

## THE ACTION OF TWO-JOINT MUSCLES: THE LEGACY OF W. P. LOMBARD

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### **Introduction**

In a five-page missive, W. P. Lombard (1903a) helped lay the foundation for a new and rational approach to understanding how muscles are coordinated. Although investigators dating to the time of Borelli (1685) had described the relationship between muscle, joint, and limb in a logical, intuitive, and physically meaningful manner, Lombard made an astonishing, counter-intuitive claim. He asserted that muscles with apparently opposing actions, termed *pseudo-antagonists*, can be used together in a productive way (see also Lombard, 1903b and 1907). This concept came from a remarkable intuition, one that has challenged us to state more quantitatively and objectively for nearly 100 years. Though his ideas were expressed qualitatively, the definitive nature of his statements implied that they could be translated into precise mathematical descriptions. The enduring influence of his work stems perhaps from its tantalizing character, a mix of biological description and bold prediction. We might classify Lombard's theory of the pseudo-antagonist as a conjecture—a theorem waiting for definitive proof. In our attempts to find the proof, we have adopted the principles of mechanics, carving out a new field of biomechanics. In our review of Lombard's influence, it will be seen that much of our as yet incomplete understanding of muscular function has been fed and driven from this remarkable source.

As with any developing field, advances in biomechanics have come on a variety of fronts. Motor tasks have been differentiated in terms of upper and lower extremity, unconstrained and constrained, isometric and otherwise. In taking stock of Lombard's impact on biomechanics, it is expedient to use a single mathematical notation to assess and compare the many contributions made by different researchers studying different motor tasks. Mathematics is a precise and unambiguous language, but it is also somewhat inaccessible. Fortunately, there is a geometric interpretation that

will be both accessible and mathematical. We will use Lombard's main thesis as a vehicle for introducing the mathematics and geometry, after which contributions by others will motivate appropriate extensions but all within a single framework.

### **Historical Perspective**

The existence of two-joint muscles was remarked upon long before the time of Lombard. For example, Ingen Schenau (1990) cites Galen's (131-201 AD) *De usu partium* (translated by May, 1968) as an early description of the effect of the rectus femoris in flexing the hip and extending the knee. Borelli (1685, cited by Fick, 1879) described the dependence of knee joint torque on the hip angle, and his mechanistic arguments are thought by some to pioneer the field of biomechanics. Prior to (Hunter, 1797) and more contemporary to Lombard, several authors (Cleland, 1867; Fick, 1879; Langer, 1879) remarked on how two-joint muscles could link movement of joints, and how this would enable transfer of energy from other muscles closer to the trunk to the periphery. Of particular interest was the fact that by extending one joint and flexing another, a two-joint muscle is able to maintain a constant length and thereby stay within a length range for which significant force can be produced (Hüter, 1863 and 1869; Fick, 1879).

Up to the early 20<sup>th</sup> century, however, muscle coordination was mostly discussed in terms of combinations of muscles all located at different joints. There was scant consideration of how the joint coupling provided by a two-joint muscle could be harnessed by another muscle crossing the opposite side of one of the same joints. One early exception was Duchenne (1885), who described the ability of hamstrings to work with rectus femoris to extend the knee, which will see is identical to Lombard's concern. Although earlier this century he was cited more extensively in the literature than Lombard was, he is presently given little credit for his earlier observation. (This

unfortunate fact is not due to any inferiority in Duchenne's contribution; the primacy of English as a scientific language is a likely reason.) In his time, Duchenne was well known for his treatises on muscle function (1867 and 1885), in which he also described the ability of one- and two-joint muscles to stabilise a joint, which is a relevant concern to this day (e.g., Markee et al., 1955; Baratta et al., 1988). Another early contributor was Hering (1897), who predated Lombard in the use of the term pseudo-antagonist and made very similar observations to Lombard.

### Lombard's Conjecture

Lombard stated that opposing two-joint muscles can reinforce each other, and that "A muscle can cause the extension of a joint which it can flex." He gave the following necessary conditions for such behavior:

- It must have the better leverage at the end by which it acts as extensor.
- There must be a two-joint muscle that flexes the joint which the muscle in question extends, and extends the joint which it flexes.
- It must have sufficient leverage and strength to make use of the passive tendon action of the other muscle.

He further asserted:

When all the two-joint muscles are contracting at the same time...the energy is transmitted by the muscles, as by an endless chain, having the form of a figure 8... Thus each muscle helps all the rest to produce the extension of hip, knee and ankle, and all the two-joint muscles act as a unit...

Lombard gives no specific definition for condition (c) regarding *passive tendon action*. It appears to refer to a muscle that is activated isometrically, in a manner similar to Cleland's (1867) "ligamentous action" and to the work of others (Hueter, 1869; Fick, 1879; Duchenne, 1885; Hering, 1897; Strasser, 1917; Baeyer, 1921 and 1922). We will see that an isometric interpretation of passive tendon action is merely a simplifying assumption and not critical to the understanding of muscle coordination.

We will first apply the concept of passive tendon action to studying the ability of a muscle to

Two-joint, six-muscle system

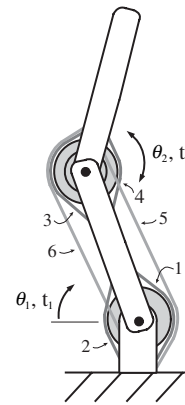


Figure 1. Example system illustrates muscle actions. (a) Two-joint, six-muscle system illustrates many principles of muscle function. Four of the muscles cross a single joint, each with a moment arm that is independent of configuration (moment arms in diagram are for schematic purposes and are not to drawn to scale). Muscles 5 and 6 are two-joint muscles and illustrate Lombard's (1903) concept of pseudo-antagonists. Muscle 5 has a larger extension moment arm about joint 1 than its flexion moment arm about joint 2, and muscle 6 has a larger flexion moment arm about joint 2 than its extension moment arm about joint 1. Joint torques  $t_1$  and  $t_2$  and joint angles  $\theta_1$  and  $\theta_2$  are defined to be positive in the direction corresponding to joint extension.

couple joints kinematically, using a simple two-joint system as an example (see Figure 1). Kinematic behavior serves as an ideal introduction to musculo-skeletal function, but for many motor tasks, intersegmental dynamics will also be relevant. We will therefore consider the role of dynamics of both unconstrained and constrained two-joint systems in the coordination of muscles. As alluded to by Lombard, there are also energetic concerns in coordination, leading us to study the production and absorption of power, and its relation to efficiency. Finally, we will examine the actions of muscles of all types and arrive at a more general interpretation, based on the summation of vector contributions, of muscle function in any type of motor task.

### Kinematic Coupling of Joints

The first concern in studying a multi-joint system is the set of kinematic constraints that connect the links. Joints and muscles both constrain movement. A ball and socket joint, for example, constrains the relative positions of the ends of two limbs. A muscle viewed kinematically, that is to say with an output in

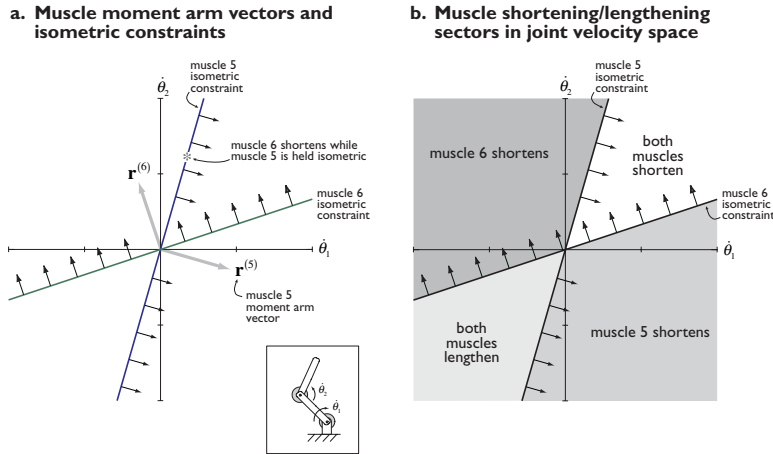


Figure 2. Isometric constraints for two-joint muscles. (a) Each muscle's moment arm vector defines a constraint surface (a line in this example) in joint velocity space. The constraint divides joint velocity space into motions for which the muscle shortens (denoted by arrows) lengthens. Motions that lie on the constraint are achieved with the corresponding muscle held isometric. Only constraints for muscles 5 and 6 are shown. (b) Isometric constraints in combination divide joint velocity space into sectors. Depending on the direction of motion, any combination of lengthening or shortening of the two muscles is possible. The constraints show that a muscle with an extension moment arm about joint 1, such as muscle 5, can still shorten while joint 1 flexes.

terms of length rather than force, also constrains movement. This is the case regardless of the rate of shortening or lengthening, although it is conceptually easier to understand when the length is fixed. Physiologically, there is some significance to isometric action, because a muscle can stay near optimum length and thereby generate large forces. More generally, the force-velocity relationship limits muscle's ability to generate force when the rate of shortening is high. This implies that there is some advantage to be gained from the ability of a two-joint muscle to act nearly isometrically during a motion in which two joints are simultaneously extended or flexed (Cleland, 1867; Duchenne, 1867; Fick, 1879; Fischer, 1902a, b; Fenn, 1932; Fenn, 1938).

In interpreting kinematic coupling, we must first establish a few definitions. First, instead of leverage, we will prefer to discuss a muscle's moment arm about each joint. Supposing that there are  $m$  muscles and  $n$  joints, we define the moment arm  $r_{ij}$  as the moment arm of muscle  $j$  (numbered from 1 to  $m$ ) about joint  $i$  (numbered from 1 to  $n$ ). It will also be convenient to place the list of all  $n$  moment arms for a muscle  $j$  into a vector, denoted

$$\mathbf{r}^{(j)} \equiv [r_{1j} \quad r_{2j} \quad \cdots \quad r_{nj}]^T.$$

In each muscle's moment arm vector, the moment arm will be zero for each joint the muscle does not cross. There will be only one non-zero moment arm for one-joint muscles, and two for two-joint muscles (see Figure 1).

The moment arms determine the shortening speed of a muscle-tendon unit as a function of the joint angular velocities. (For convenience, we will use "muscle" informally to refer to the muscle-

tendon unit.) Defining the shortening speed  $v_j$  to be positive when the muscle is contracting,

$$v_j \equiv \{\mathbf{r}^{(j)}\}^T \dot{\boldsymbol{\theta}}$$

where

$$\dot{\boldsymbol{\theta}} \equiv [\dot{\theta}_1 \quad \dot{\theta}_2 \quad \cdots \quad \dot{\theta}_n]^T$$

is the vector of joint angular velocities. Of particular interest, primarily for simplicity, is the case when the muscle is held isometric,

$$v_j = 0, \tag{1}$$

which specifies the relationship between joint angular velocities necessary for the muscle to be kept at constant length. This isometric case is what was studied by Cleland (1867) and his contemporaries and later used as a requirement for Lombard's conjecture.

Graphical methods are particularly well suited to describing these kinematic constraints. Enklaar (1954) used graphical methods to show that, in the space of joint positions or velocities, equation (1) can be expressed as a straight line constraint (for a two-joint system). More generally,  $\mathbf{r}^{(j)}$  may be regarded as a vector normal to an *isometric constraint* surface passing through the origin in the coordinate system of joint angular velocities. When the joints move such that the joint velocities lie on the constraint surface, muscle  $j$  is held at constant length (see Figure 2a). It is obvious that the constraint surface for a one-joint muscle requires that one joint be held stationary. For a two-joint muscle, however, the corresponding constraint requires that the two joints move in a

fixed proportion. As noted by Landsmeer (1961), constant, non-zero muscle speeds appear as additional constraints parallel to the isometric constraint surface.

Enklaar (1954) examined the case of two muscles, both crossing two joints on the same side but with different moment arms (see Figure 2*b*). Not only are there motions in which both muscles must shorten or lengthen, but there are also motions in which one muscle must lengthen while the other shortens, even though these muscles might naively be labeled agonists. We can see that this phenomenon takes place whenever two muscles have differing moment arms, and that the more the moment arms differ, the larger the set of such lengthening/shortening motions.

This same example also applies directly to Lombard's Conjecture. Here we will use muscle 6 as the primary muscle, which produces flexion torque about joint 1 (i.e.,  $r_{16} < 0$ ) and extension torque about joint 2 (i.e.,  $r_{26} > 0$ ), satisfying condition (a). The pseudo-antagonist, muscle 5, satisfies condition (b) and when held isometric, constrains joint 1 to extend when muscle 6 shortens (see Figure 2*a*). Stated mathematically, the isometric constraint from equation (1) is

$$v_5 = \{\mathbf{r}^{(5)}\}^T \dot{\boldsymbol{\theta}} = r_{15} \dot{\theta}_1 + r_{25} \dot{\theta}_2 = 0 \quad (2)$$

and if muscle 6 is shortening,

$$v_6 = \{\mathbf{r}^{(6)}\}^T \dot{\boldsymbol{\theta}} = r_{16} \dot{\theta}_1 + r_{26} \dot{\theta}_2 > 0. \quad (3)$$

Together, these equations imply that joint 1 will extend, that is,  $\dot{\theta}_1 > 0$ , if

$$r_{16} - r_{26} \frac{r_{15}}{r_{25}} > 0. \quad (4)$$

Condition (b) guarantees that  $r_{15}/r_{25} < 0$ , and the requirement that muscle 6 be an extensor of joint 2, implies that  $-r_{26}r_{15}/r_{25}$  is positive, and must outweigh  $r_{16}$ , which is negative. We can therefore see that Lombard was very nearly correct in his conjecture. With condition (a) he stated the equivalent of  $r_{26} > |r_{16}|$ , which is helpful but not actually necessary for inequality (4) to be true.

We therefore conclude that a more precise version of Lombard's claim is that the two-joint muscle can extend a joint for which it has a flexor

moment arm, if inequality (4) is true. Geometrically, this is equivalent to stating that the other two-joint muscle, the pseudo-antagonist, must have an isometric constraint line that both passes through the quadrant in joint velocity space for which both joints extend (the first quadrant in Figure 2*a*), and that it does so on the shortening side of the primary muscle's isometric constraint.

Stated in this way, Lombard's claim is neither mysterious nor paradoxical. What might make it seem paradoxical is the potential misconception that the direction of torque produced by a muscle is equivalent to the direction of the resulting motion. Any two-joint muscle with such a flexor moment arm is capable of a range of motions that include extension of that same joint (see Figure 2*b*). This property is moreover not exclusive to pairs of two-joint pseudo-antagonists. A two-joint muscle, held isometric, can act as a pseudo-antagonist to a single-joint muscle so that the latter can extend a joint that it does not even cross. Again, the joint velocities are constrained to the pseudo-antagonist's isometric surface, and joint extension lies on the shortening side of the primary (single-joint) muscle's isometric constraint.

### **Joint Torques due to Muscle**

Although Lombard did not explicitly consider joint torques in his paper, his secondary claim regarding transfer of energy hinted at the importance of kinetic variables. In fact, many later developments regarding two-joint muscles have discussed joint torques. We will first review the relationship between torques and the isometric constraint lines discussed above, and then apply this relationship to the coordination of multiple muscles in order to gain additional insight about the action of single- and two-joint muscles.

Landsmeer (1961) was the first to observe that the same moment arm relationship that determines the joint motions for which a muscle is isometric, applies to joint torques as well. Plotted in joint torque space, the individual torque vectors for each muscle are perpendicular to the isometric constraint lines (see Fig. 2*a*), and the joint torque vector associated with the activation of muscle  $j$  is

$$\mathbf{t}^{(j)} = \mathbf{r}^{(j)} \cdot f_j^{\max} \cdot f_j \quad (5)$$

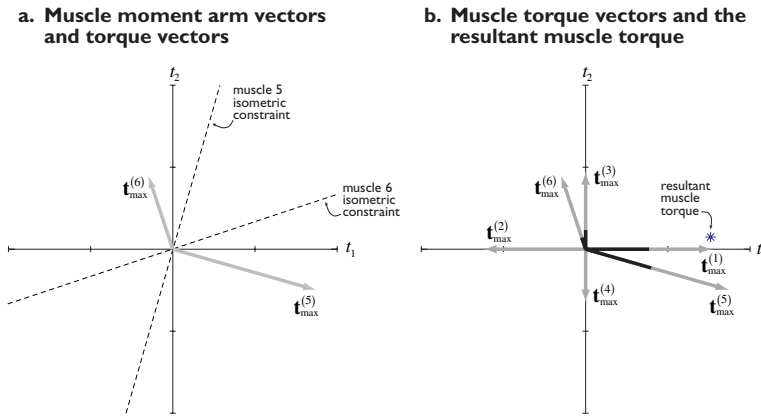


Figure 3. Muscle joint torque vectors. (a) Each maximal torque vector  $t_{max}^{(j)}$  is proportional to that muscle's moment arm vector and perpendicular to the isometric constraint line. (b) Maximal torque vectors for all six muscles, of which a weighted sum is formed to produce a net muscle torque  $t^M$ . Shown in black are normalized activation levels  $f_j$  for each muscle such that a net joint torque (denoted by asterisk) of almost pure extension about joint 1 is formed. There is no unique solution, but this example distributes forces between muscles so as to minimize the sum of squared activation levels. Both single- and two-joint muscles will typically be activated to minimize this objective.

where  $f_j^{max}$  is the maximum isometric (tensile) muscle force, and  $f_j$  is the normalized force or activation level allowed to vary between 0 and 1 (Kuo, 1994). The net torque due to muscle,  $t^M$ , is the sum of the individual muscle torques, which can alternatively be interpreted as a weighted sum of maximal torque vectors  $t_{max}^{(j)}$ ,

$$\begin{aligned}
 t^M &= \sum_{j=1}^m r^{(j)} \cdot f_j^{max} \cdot f_j = \sum_{j=1}^m (r^{(j)} \cdot f_j^{max}) \cdot f_j \\
 &= \sum_{j=1}^m t_{max}^{(j)} \cdot f_j
 \end{aligned}
 \tag{6}$$

where

$$t_{max}^{(j)} \equiv r^{(j)} \cdot f_j^{max}.$$

Landsmeer (1961) noted that the net torque due to muscle is simply the vector sum of individual muscle torque vectors. Static equilibrium can therefore be achieved if the torques due to gravity and other external forces balance those produced by the muscles.

A geometric interpretation of this vector summation demonstrates that each muscle's torque vector has a unique direction in torque space, determined by the relative moment arms (An et al., 1981; Kuo, 1994). The maximal torque vectors  $t_{max}^{(j)}$  are equal to the moment arm vectors scaled by the maximal muscle force, and so must also be perpendicular to the isometric constraint lines (see Figure 3a). The vector lengths depend on both the absolute moment arms and the maximal force that can be generated by each muscle. The CNS must weight each of the possible torque vectors in such a way as to achieve a desired task (see Figure 3b).

Elftman (1939a, 1939b, 1966) also understood the summation properties of joint torques, although without using a geometric interpretation. He calculated the joint torques associated with human locomotion, and noted that a single muscle crossing the ankle, knee, and hip with appropriate moment arms could potentially replace the actions of the existing muscles and with greater efficiency. The torques associated with such a muscle would require the minimum summed absolute torque as opposed to any group of less specialized muscles with the same resultant torque. This point was also illustrated by Herzog and Binding (1994).

We can use the vector summation approach to gain a greater understanding of Elftman's observation. Any two collinear vectors that are not collinear will have components both normal and tangent to their resultant. The normal components cancel out, and because there is a metabolic cost to producing force or torque whether or not work is performed, this cancellation is costly. Any net muscle torque produced by a group of muscles could therefore be replaced by a single more efficient but more specialized muscle, with moment arms chosen so as to produce the same torque without cancellation. Elftman recognized that even single-joint muscles that cross different joints will act together such that there is a normal component to the resultant, an inefficiency that can be eliminated by substituting an appropriately located muscle in their stead.

It is therefore advantageous, in terms of the efficiency of vector summations, to have a large variety of muscles pointing in many directions in joint torque space (Kuo, 1994). These advantages are of course weighed against possible difficulties in routing a physical muscle to achieve some

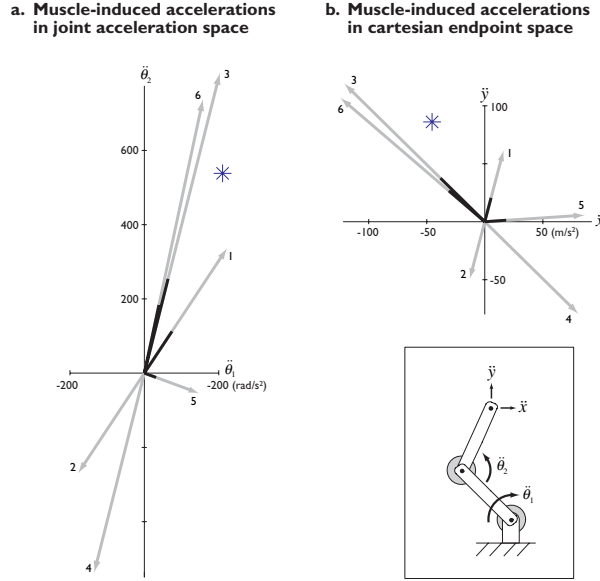


Figure 4. Muscle-induced acceleration vectors. (a) Each maximal joint angular acceleration vector  $\ddot{\theta}_{\max}^{(j)}$  points in a direction in acceleration space which generally differs in direction from  $\mathbf{t}_{\max}^{(j)}$ . As in torque space, a weighted sum of muscle-induced acceleration vectors (plus Coriolis, centripetal, and gravity terms, if any) produces the net joint angular acceleration  $\ddot{\theta}$ . Shown in black are normalized activation levels necessary to produce the net acceleration denoted by an asterisk. This acceleration vector is the result of the net torque produced in Figure 3. (b) An alternative output space can be defined in terms of accelerations of the endpoint in the Cartesian plane. Maximal acceleration vectors are shown, along with normalized activation levels necessary to form the net acceleration, which is upward and to the left (denoted by asterisk). Inset diagram shows configuration of links. See Appendix for details.

torque combinations, and the probable fact that efficiency is a concern in only a subset of all motor tasks. Nevertheless, the significance of different muscle types is partially to provide a repertoire of directions in torque space that are both achievable and efficient when used with other muscles. This basis of this observation, properly attributed to Elftman (1939a) has never been properly recognized, perhaps because it was overshadowed by the many other contributions from that same paper, not least of which was his original use of dynamics to study human movement.

### Dynamic Coupling at Joints

Most studies of two-joint muscles have been concerned with the flexion or extension of joints without consideration of the role of dynamics. A

flexion or extension *torque* is therefore implicitly assumed to equate to flexion or extension *motion*. However, intersegmental forces can couple joint motion such that a pure flexion torque about a single joint will generally be expected not only to cause that joint to accelerate in flexion, but also to cause other joints to move as well. We can distinguish between joint torques and joint accelerations by regarding them as separate sets of outputs and examining the action of muscles in each case. This examination is made simpler by the fact that the dynamical equations of motion map torque space to acceleration space.

The control implications of dynamical coupling were not appreciated until relatively recently. Dynamical equations of motion were first employed by Elftman (1939a) and later applied by many others to perform inverse dynamics and other calculations. Interest in motor control led Hollerbach and Flash (1982) to examine the problem of compensating for intersegmental forces. Zajac and Gordon (1990) offered an accessible tutorial on dynamical coupling based on a simple two-segment system, showing how muscles can accelerate joints they do not cross. This concept can be illustrated by examination of the dynamical equations of motion,

$$\ddot{\theta} = \mathbf{M}(\theta)^{-1} \cdot (\mathbf{t}^M + \mathbf{v}(\theta, \dot{\theta}) + \mathbf{g}(\theta)) \quad (7)$$

where  $\mathbf{M}(\theta)$  is the mass matrix,  $\mathbf{g}(\theta)$  is a vector of gravitational terms, and  $\mathbf{v}(\theta, \dot{\theta})$  is a vector of Coriolis and centripetal terms (see Appendix for details). Neglecting the latter two terms for now, combining equations (5)-(7) with the following definition for *maximal acceleration vectors* for each muscle  $j$ ,

$$\ddot{\theta}_{\max}^{(j)} \equiv \mathbf{M}(\theta)^{-1} \cdot \mathbf{r}^{(j)} \cdot f_j^{\max}$$

yields

$$\begin{aligned} \ddot{\theta} &= \mathbf{M}(\theta)^{-1} \cdot \mathbf{t}^M + \dots \\ &= \mathbf{M}(\theta)^{-1} \cdot \sum_{j=1}^m \mathbf{t}_{\max}^{(j)} \cdot f_j + \dots \\ &= \sum_{j=1}^m (\mathbf{M}(\theta)^{-1} \cdot \mathbf{r}^{(j)} \cdot f_j^{\max}) \cdot f_j + \dots \\ &= \sum_{j=1}^m \ddot{\theta}_{\max}^{(j)} \cdot f_j + \dots \end{aligned} \quad (8)$$

This equation demonstrates that the mass matrix transforms  $\mathbf{t}_{\max}^{(j)}$  from joint torque space to  $\ddot{\boldsymbol{\theta}}_{\max}^{(j)}$  in joint acceleration space (Hogan, 1985; Kuo, 1994), and that each muscle induces an acceleration that contributes to the net acceleration vector.

In the output space of joint accelerations, the inverse of the mass matrix  $\mathbf{M}(\boldsymbol{\theta})$  transforms the effect of each muscle. Figure 4a shows the muscle-induced joint angular acceleration vectors for our two-segment example, demonstrating that none of the muscles accelerate the joints in the same direction that they do in joint torque space, nor in the same relative amounts. For example, muscle 1, which produces a pure extension torque about joint 1, accelerates both joints in nearly equal proportions. Muscle 6 produces a flexion torque about joint 1 and an extension torque about joint 2, but actually accelerates both joints into extension. Muscle 3 produces significantly less torque than muscle 1, yet it accelerates the joints by nearly twice as much for the configuration shown. The torques produced by a muscle can therefore not be assumed to indicate the direction or amount of accelerations that actually occur. It is also interesting to note even the extension of a single joint requires coordination of many muscles, including some that do not even cross that joint (Fujiwara & Basmajian, 1975).

Despite the effect of these transformations, however, the fundamental role of the CNS is still to form (at each instance in time) an appropriate weighted sum of vectors. In acceleration space, it is typical that the mass matrix  $\mathbf{M}(\boldsymbol{\theta})$  will exhibit significant dependence on the configuration of the limbs, more so than the moment arms. Added to the muscle-induced accelerations are those due to gravity, Coriolis, and centripetal terms.

The choice of output space is somewhat arbitrary. Joint torques, joint angles, and other coordinates are quantities that are derived, measured, and analyzed for the convenience of the observer, and are not necessarily relevant to the central nervous system (CNS). An alternative to joint angular acceleration output space for our two-joint system is the accelerations of the endpoint or body center-of-mass in the plane (see Figure 4b), which is of interest in cases such as reaching (Sergio and Ostry, 1994; Gielen et al., 1990), leg extension or sit-to-stand tasks (Soest et al., 1993; Ingen Schenau et al., 1995; Jacobs and

Ingen Schenau, 1992), and posture (Kuo and Zajac, 1993). The vector summation properties apply here as well, and as in most output spaces, it is difficult (and not necessarily helpful) to distinguish between single- and two-joint muscles. Again, there is an advantage to be gained from the availability of muscles distributed in a variety of directions in the output space of interest.

### **Contact and Force Tasks**

When a motor task involves contact with the environment, the consequences of muscle activation are different than for unconstrained tasks. This is because external constraints limit the kinematically admissible motions. Several investigators (Fischer, 1927; Donskoi, 1961; Molbech, 1966; Carlsöö & Molbech, 1966; Ingen Schenau, 1989a-c; Ingen Schenau et al., 1990; Gielen et al., 1990; Jacobs, and Ingen Schenau, 1992; Doorenbosch et al., 1994 and 1995; Prilutsky and Gregor, 1997) reported various kinematic constraint analyses demonstrating cases in which a two-joint muscle can exhibit behaviors similar to those reported by Lombard, and some (Ingen Schenau, 1990) thought that these behaviors were due to the presence of constraints. Closer analysis, however, reveals that these behaviors are fundamentally no different than those for unconstrained tasks, and can be understood using the same mathematics as considered previously. We will begin by adopting the kinematic analysis of Enklaar (1954) before considering the role of dynamics and reaction forces in contact tasks.

When the limbs form a closed kinematic chain with the environment, the joints are kinematically constrained. If this constraint is expressed as

$$\boldsymbol{\gamma}(\boldsymbol{\theta}) = 0 \quad (9)$$

the corresponding constraint on joint velocities is

$$\frac{d\boldsymbol{\gamma}(\boldsymbol{\theta})}{dt} = \frac{\partial \boldsymbol{\gamma}}{\partial \boldsymbol{\theta}} \cdot \dot{\boldsymbol{\theta}} = \mathbf{J} \cdot \dot{\boldsymbol{\theta}} = 0 \quad (10)$$

where  $\mathbf{J}$  is referred to as the Jacobian matrix (see Appendix for details). This constraint is similar in form to that of isometric constraints such as equations (2) and (3), and can therefore be interpreted as a surface in joint velocity space.

As an example, let us consider the previous two-joint system except with the endpoint constrained to move in a slot (see Figure 5). For the particular configuration shown, the corresponding constraint

**External kinematic constraint**

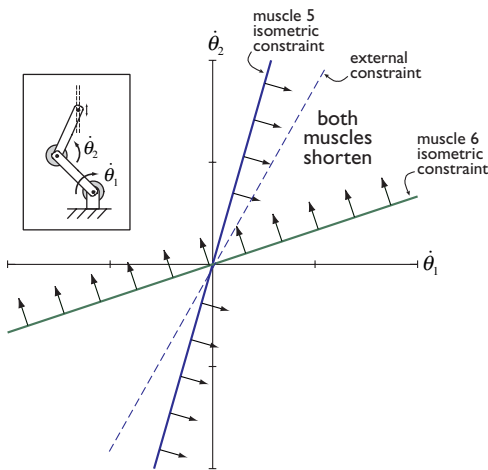


Figure 5. External kinematic constraints limit possible joint motions. A slot constrains motion of the endpoint as shown, limiting joint motions to the dashed line. Upward motion of the endpoint corresponds to shortening of both muscles 5 and 6. Muscle 6, which produces a flexion torque about joint 1, actually extends that joint while shortening with the constraint is in place. Inset diagram shows configuration of links and slot constraint.

in joint velocity space is nearly aligned with the isometric constraint of equation (2). (In fact, an appropriately designed slot could have an identical constraint surface in joint velocity space.) Just as in the isometric case, the slot constrains motion so that joint 1 must extend when muscle 6 is shortening, even though that muscle produces a flexion torque about that joint.

These kinematic observations must in any case be discarded when dynamic coupling is significant. In contact tasks, the external constraints produce reaction forces that are a function of the joint torques and velocities similar in nature to the equations of motion (7). These reaction forces could be thought of as producing virtual accelerations that add to the unconstrained accelerations to produce motion along the constraint. Mathematically, the constrained accelerations must satisfy the constraint

$$\mathbf{J} \cdot \ddot{\boldsymbol{\theta}} + \dot{\mathbf{J}} \cdot \dot{\boldsymbol{\theta}} = 0 \tag{11}$$

which is simply the time-derivative of equation (10). The fact that the reaction force equations are similar in form to the equations of motion allows for a remarkably simple interpretation of constrained motion.

We illustrate the dynamical effects of a constraint with our two-joint example with the endpoint moving in a slot (see Figure 6a). To each

muscle's unconstrained acceleration vector is added the virtual acceleration imposed by the constraint reaction force. The effect is such that muscle 1, which produces no torque about joint 2 but accelerates that joint into extension in the unconstrained case, will actually reverse the direction of that acceleration in the presence of the slot. In fact, with the exception of muscle 5, every muscle exhibits a change in direction of acceleration about one joint or another. As tempting as it may be to attribute these changes to the properties of two-joint muscles, they are merely changes in degree rather than type. The fundamental significance is that constraints generally produce quantitative changes in the muscle-induced acceleration vectors.

Even though the constraint reduces the number of degrees of freedom of acceleration, there remain other degrees of freedom that are of interest. For contact tasks, most significant of these is typically the actual reaction force against the constraint, such as the pedal force (Gregor et al., 1985). Andrews (1985, 1987) advocated using

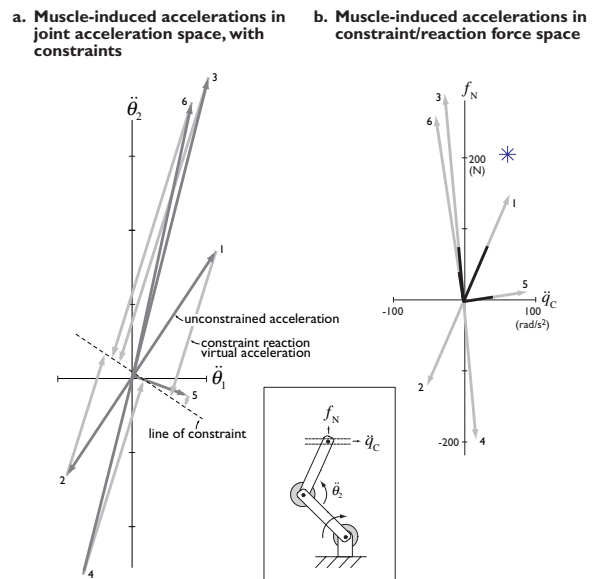


Figure 6. Muscle-induced acceleration vectors for constrained system, in two output spaces. In joint angular acceleration space (a), slot constraint adds virtual accelerations in reaction to the accelerations of the unconstrained system, so that each muscle induces an acceleration that must satisfy constraint (11). Constraint/reaction force space (b) defines the horizontal axis for accelerations along constraint (11), and the vertical axis for the normal force exerted against the slot. Also shown in both (a) and (b) are normalized activation levels necessary to produce a net output of 60 m/s<sup>2</sup> endpoint acceleration to the right and 200 N normal force, with an instantaneous endpoint velocity of 1 m/s to the right. Inset diagram shows configuration of links and slot constraint.



alternative outputs such as motion along a constraint. Because there is acceleration but no reaction force *along* the constraint (11), and reaction force but not acceleration normal to the constraint, we will combine acceleration along and reaction force normal to the constraint as outputs of interest. For our example system we denote these two quantities  $\ddot{q}_C$  and  $f_N$ , respectively. The corresponding output equations are

$$\begin{bmatrix} \ddot{q}_C \\ f_N \end{bmatrix} = \begin{bmatrix} K_1 & K_2 \\ S_1 & S_2 \end{bmatrix} \cdot \mathbf{t}^M + \begin{bmatrix} K_0 \\ S_0 \end{bmatrix} \quad (12)$$

where the quantities  $K_0, K_1, K_2, S_0, S_1,$  and  $S_2$  are defined in the Appendix.

Combining equations (6) and (12), it is apparent that whether viewing accelerations, reaction forces, or some combination thereof, that at each instance in time each muscle produces a vectorial contribution to these outputs. As shown in Figure 6b, the directions of each muscle-induced acceleration vector in this output space are again different from those of the torque vectors of Figure 3. Each muscle produces components of both acceleration and reaction force. For the configuration shown, muscle 1, with an extension moment arm about joint 1 only, produces a substantial reaction force in addition to its acceleration. It is evident that even a motor task calling for no reaction force will require that the muscles be coordinated carefully, so that the weighted sum of the output vectors produces no resultant reaction force.

It is therefore clear that even though contact tasks produce different motions of the limbs, they do so in a manner no different from isometric or other constraints. In fact, so-called unconstrained tasks are subject to the action of *joint* constraints that are fundamentally no different from the *external* constraints considered here. Again, we must be careful not to assume that the direction of torque produced by a muscle is equivalent to the direction of the resulting motion. It is advantageous to rely on the mathematics to determine how the joints will move, whether we are considering kinematics alone or dynamics and reaction forces.

### **Positive and Negative Muscle Power**

Although Lombard's Conjecture describes the isometric behavior of a two-joint muscle, it is clear

that Lombard was also concerned with the production and transfer of energy. Quantitative estimates of the net power produced at each joint were first made by Elftman (1939a), who was studying human gait. A number of methods have been proposed to quantify the transfer of energy between body segments (e.g., Aleshinsky, 1986; Ingen Schenau and Cavanagh, 1990; Bobbert et al., 1986a and b; Bobbert et al., 1987; Prilutsky and Zatsiorsky, 1994), although not without controversy (Wells, 1988; Ingen Schenau, 1998). The controversy arises because there is no objective means to attribute energy transferred from one body segment to another to a particular muscle, when there are multiple joints and muscles. Fortunately, for the purpose of understanding two-joint muscles, it is sufficient merely to determine whether a muscle is producing or absorbing power rather than where the power goes. From this information it can be demonstrated that a wide selection of muscles, with a variety of directions in torque space, allows many tasks to be performed efficiently.

One of the most-studied tasks in which power is a concern is cycling, which has typically been regarded as an example of Lombard's Conjecture. The quadriceps must lengthen during the propulsive phase and might be thought of as antagonists to the hamstrings. However, Gregor et al. (1985) showed that when pedal forces are considered along with crank motion, the muscles contribute to the overall output in a consistent manner, especially in the second half of the propulsive phase, during which knee flexor moments were reported. Andrews (1987) also found more agreement when considering crank rather than knee motion. Ingen Schenau (1990) demonstrated that for some motor tasks, two-joint muscles make it possible to perform tasks more efficiently, in metabolic terms, than would be possible using single-joint muscles alone. In the absence of two-joint muscles during a positive work task such as cycling, it would be necessary for some single-joint muscles to actively lengthen and therefore absorb power even as others produce power. With two-joint muscles, this inefficiency can be avoided.

These phenomena can be understood by applying our mathematical approach. The net power produced or absorbed at a joint is given by the equation

$$p_i^M = t_i^M \cdot \dot{\theta}_i \quad (13)$$

where  $p_i^M$  is the total muscle power produced or absorbed at joint  $i$  and  $t_i^M$  is the  $i$ 'th component of  $\mathbf{t}^M$ . An individual muscle produces power when it actively shortens, and absorbs power when it actively lengthens. The power produced by muscle  $j$  is

$$p^{(j)} = f_j \cdot v_j . \quad (14)$$

An equivalent relation is found using joint torque space, where the power is equal to the inner (dot) product of the muscle torque and angular velocity vectors (Kuo, 1994),

$$p^{(j)} = \{\mathbf{t}^{(j)}\}^T \dot{\boldsymbol{\theta}} . \quad (15)$$

The net power generated by all muscles is

$$p^M = \sum_{j=1}^m p^{(j)} = \sum_{j=1}^m \{\mathbf{t}^{(j)}\}^T \dot{\boldsymbol{\theta}} = \{\mathbf{t}^M\}^T \dot{\boldsymbol{\theta}} . \quad (16)$$

The inner product equation allows us to ascertain whether a muscle is producing or absorbing power by plotting the angular velocity and muscle torque vectors and measuring the angle between the two. If the included angle is less than  $90^\circ$ , the muscle is producing power; if greater, it is absorbing power.

These equations can be applied to the case of a positive work task performed first using single-joint muscles alone, and then with the addition of two-joint muscles. Even though the net torque vector has an acute angle with the angular velocity vector  $\dot{\boldsymbol{\theta}}$ , as is required of positive work tasks, this need not be true for each individual muscle's

torque vector (see Figure 7a). When two-joint muscles are included (see Figure 7b), they may make it possible to perform the same task, but with all muscles producing positive work. There is therefore no need for one muscle to dissipate the power produced by another.

Ingen Schenau et al. (1987, 1988) claimed that this efficiency advantage is a unique property of two-joint muscles during contact and force tasks, but in fact the advantage can apply to other situations. First, any task can be described in terms of joint angles and angular velocities, which comprise the state of the system, and individual muscle torques, which may be thought of as the control input. The same power relationships demonstrated by Ingen Schenau, which are mathematically described in terms of torque and angular velocity vectors here, can occur in non-contact tasks. In fact, the example of Figure 7 can apply to both contact and non-contact tasks, because the presence of a constraint has not been specified and is not necessary. Second, because the efficiency advantage simply depends on the availability of a muscle whose torque vector is more closely aligned to the angular velocity vector than the torque vector of another muscle, a single-joint muscle can also function in this capacity.

We may therefore augment our previous conclusion regarding the distribution of torque vectors. A selection of torque vectors provides the potential to produce certain net torques with increased efficiency. Poor efficiency is not only a case of muscles producing torques that cancel each other, it may also involve some muscles absorbing the power produced by others. A wider variety of

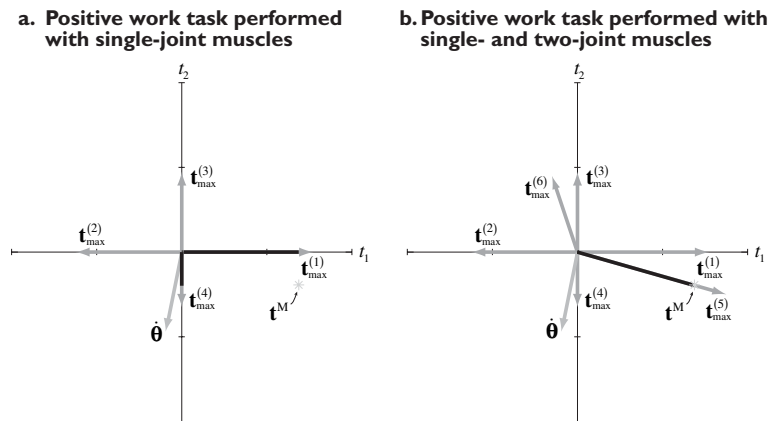


Figure 7. Example of a positive work task that is more efficient with a wider variety of muscles. (a) The task involves exerting an extensor torque about joint 1 and a flexor torque about joint 2, at a point in time when the joint angular velocities are such that both joints are flexing. This is a positive work task because the inner product of the net muscle torque vector  $\mathbf{t}^M$  and angular velocity  $\dot{\boldsymbol{\theta}}$  is positive. In a system with few muscles—in this case single-joint muscles only—it is necessary for muscle 4 to perform negative work, thereby absorbing power produced by muscle 1. (b) When the variety of muscles is increased—in this case with two-joint muscles—the same task can be performed with only one muscle. Here, muscle 5 performs positive work only. In other tasks, the roles of the single- and two-joint muscles can be reversed.

muscles increases the potential for the CNS to avoid both torque and power cancellation.

### ***Are Two-Joint Muscles Really Unique?***

We have reviewed a substantial body of literature discussing the role of two-joint muscles in a variety of tasks. The analyses we have considered are by and large correct. However, the majority of them have been used to argue that two-joint muscles have unique properties not shared by single-joint muscles, one which we feel is too narrow of a conclusion. It may seem contradictory, given the body of evidence, that we could arrive at such a conviction, but it is because we prefer the conclusion that uniqueness is not exclusive to two-joint muscles. All muscles, by nature of unique origins, insertions, and moment arms, have unique actions. The differences between muscles are a matter of degree rather than of type, and to emphasize a classification based on the number of joints crossed is to disregard more important issues. Moreover, we conclude that different types of motions, such as contact or reaching tasks, alter the quantitative effect of each muscle, but can all be explored using the same fundamental mathematics, for which all relevant distinctions are also a matter of degree rather than of type.

The mathematical approach reveals the fundamental principles of muscle coordination. Whether viewed in an output space of joint torques, joint angular accelerations, contact or reaction forces, or any combination of these outputs, each muscle contributes a vector component to a net output. These vectors point in different directions in output space, and there is an advantage, in terms of efficiency of force, torque, or power production, to having a variety of vector directions to choose from. Differences in vector directions are quantitative rather than qualitative in nature, and in most output spaces such as joint acceleration, there are no distinguishing features between single- and two-joint muscles.

Torque space is one output space in which single- and two-joint muscles might superficially appear to be distinct, because single-joint muscles have output vectors aligned to the coordinate axes, and two-joint muscles do not. The choice of joint torque space is however somewhat arbitrary and subjective. The concept of joint torque is an abstraction made for our convenience of analysis rather than a quantity that is measurable or known to the CNS. In an alternative and equally sensible

output space of segment torques, the distinction between single- and two-joint muscles is lost. Moreover, the classification of single joints is subjective because common “single joints” as the ankle or shoulder are actually composed of multiple joints. Mathematically, an accurate and complete dynamical representation of a biomechanical system must in any case be given in terms of degrees of freedom (DOFs) rather than number of joints. Even when considering the knee, there are varus/valgus moments that are relevant to joint and ligament loading. Lombard himself was careful, in his first paragraph, to qualify his claims by recognizing that he was neglecting DOFs such as abduction/adduction and inward/outward rotation. In nature, single-DOF or single-joint muscles are quite rare. In the human, the numerous muscles of the spine cross many DOFs, and in the lamprey there may not be any single-DOF muscles to be found; they may very well be a relatively recent evolutionary improvement upon multi-DOF muscles. There are significant concerns of objectivity and relevance for a classification scheme based on the number of joints crossed.

The legacy of Lombard’s findings should therefore be that in explaining the function of two-joint muscles, he was exposing two common misconceptions. The first is the assumption that antagonism occurs only when two muscles cross opposite sides of a joint, and the second is that the moment arms or directions of torque production associated with a muscle automatically the motion that occurs of the muscle shortens. Lombard’s example disproves both of these assumptions. The further analyses of Elftman (1939a), Enklaar (1954), and Landsmeer (1961) have further shown that in a multi-joint system, the consequences of muscle activation occur about joints that are not even directly actuated.

These misconceptions arise because what is true of a single-joint system does not apply to multi-joint systems. In a single-joint system, antagonism can only occur when two muscles are on opposite sides of the joint, and shortening of a muscle implies movement of the joint in the shortening direction. However, when considering a multi-joint system, no joint can be considered in isolation. Vectors are useful for representing muscle function because they summarize how a muscle affects motion about all joints simultaneously. Using the vector approach, it is evident that there is an antagonistic component

when practically any pair of muscles is activated. Even two single-joint muscles, crossing two different joints but on the same side, produce components of torque that cancel each other. Isometric constraint lines (Figure 2) show that shortening of a muscle only restricts movements to a region of angular velocity space, admitting, as Lombard states, “extension of a joint which [the muscle] can flex.”

One advantage to the mathematical approach is that understanding of muscle coordination is made simpler. Instead of examining each muscle individually to decide if it should be activated as part of a motor task, it is more comprehensive simply to consider whether a muscle produces moments that contribute to the desired net action (Capozzo et al., 1976; Wells & Evans, 1987; Wells, 1988). In fact, a number of researchers have employed optimization to successfully predict muscle activation patterns without special classifications of two-joint muscles at all (Crowninshield and Brand, 1981; Dul et al., 1984; Pederson et al., 1987; Patriarco et al., 1981; Herzog & Binding, 1992; Prilutsky et al., 1997). The large number of muscles and the large number of motor tasks makes it unwieldy to classify and qualitatively describe how these muscles are used, when compared to an approach that requires no special classification and can make quantitative predictions.

This approach is not to say that two-joint muscles are not interesting. The rectus femoris, hamstrings, and gastrocnemius muscles, for example, surely contribute to human gait in a unique manner (Elftman, 1939b; Prilutsky et al., 1998). However, for other motor tasks, there are other muscles crossing one or more joints that will act efficiently, allow transfer of energy, or exhibit other phenomena that have sometimes been attributed to two-joint muscles alone. Each muscle is unique and provides a unique contribution to certain motor tasks. In turn, many motor tasks are likely to be coordinated so as to harness the action of these muscles. It is this important realization that has come about from the pioneering studies of Duchenne, Lombard, and their contemporaries.

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## Appendix

The example system consists of two links that rotate in the plane about two hinge joints. There are six muscles actuating the links, four of which cross a single joint, and two of which cross two joints. The mass of each link is  $M = 3$  kg, and the length is  $L = 0.3$  m. Each link is also of uniform density with the inertial properties of a thin rod. The equations of motion are described in equation (7), where the following quantities are defined:

$$\mathbf{M}(\boldsymbol{\theta}) \equiv ML^2 \begin{bmatrix} \frac{5}{3} - \cos\theta_2 & -\frac{1}{3} + \frac{1}{2}\cos\theta_2 \\ -\frac{1}{3} + \frac{1}{2}\cos\theta_2 & \frac{1}{3} \end{bmatrix},$$

$$\mathbf{v}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \equiv \begin{bmatrix} \sin\theta_2 \left( -ML^2\dot{\theta}_1\dot{\theta}_2 + \frac{1}{2}ML^2\dot{\theta}_2^2 \right) \\ \frac{1}{2}ML^2\dot{\theta}_1^2 \sin\theta_2 \end{bmatrix},$$

$$\mathbf{g}(\boldsymbol{\theta}) \equiv \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The muscles are specified as follows. The moment arm matrix is

$$\mathbf{R} \equiv \begin{bmatrix} 0.03 & -0.03 & 0 & 0 & 0.035 & -0.01 \\ 0 & 0 & 0.03 & -0.03 & -0.01 & 0.03 \end{bmatrix} \text{m}.$$

As an example of the sign convention, muscle 1 has an extension moment arm about joint 1, as does muscle 5 but with the addition of a flexion moment arm about joint 2. The maximum isometric forces are

$$\mathbf{f}^{\max} \equiv [2500 \ 2000 \ 1500 \ 1000 \ 2500 \ 1500]^T \text{ N.}$$

The transformation from joint angular accelerations to endpoint (or center of mass) accelerations involves a simple matrix multiplication. The endpoint accelerations are

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \mathbf{J}(\boldsymbol{\theta}) \cdot \ddot{\boldsymbol{\theta}} + \mathbf{J}(\boldsymbol{\theta}) \cdot \dot{\boldsymbol{\theta}}$$

where  $\ddot{\boldsymbol{\theta}}$  is found using equation (7) and

$$\mathbf{J}(\boldsymbol{\theta}) \equiv L \begin{bmatrix} \sin \theta_1 - \sin(\theta_1 - \theta_2) & \sin(\theta_1 - \theta_2) \\ \cos \theta_1 - \cos(\theta_1 - \theta_2) & \cos(\theta_1 - \theta_2) \end{bmatrix}.$$

Two slot constraints are employed. In the example of Figure 5, the slot constrains prevents horizontal motion, and the constraint equations (9) and (10) use the quantities

$$\gamma_x(\boldsymbol{\theta}) \equiv -L \cos \theta_1 + L \cos(\theta_1 - \theta_2) - c_x$$

and

$$\mathbf{J}_x(\boldsymbol{\theta}) \equiv L [\sin \theta_1 - \sin(\theta_1 - \theta_2) \ \sin(\theta_1 - \theta_2)].$$

In the example of Figure 6, the slot constrains vertical motion, and the corresponding quantities are

$$\gamma_y(\boldsymbol{\theta}) \equiv L \sin \theta_1 - L \sin(\theta_1 - \theta_2) - c_y,$$

$$\mathbf{J}_y(\boldsymbol{\theta}) \equiv L [\cos \theta_1 - \cos(\theta_1 - \theta_2) \ \cos(\theta_1 - \theta_2)].$$

For the constrained system of Figure 6, the equations of motion (7) are combined with the constraint (11), yielding the constrained accelerations (Figure 6a)

$$\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{12} & C_{22} \end{bmatrix} \cdot \mathbf{t}^M + \begin{bmatrix} C_{10} \\ C_{20} \end{bmatrix}$$

where

$$\begin{aligned} C_{11} &\equiv \frac{12}{ML^2 d} \cos^2(\theta_1 - \theta_2), \\ C_{12} &\equiv \frac{12}{ML^2 d} (\cos \theta_1 \cos(\theta_1 - \theta_2) - \cos^2(\theta_1 - \theta_2)), \\ C_{22} &\equiv \frac{24}{ML^2 d} (\cos \theta_1 - \cos(\theta_1 - \theta_2)) \cdot \\ &\quad \sin\left(\theta_1 - \frac{\theta_2}{2}\right) \sin \frac{\theta_2}{2}, \end{aligned}$$

$$C_{10} \equiv \frac{1}{d} \begin{pmatrix} (-\sin 2\theta_1 + \sin(2\theta_1 - \theta_2) +) \\ 5 \sin \theta_2 - 3 \sin 2\theta_2 \\ (\sin(2\theta_1 - \theta_2) + 5 \sin \theta_2) \end{pmatrix} \dot{\theta}_1^2 + \begin{pmatrix} \\ \\ (2\dot{\theta}_1 \dot{\theta}_2 - \dot{\theta}_2^2) \end{pmatrix}$$

$$C_{20} \equiv \frac{1}{d} \begin{pmatrix} (\sin 2\theta_1 + 5 \sin 2(\theta_1 - \theta_2) -) \\ 6 \sin(2\theta_1 - \theta_2) - 16 \sin \theta_2 + \\ 6 \sin 2\theta_2 \end{pmatrix} \cdot \dot{\theta}_1^2 + \begin{pmatrix} \\ \\ (2\dot{\theta}_1 \dot{\theta}_2 - \dot{\theta}_2^2) \end{pmatrix}$$

$$d \equiv -7 + \cos 2\theta_1 - 5 \cos(2\theta_1 - 2\theta_2) + 3 \cos 2\theta_2.$$

In terms of the output space of motion along the constraint (11) and the normal force against the constraint, the quantities in equation (12) are

$$K_1 = \frac{-12}{MLd} \cos(\theta_1 - \theta_2) \sin \theta_2,$$

$$K_2 = \frac{12}{MLd} (\cos \theta_1 \sin \theta_2 - \cos(\theta_1 - \theta_2) \sin \theta_2),$$

$$K_0 = \frac{L}{d} \begin{pmatrix} 4 \left( \cos \theta_1 + 5 \cos(\theta_1 - \theta_2) \sin\left(\frac{\theta_2}{2}\right)^2 \right) \\ \dot{\theta}_1^2 + \\ (-9 \cos(\theta_1 - \theta_2) + \cos(\theta_1 + \theta_2)) \cdot \\ (2\dot{\theta}_1 \dot{\theta}_2 - \dot{\theta}_2^2) \end{pmatrix},$$

$$S_1 = \cos \theta_1 - 3 \cos(\theta_1 - 2\theta_2),$$

$$S_2 = \cos \theta_1 - 3 \cos(\theta_1 - 2\theta_2) + \\ 13 \cos(\theta_1 - \theta_2) - 3 \cos(\theta_1 + \theta_2),$$

$$S_0 = \frac{ML}{6d} \begin{pmatrix} (\sin \theta_1 - 15 \sin(\theta_1 - 2\theta_2) +) \\ 17 \sin(\theta_1 - \theta_2) - 3 \sin(\theta_1 + \theta_2) \end{pmatrix} \dot{\theta}_1^2 + \begin{pmatrix} \\ \\ 2(-17 \sin(\theta_1 - \theta_2) + 3 \sin(\theta_1 + \theta_2)) \cdot \\ (2\dot{\theta}_1 \dot{\theta}_2 - \dot{\theta}_2^2) \end{pmatrix}.$$

The constraint forces are derived using application of Newton's Law, and is described in more detail by, for example, Kuo (1998).