ME 599-1 Homework 2

1. The arch of the human foot contains connective tissue with elastic properties. This system can be modeled crudely as a suspension spring as shown below (unloaded case):

   ![Spring Diagram](image)

   a. Find the deflection of the spring (the plantar fascia of the foot) from the unloaded situation as a function of the vertical displacement of the ankle, using basic kinematics.
   b. Assume that the vertical displacement reaches a peak of 7 mm during mid-stance for a running person. Calculate the corresponding displacement of the spring.
   c. Use the Jacobian of the kinematic function from a. to find the linearized relation between the vertical ankle displacement and the spring displacement. Do you consider the linear approximation adequate?
   d. Using the static, unloaded case as an operating point, calculate the force in the spring if the vertical force reaches a peak of 6.4 kN.
   e. Using a linear spring approximation, calculate the potential energy stored in the spring at the force in part d. If the linear spring over-estimates the actual energy by a factor of 2, what is the actual energy stored? What could account for the error in the linear spring approximation?

2. Derive the forward kinematics of fingertip position in one plane. Use the following lengths of the individual segments: 55 mm, 30 mm, 25 mm, or use the lengths of your own finger. Devise any desired convention for describing the angles of the joints or segments. Consider the situation in which the first knuckle is 70 mm above a horizontal surface.

   ![Fingertip Diagram](image)

   a. Derive the position & velocity of the fingertip in Cartesian coordinates.
   b. Calculate the Jacobian of the fingertip position, and verify against the velocity calculated above.
   c. Calculate the vertical force exerted by the finger in the static case, as a function of the joint torques and angles.
   d. Suppose the third joint is permanently held at 15 degrees flexion (measured from full extension). For any given lateral position of the fingertip with respect to the first knuckle, it is possible to determine the corresponding angles at the first and second joints. This is known as the inverse kinematics problem, and is sometimes quite difficult. Solve the
inverse kinematics problem for lateral position between 0 and 84 mm from the first knuckle. Hint: use geometry, and make use of the law of cosines and the law of sines.
e. Considering the joint torques to be a vector, at what lateral position in part d. is the finger able to exert the maximum static force at the tip, relative to the magnitude of the joint torque vector? Consider the maximum force in any direction.
f. What about the maximum force vertically?
g. At what lateral position is the fingertip most “manipulable?” Discuss the trade-off between manipulability and the ability to exert large forces in a single direction.
h. Give an interpretation of the columns of the Jacobian, with respect to the joint velocities.
i. Give an interpretation of the rows of the Jacobian, with respect to fingertip forces.

3. One interesting application of the singular value decomposition is to perform image compression. An image can be viewed as simply a matrix of values, each value denoting the brightness or color of a pixel. MATLAB has a sample image of a penny, which can be displayed using the following commands:

```matlab
load penny
image(P); axis off; axis equal; colormap(copper);
```

Think of this image as being the weighted sum of a number of (output) vectors, with the weights equal to the matrix’s singular values. Since most of the singular values are much smaller than the largest ones, the matrix can be approximated by the sum of its most significant vectors.

a. To test this notion, perform a singular value decomposition of the matrix P. Then construct a new matrix constructed from the product of the first $k$ columns of the output matrix, the first $k$ singular values, and the first $k$ input rows.

b. How many singular values are necessary to provide a reasonable approximation of the original image? If only those singular values and their corresponding input and output vectors are to be stored, what is the percentage compression?