swing about the hip. Realistically, however, ground clearance must be provided, and knees perform this function naturally. As with previous dynamic walking machines, the knees need not be actuated, although it is helpful to provide a locking mechanism and/or tunable compliance about each knee. Active powering on level ground also requires actuation about the hips or ankles, but not necessarily both. Whether using hip power or push-off, this actuation must be capable of performing significant mechanical work. Again, tunable compliance about these joints can aid leg swing and decrease collision losses.

A versatile biped will probably also perform active balancing from side to side; for example, through lateral foot placement [18]. This requires the ability to actively splay the legs laterally (also known as hip abduction/adduction) during each swing phase. This need be performed only against the weight and inertia of the leg, and so power requirements may be small. However, this is only the case if the same degree of freedom can be locked during the stance phase. Otherwise, the same actuator must be able to support body weight across the moment arm between the center of gravity and the splay axis. When the legs are splayed, the ankles must also accommodate inversion and eversion of the feet, but this may be accomplished passively.

The task of standing poses a considerable challenge to the economical bipedal walker. Standing puts the curved feet of many dynamic walking machines at immediate disadvantage. Fortunately, the sprung ankles of Delft's Denise biped can provide many of the same energetic advantages, but with flat or nearly flat feet. With the addition of either ankle actuation or simply the ability to lock the ankle, an economical walking machine will gain the ability to stand. The existence of a torso has not yet been considered. A torso may be balanced mechanically fore-aft with a bisecting mechanism that keeps it aligned between the two legs. However, the torso can alternatively be balanced actively, using hip actuation.

Turning has not yet been demonstrated in dynamic walking machines but can in principle be achieved with very minimal actuation. It is possible that a turn can be accomplished simply by rotating the leg about its longitudinal axis (also known as internal/external rotation of the hip) during the swing phase. The subsequent collision will not only redirect the body COM upward and forward, but it may also alter the redirection laterally. Perhaps coupled with added leg splay, the step-to-step transition might place the robot's state within the region of attraction for a continued gait that is rotated left or right of the original. The full analysis of turning by this means is not trivial, but the concept is plausible. The actuation requirements are low, because the leg rotation may be performed slowly through the swing phase, with no load other than the mass of the leg or foot. It is also possible to actively rotate the leg during the stance phase, but this would entail a greater load.

Dynamic walking machines that can walk stably can easily be adapted to cross a succession of stepping stones. Here, ZMP principles are at disadvantage, because continued controllability of the ZMP requires that stepping stones be large, so that the foot can exert torque on the ground, and that they be relatively closely spaced. Humans, in contrast, can easily handle a variety of stone spacings. As discussed by McGeer [19], dynamic walking machines can similarly adjust step lengths dynamically by modulating the torques produced at the hips. Here the torso becomes helpful, because it provides a large inertia against which torques can be produced. It can be leaned forward or backward to modulate speed in the absence of ankle push-off.

Perhaps the most challenging task for a dynamic walking machine is stair climbing. Climbing or descending a slope is in fact quite easy, as a machine with ankle push-off need only adjust the amount of positive mechanical work. Alternatively, a machine with a controllable torso need only lean it forward or backward to add or subtract energy during each stance phase. The challenge of stair climbing is in the relatively steep slope and requirement for step clearance. A steep slope increases the power requirements of the actuators, which implies indirect disadvantages such as motors with greater inertia or more gearing and hence more friction. Step clearance, where the swing leg must not trip as it rises to the next step, is almost certainly not compatible with passive dynamics. This means that considerable active control must be performed to lift the leg, and to balance the body during each step. This control does not exceed that already in place for other tasks, but the power requirements for ascending stairs will affect nearly every substantive actuator.

Examining these various degrees of freedom, a minimal powered walking machine that can accomplish all of these tasks might possess ten degrees of freedom. The production of forward walking with lateral balance involves six degrees of freedom in the sagittal plane, for each ankle, knee, and hip, and two degrees of freedom for abduction/adduction at each hip. The ability to turn entails two more degrees of freedom, for internal/external rotation of each leg. Of these, only the sagittal plane ankle and hip motions must be actuated with high power capability; the knees could function with only latching and/or tunable compliance. With additional locking mechanisms, abduction/adduction and internal/external rotation need not be powerful enough to act against entire body mass. These are of course minimal requirements. For side-to-side balancing it is helpful to have additional degrees of freedom, such at the ability to move the trunk laterally, or the ability to actively flex and extend the knees. Many robot designers will favor more degrees of freedom for more versatility, but each addition will likely entail a direct or indirect energetic penalty.

Actuation Challenges

From a mechanical design standpoint, several new technologies can contribute to an economical and versatile robot. The most obvious, yet most challenging, approach is to develop muscle-like actuators. Current actuators such as McKibben muscles have advantages such as back-drivability and high force production at low speeds, but pneumatic muscles are not energetically economical. Alternatives such as piezoelectric actuators or shape memory alloys also have energetic disadvantages. Emerging technologies, such as those based on electroactive polymers, may offer high power density with reasonable economy. At the time of this writing, such actuators are becoming commercially available, and robotic applications will soon test their suitability for walking.

New design features may make conventional actuators more suitable for walking. Of particular note is a lockable joint that allows load to be supported with minimal energetic cost. Several dynamic walking machines employ a mechanical stop to prevent knee hyperextension, and some machines also use a powered latch to lock the knee in full extension. These mechanisms allow some control to be exerted about the knee, but without the energetic cost of producing force to support a load, and without the ancillary disadvantages of higher inertia and friction in a fully-actuated joint. Similar mechanisms might find application in the added degrees of freedom of more advanced walking machines. For example, a lockable ankle might enable the foot to exhibit different behaviors depending on the task, such as standing, walking, or climbing stairs.

Another approach is to make conventional actuators more suitable for walking. The main drawbacks of electric motors are inertia and friction about the joint, which adversely affect back-drivability. However, motors are also quite economical when performing positive work. A clutch would make it possible to engage a motor only when power is to be delivered, and to disengage it when a limb is to swing freely. This would allow for inclusion of even a heavily-geared transmission, where high torques could be produced, with the associated inertia and friction decoupled mechanically from the limb as necessary. Here, the lack of back-drivability could even be used to advantage, to effectively lock a joint and support a load with little energetic cost, yet still allow for pendulum-like limb swinging.

Additional advantages can be gained through the strategic use of springs. Human tendon is highly elastic, storing and returning energy while acting in series with muscle fibers. In robot manipulation, series compliance poses a serious problem for positioning accuracy and settling time, and it is generally to be avoided. However, walking—and in particular dynamic walking—need not be performed with high positional accuracy. Used judiciously, series compliance can reduce the power requirements for active actuation. The most widely demonstrated application is running, where tendons may be responsible for half or more of the overall work of muscle-tendon actuators [20]. However, a similar case applies to leg swinging, where tendons could help speed leg motion while requiring little active work production [8]. Human push-off also benefits from elastic energy storage in the Achilles tendon. Similar benefits would be expected in dynamic walking machines.

Springs could also be applied to advantage in a parallel arrangement. Although animals store elastic energy mostly in series with muscle, machines need not be restricted in a similar manner. The potential advantage of putting springs in parallel with actuators is that no active force would be needed to engage the springs. The Delft biped employs relatively flat feet with ankle springs, which bias the stance leg to remain upright and also help to reduce collision losses. Of course, the disadvantage is that parallel springs always act about a joint, whether or not that action is desired. Springs acting between the torso and the legs would help to balance the torso, but would hinder the robot's ability to sit down. For a machine designed for a specific set of tasks, springs could likely provide overall benefit.

Although these design features can improve energetics, they tend to be ill-suited for theoretical control design. Mechanical stops, clutches, and latches introduce nonlinearities that are largely incompatible with feedback control techniques, which favor systems with continuous dynamics. Linear springs that store and return energy do have continuous dynamics, but they also introduce additional states to be tracked and controlled. The unilateral force production of muscle-like actuators is also nonlinear, which is mechanically an advantage for back-drivability but not for mathematical analysis. The reason why these features are mechanically advantageous may be attributed largely to the fact that walking itself is a nonlinear task, with periodic discontinuities. Wheeled vehicles, having continuous and linear dynamics, benefit less from discontinuous features. The difficulties that discontinuities pose for control design may also be due to lack of attention, with greater research effort historically devoted to continuous systems. With new theoretical frameworks, discontinuous design features may yet prove mathematically tractable.

These same features are also challenging to employ in practice. Few robots currently incorporate clutches or locking mechanisms, and it remains to be determined whether practical devices could be made lightweight and economical enough to yield overall economy benefits. One notable example is the use of a latching mechanism to allow the slow wind-up of a push-off spring in the Cornell Efficient Biped [2]. Mechanical design has historically been devoted to improving

It is interesting to note that humans convert chemical energy to mechanical work using a respiratory cycle that differs significantly in operation from internal combustion engines but is remarkably similar in efficiency.

the efficiency of power delivery in wheeled devices. The intermittent behavior of clutches and locks, in the context of walking robots, may require new mechanisms that have yet to be designed and refined. But just as humans have evolved actuation methods and mechanisms quite distinct from that of a GM EV-1 and yet still reasonably economical, a walking robot might also benefit from similar refinement of mechanical actuation devices that may differ substantially from conventional technology.

Control Challenges

We next consider the means by which a robot based on both dynamic walking and ZMP principles may be controlled. Machines based on dynamic walking are distinguished by the nearly complete lack of feedback control. With mechanical design that by itself produces stable walking, such robots require control only for those features not directly related to walking. For example, pneumatic bipeds require control of artificial muscles, and some kneed bipeds require control of knee locking. Active push-off in the Cornell Efficient Biped requires control of a motor to wind up a spring during the stance phase, followed by a triggered release of this energy at the end of stance. All of these components may be controlled locally, with their operation with the gait cycle often triggered discretely with very simple logic. As a result, local control is largely decoupled from the dynamic stability of the gait. Avoidance or minimal use of control remains an attractive strategy for a robot's walking. However, a machine with multiple actuated joints may also be capable of efficient gaits that are only partially driven passively. The presumption of dynamic walking might therefore limit exploration of other economical strategies. Another disadvantage of dynamic walking is that it has thus far been designed ad hoc, with little in the way of a standard procedure or theory. There lacks a set of engineering guidelines or strategies that allow construction of a dynamic walking gait.

A possible solution is to utilize ZMP control principles for all tasks that are not addressable through dynamic walking. ZMP is, after all, a principle that concentrates on control of the center of pressure of foot-ground contact and is therefore not expressly designed for, nor limited only to, walking. A prospective robot might therefore walk economically with motion governed primarily by passive dynamics and then switch to ZMP control when stopping, turning, or executing less dynamic maneuvers. From a theoretical standpoint, however, ZMP control is no more analytical or constructive than dynamic walking. Design therefore proceeds in an ad-hoc basis, largely dependent on the individual skill of the designer. Nevertheless, ZMP control has an undeniably positive track record, serving as the basis for the most versatile robots to date.

More theoretical approaches may be applicable to economical and versatile walking. Nonlinear control theory provides analytical tools for addressing the hybrid dynamical features of walking, where intervals of continuous dynamics are punctuated by discrete events. Promising approaches include virtual model control, hybrid zero dynamics control, controlled symmetries/potential shaping, and geometric reduction. Virtual model control refers to the use of a simplified representation such as an inverted pendulum, along with virtual springs, to drive a more complicated mechanism [21]. Hybrid zero dynamics refers to the application of virtual kinematic constraints, enforced by feedback laws, to reduce the dimensionality of walking [22]. The dimension reduction makes it simpler to test stability and robustness. Hybrid zero dynamics methods allow for and, in fact, take advantage of the underactuated nature of the stance leg. Controlled symmetries refer to a means to design methods for powering and stabilizing a dynamic walking machine [23]. It transforms a passive gait designed to descend a slope into the appropriate joint torques for level ground. Potential shaping generalizes the controlled symmetry approach so that the reference may have features beyond those of a slope-descending gait. Geometric reduction takes advantage of symmetries in the dynamics to reduce dimensions [24]. It can be used in combination with the other techniques to produce a high-dimensional gait that is extended from a simpler reference gait. All of these approaches are theoretically appealing, because they provide a more constructive basis for producing gaits than the ad-hoc dynamic walking approach. They also simplify the verification of stability, typically by referring a high-dimensional gait to a simpler, low-dimensional proxy or analogue.

These approaches also presently suffer from a few disadvantages. For example, the hybrid zero dynamics approach is dependent on enforcement of the virtual kinematic constraint, implying the application of high gain control. High gains are conducive to robust behavior, but they almost certainly come with a high energetic cost for control. However, the approach also allows for the zero dynamics to coincide closely with a passive dynamic motion [25], so that control can in principle be quite minimal during steady-state operation. The remaining approaches typically assume the ability to exert arbitrary torques on each degree of freedom. This assumption is incompatible with many walking robots, where the torque exerted between ground and the stance leg is quite limited or even unactuated. Because all of these control approaches are relatively early in their development, it remains to be seen the degree to which their respective constraints can be relaxed.

Nevertheless, all of these approaches show great promise for understanding and controlling robot locomotion.

Several control design goals emerge for a future robot that is both economical and versatile. For economy, it is desirable that much of the gait be produced passively, thereby minimizing control costs. (This is predicated on the assumption that the robot's limbs move passively with low friction.) It is furthermore desirable that deviations from the reference motion be resisted with low or minimal feedback gains where practical, again to minimize control costs. Dynamic walking has already demonstrated that stability—albeit with poor robustness—can be achieved with low gains. However, for very large disturbances that exceed the basin of attraction of passive dynamics, it may be prudent to switch to a higher gain control strategy to recover stability. Finally, for more static tasks or those where passive dynamics cannot produce the required motion, ZMP principles may be most applicable.

Conclusions

Robot walking, while appealing for its resemblance to human motion, is not an obvious choice when both economy and versatility are desired. Wheeled vehicles are surprisingly capable on different terrains and are nearly unbeatable in terms of economy. In specialized situations, legged locomotion may become preferable. But legged locomotion entails inertial and other energetic costs that do not appear in wheeled machines. The force and work requirements of legged locomotion also only appear energetically economical when considering the unique features of the human body and human muscle. The attainment of high economy in a legged robot requires either actuators similar to humans' or discontinuous nonlinear mechanisms that can reduce energetic losses to support a load. The attainment of high versatility indicates that the ZMP is likely to remain applicable, unless serious advances are made in other control theoretical approaches.

KEYWORDS

Locomotion, gait, passive dynamic walking, biomechanics, energy, efficiency.

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