ME 599-1 Homework 5

1. Problem 4: When $b=0$, the two springs are in series, with an effective stiffness of $k_e = k_m k_t / (k_m + k_t)$. Then $\omega_n^2 = k_e / m$ is the resonant frequency of the equivalent mass-spring system. When the track is rigid, the system is equivalent to a mass-spring-damper, which has natural frequency equal to $\omega_0^2 = k_m (1 - \zeta^2) / m$.

Problem 5: When $L=0$, the step length is at its maximum, equal to twice the leg length. The stiffness at which this occurs is $k^*_t = mg / \sqrt{\ell^2 - L_0^2 / 4}$.

2. a. The proper command for finding the limit cycle is

\[
x0 = \text{fsolve('simwalk', xguess)}
\]

which returns an $x0$ which is a very accurate initial condition which is repeated by the walking cycle. Small deviations in any element of $x0$, such as 0.01, are sufficient to make the system diverge.

b. Starting with $x0$, the Jacobian may be determined from a few simple lines. First note that $\text{simwalk}$ actually returns the difference between the following step’s initial conditions and the provided initial conditions, so $\text{simwalk}(x0) + x0$ is the expression for the following step’s initial conditions.

\[
\text{for } i = 1:4,
\text{ } dx = [0; 0; 0; 0]; dx(i) = 1e-6; \quad \% \text{ a small deviation in the } i’\text{th position}
\text{ } J(:,i) = (\text{simwalk}(x0+dx) + x0+dx) - x0) / 1e-6; \quad \% \text{ note that } x0'\text{s cancel in this line}
\end{\text{for}};
\text{[v,d] = eig(J); \quad \% \text{eigenvectors and eigenvalues are provided here}}
\]

The eigenvalues are 0.46, -0.41, -0.17, 0. The corresponding eigenvectors are

\[
v1 = [0.61; -0.61; -0.50; 0.13] \text{ “speed”}
v2 = [-0.65; 0.65; 0.22; 0.31] \text{ “totter”}
v3 = [-0.44; 0.44; 0.23; 0.74] \text{ “swing”}
v4 = [0.37; 0.27; -0.36; -0.81] \text{ “swing”}
\]

The eigenvectors have been somewhat arbitrarily named to describe how they behave. The “speed” mode is named so because its eigenvalue of 0.46, corresponds to the dissipation of speed similar to that of the rimless wheel. The “totter” mode is another energy dissipation mode, except with a negative eigenvalue which means there is some sort of oscillation as it dies out. It is the mode by which the system matches the step length to the downward slope. The two swing modes are named because they are dominated by the angular velocity of the swing leg. Note that one of these modes has a value of zero, and corresponds to the perturbation which is immediately canceled by ground contact. The other swing mode shows that perturbations to the swing leg angular velocity are canceled quite quickly, with a small negative eigenvalue.

c. Using practically any $x0$ in the lines above will result in at least one eigenvalue which has magnitude larger than 1.
Mid-swing: Prior to heel lock, there are three separate velocities for the stance leg, and thigh and shank of the swing leg. After heel lock, the thigh and shank have identical velocities, so that there are two unknowns—stance leg angular velocity and swing leg (thigh and shank) angular velocity. Two equations are therefore necessary. Angular momentum is conserved for the swing leg about the hip, and for the whole system about the point of contact of the stance leg. Both of these momenta are linear in the angular velocities (but are not equal to the moments of inertia of the segments), and may be combined into matrix equations for before and after heel lock, \( H^- = M^- \dot{\theta}^- \) and \( H^+ = M^+ \dot{\theta}^+ \), where \( \dot{\theta}^- \) is a vector of the three angular velocities, and \( \dot{\theta}^+ \) contains the two unknowns. The post-heel-lock angular velocities are found through \( \dot{\theta}^+ = (M^+)^{-1} M^- \dot{\theta}^- \).

Ground contact: Prior to ground contact, there are only two separate velocities for the stance and swing legs, but the stance leg may be thought of as being composed of a thigh and shank which share the same angular velocity. After ground contact, the thigh and shank of the former stance leg become the thigh and shank of the new swing leg, and these two segments have differing angular velocities. This gives three unknowns post-contact—angular velocities of the new stance leg and of the thigh and shank of the new swing leg—which require three equations. Angular momentum is conserved for the whole machine about the point of contact, for the trailing leg about the hip, and for the trailing leg’s shank about the knee. The matrix equations are similar to those above, except that \( \dot{\theta}^- \) is a vector of two angular velocities, \( \dot{\theta}^+ \) contains three angular velocities, and the matrices have corresponding dimensions.