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THE NORMATIVE IMPACT OF CONSUMER PRICE EXPECTATIONS FOR MULTIPLE BRANDS ON CONSUMER PURCHASE BEHAVIOR

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Empirical research indicates that some consumers form price expectations which may impact their purchase behavior. While literature in operations research has built purchase policy models incorporating uncertain price expectations, these models have been built for commodities. Consumers face an environment with multiple brands. In this paper, we develop a model that incorporates consumer preferences and price expectations for multiple brands as determinants of normative consumer purchase behavior. The model demonstrates how commodity purchase policy models recognizing price uncertainty can be adapted to the study of multi-brand markets. The model is used to analyze the normative impact of changes in price promotion policies and holding costs on individual purchase behavior. It is also used in a Monte-Carlo market simulation that illustrates some scenarios where a post-promotion dip is more or less evident, and provides an explanation for the nonexistent post-promotion dip.

(Price Expectations; Stockpiling; Inventory Models; Decision Making Under Uncertainty)

1. Introduction

Empirical research indicates that consumers form price perceptions (Dickson and Sawyer 1990; Krishna, Currim and Shoemaker 1989). While literature in operations research (Golabi 1985), economics (Bucovetsky 1983), and marketing (Assuncao and Meyer 1990, Helsen and Schmittlein 1989, Meyer and Assuncao 1990) has modelled the effect of uncertain price expectations on purchase policy, most of these models have been developed for the purchase of commodity products (Magirou 1982, Assuncao and Meyer 1991), or for the purchase of raw materials (Fabian et al. 1959, Morris 1959, Golabi 1985).

Consumers purchase multiple brands in a product category, and form price perceptions for multiple brands (Conover 1986; Krishna, Currim and Shoemaker 1989). Price expectations for different brands may have important implications for how consumers react to price promotions, since the likelihood of their buying a brand may be a function not only of expectations of future prices for that brand, but also of future prices of competing brands. In this paper, we develop a model that incorporates consumer preferences and price expectations for multiple brands as determinants of normative consumer purchase behavior.

The model extends Golabi's (1985) single-commodity model to the multi-brand case, and demonstrates how commodity purchase policy models recognizing price uncertainty
can be adapted to the study of branded goods markets. This is done by collapsing a multivariate distribution of prices across a set of brands into a univariate distribution, allowing direct application of a single-commodity model to the multi-brand environment.

We also show how the model can be used by an analyst to examine the possible effect of changes in pricing policies and other factors on consumer response to these policies. It is used to explore the normative impact of deal frequency and holding cost on individual purchase quantity, probability of deal response, and share of purchases, for different brands.

The model is also used in a Monte-Carlo market simulation to analyze the normative effect of deal probability and brand loyalty on nonpromotion baseline sales and the post-promotion dip. There has been a premise in marketing that there should be a dip in sales after a promotion. However, this has not been empirically observed. Many explanations for this phenomenon have been offered. But, as Blattberg and Neslin (1990, p. 360) note “. . . there has been very little theoretical work trying to describe how and when a trough after a deal should be evident.” The model simulation illustrates some scenarios where a post-promotion dip is more or less likely, and provides an economic explanation for the expected but nonexistent post-promotion dip.

The paper concentrates on the consumer decision. We do not consider the firm decision. The remainder of this paper is organized as follows. §2 develops the optimal policy for purchasing under price uncertainty for multiple brands. In §3 we show how the purchase policy model can be used to explore the normative effect of changes in promotion policies and other variables on purchase behavior. In §4, we derive individual and market level implications using the model.

2. Optimal Purchase Policy

2.1. Overview

We first set up the problem for the multiple-brand scenario. Then we show how we can adapt the problem to Golabi’s (1985) single-commodity problem framework.

We assume that the buyer goes to the store every period (for example, every week). In the store he observes prices for many brands. The price of each brand is assumed to be randomly drawn from a known stationary price distribution for that brand.\(^1\) Also, the prices of different brands are assumed to be uncorrelated so that at each purchase occasion the price for each brand is independently drawn from its distribution. In some highly promoted product categories retailers might force price dependence among brands by offering price discounts on exactly \(n\) brands in the product category per week. However, in many other product categories this need not be the case, and the timing of price changes for one brand may be independent of the price changes for other brands. We also assume that there is no variety seeking and no satiation, and that there is a constant consumption rate for the product class.

The buyer’s task is to decide how many units of each brand to purchase on a shopping trip so that he minimizes the total cost of purchasing the good and storing it, while taking his brand preferences into consideration.

Let

\[
\begin{align*}
z_i & \text{ = inventory of brand } i \text{ at the beginning of a given period,} \\
q_i & \text{ = quantity consumed of brand } i \text{ in a period,} \\
c_i & \text{ = observed price for brand } i \text{ at the beginning of period,}
\end{align*}
\]

\(^1\) Temporal price independence in prices would hold for many product categories where deals did not occur regularly. We found one-period serial correlation in prices to be insignificant \((p < 0.05)\) for Brawny paper towels, Mr. Big and Charmin bathroom tissue in a Grand Union store.
$F_i(x_i)$ = cumulative probability distribution of prices for brand $i$,
y = quantity purchased of brand $i$,
$\alpha$ = rate of discount per period (for cost),
$\bar{q}$ = constant consumption rate per time period for the product class = ($\sum_i q_i$),
s = substitution cost for brand $i$ ($\infty > s_i \geq 0$).

To allow for consumer preferences among brands, we include substitution costs for less preferred brands. $s_i$ reflects the loss in utility to the consumer by purchasing one unit of brand $i$, instead of one unit of the consumer’s favorite brand. $s_i$ is assumed to depend only on brand preferences, and to be independent of prices. Since there is no variety seeking or satiation, it is also independent of the current inventory.

Substitution costs for less preferred brands have been used by prior researchers in modelling frameworks (Bawa and Shoemaker 1987). We assume that the lower utility from purchasing a less preferred brand is reflected in an increase in ‘price’ of the less preferred brand. A similar approach is taken by Narasimhan (1988) and by Raju, Srinivasan and Lal (1990) where ‘price’ of less preferred brands is effectively increased by a term which reflects brand preference. This term equals the minimum difference between the prices of two brands that is necessary to induce the consumer of one brand to switch to the other brand. Blattberg and Wisniewski’s (1989) procedure for capturing the difference in brand preference is also similar. They reflect the utility of a brand in terms of price and quality. A consumer chooses one brand over another if the utility from the higher quality that a brand offers is greater than the price difference between the two brands, or if the saving in price of a brand is less than the reduction in utility from lower quality.

2.2. Total Cost Equation

We assume that there are $m$ brands, that purchases are made at the beginning of each period 1, ..., $N$, and that there is no holding cost for consumption within the period. We further assume that holding cost is a linear function of inventory $z$ with nonnegative coefficient $h$, and is independent of the brand. There may be a variety of forces which inhibit stockpiling, besides uncertainty over future prices. These could be capacity constraints, spoilage (or a diminished preference for aged goods), and opportunity costs for space. The linear holding cost function is simply a rough approximation of these influences. Finally we assume that there is no backlogging of demand, and that there are no stockouts for any brand.

Let $V_n$ be the total cost for the remaining time from the beginning of period $n$, thus $V_N$ = cost for the last period (period $N$), $V_{N+1} = 0$, and $V_{t}$ = total cost over the $N$ period time frame. Let $V_n^*(z_1, z_2, ..., z_m, \alpha, c_1, c_2, ..., c_m)$ be the minimum expected cost when the consumer starts the $n$th period with inventory levels $z_1, z_2, ..., z_m$ of brands 1, ..., $m$, the discount rate per period is $\alpha$, and the price offered for brands 1, ..., $m$ is $c_1, c_2, ..., c_m$. The total cost is composed of two terms, the cost for the current period, and the expected cost of future periods. Both these terms include purchasing, holding, and substitution costs. The expected cost of future periods is discounted to reflect the fact that by spending now versus later, there is an opportunity cost for the consumer. This opportunity cost may be, for example, the interest rate. The consumer’s optimal purchase is given by the following equation which uses the standard Bellman’s optimality equation (1957)

Note that $x_i$ is the random variable and $c_i$ is the realized (or observed) variable.

The computation and results apply whether we take an infinite or a finite horizon, i.e. $N$ may tend to $\infty$.
\[ V^w_n(z, \alpha, c_1, c_2, \ldots, c_m) = \inf_{y \geq q - z} \left\{ \sum_{i=1}^{m} (s_i + c_i) y_i + h(y + z - \bar{q}) \right\} \]
\[ + \left( \alpha \int_0^\infty \cdots \int_0^\infty V_{n+1}(y + z - \bar{q}, \alpha, x_1, x_2, \ldots, x_m) dF_1(x_1), \ldots, dF_m(x_m) \right) \]

where
\[ z = \sum_{i=1}^{m} y_i, \quad \bar{q} = \sum_{i=1}^{m} q_i, \quad y = \sum_{i=1}^{m} y_i. \]  

The first term denotes the cost for the current period where the consumer purchases \( y_1, y_2, \ldots, y_m \), units of brands 1, 2, \ldots, \( m \) respectively. The second denotes the expected future cost with discounting at rate \( \alpha \) per period.

There is no brand distinction in inventory, since the holding cost for each brand is the same and there is no variety seeking. We call \( s_i + c_i \), the realized effective price of brand \( i \). This is denoted by \( R_i \). Since \( s_i \geq 0 \), and \( c_i > 0 \), \( R_i > 0 \). \( R_{\text{min}} \) denotes the minimum realized effective price across all brands.\(^4\)

2.3. Optimal Policy

Now we develop the optimal purchase policy which minimizes the cost as expressed in equation (1). This is done by

(a) making clear that it is optimal for the consumer to purchase a maximum of one brand.

(b) reducing the multiple price distributions for many brands into a single price distribution which captures all the price information that is relevant to the consumer’s purchase decision.

(c) using (a) and (b) to reduce equation (1) to the single-commodity total cost equation of Golabi, and invoking Golabi’s results thereafter.

2.3.1. One brand maximum. On any given purchase occasion, the consumer should optimally purchase a maximum of one brand. This will be the brand with the lowest realized effective price \( s_i + c_i \). This result is to be expected. It is a consequence of the linearity of purchase cost. Purchase cost is linear in the number of units purchased since the effective price per unit is the same. This is because the price per unit is constant, and the substitution cost per unit is also constant due to the assumption of no variety-seeking and no satiation. We ignore the degenerate case where two brands have the same effective price. In that case, the consumer is indifferent between buying different proportional quantities of the brands that have this minimum cost, as long as the total quantity adds up to \( y \).

2.3.2. Building brand differences into a single price distribution. Order statistics allow us to determine the probability distribution of the lowest realized effective price among all the brands in the market. Since the lowest realized effective price is the relevant price to the consumer, this distribution captures all the price information that the consumer needs for his purchase decision. Let \( F_i(\cdot) \) be the cumulative density function of the price of brand \( i \), \( x_i \), and \( f_i(\cdot) \) be the probability density function of \( x_i \). Since we

\(^4\) \( x_i \) is the random variable for price. \( E_i \) and \( E_{\text{min}} \) are the random variables for the effective price of brand \( i \), and the minimum effective price among all brands, respectively. Similarly, \( R_i \) is the realized price. \( R_i \) and \( R_{\text{min}} \) are the realized effective price of brand \( i \), and the minimum realized effective price among all brands, respectively.
assumed that the prices of different brands are uncorrelated, each $x_i$ is independently drawn from its distribution on each purchase occasion.

Let $G(\cdot)$ denote the cdf of the lowest effective price among all brands in the market, 

$$E_{\min} := p(E_{\min} \leq e) = G(e) = 1 - p(x_1 + s_1 > e, x_2 + s_2 > e, \ldots, x_m + s_m > e),$$

$$G(x) = 1 - \prod \{1 - F_i(e - s_i)\},$$

$$g(x) = \sum_{i=1}^{m} f_i(e - s_i) \prod_{j \neq i}(1 - F_j(e - s_j)).$$

$G(\cdot)$ and $g(\cdot)$ are the cumulative probability distribution and the probability density function respectively, of the lowest realized effective price among all brands in the market.

2.3.3. Reducing equation (1) to Golabi’s single-commodity format. Equation (1) can be rewritten so that the consumer purchases a maximum of one brand on a purchase occasion, and with the cumulative density functions for all brands ($F_1(x_1), F_2(x_2), \ldots, F_m(x_m)$) replaced by the cumulative density function for the lowest realized effective price among all brands, $G(x)$.

Equation (2) is similar to Golabi’s equation for the single-brand case, with the price distribution for the single brand, $f(x)$, replaced by the minimum price distribution across brands, $g(x)$ (see Golabi 1985, equation (10), p. 582).

Hence, we can now invoke Golabi’s results. Golabi proved that there is a sequence of critical prices (reservation prices) which determines the optimal number of units that a buyer should hold in inventory for any current price. If the buyer’s inventory prior to the purchase decision is less than this optimal level, then he should purchase up to this amount.

**Theorem 1.** For all $n, z, \alpha, c_1, \ldots, c_m$, the optimal purchase policy equation, equation (2) is satisfied by the policy

$$y^n(\alpha, z, c_1, \ldots, c_m) = \max \{0, \min \{N + 1 - n, b + 1\} \bar{q} - z\}$$

where $A_{b+1}(\alpha) < R_{\min} \leq A_b(\alpha), r \geq 0$ and $N$ is the total number of periods till the horizon, so that the purchase vector is $(y^1, y^2, \ldots, y^N)$.

In words, this means that when the minimum observed effective price ($\bar{s}_i + c_i$) lies between the critical price levels $A_{b+1}(\alpha)$ and $A_b(\alpha)$, the optimal policy dictates that the consumer purchases to bring his inventory level up to $b + 1$ periods of consumption. So, e.g., if $A_4 = $2.25 and $A_3 = $1.75, a consumer should purchase to bring his inventory up to 4 periods of consumption if the realized effective price lies between $1.75$ and $2.25$. This holds unless the consumer is close to the end of the planning horizon, and therefore should not have an inventory to last beyond that horizon.

\* See footnote 4.
Critical price sequence: The \( A_r(\alpha) \) sequence represents a set of critical price levels that establish thresholds demarcating the consumer's purchase behavior.

\[
A_0(\alpha) = \infty, \\
A_{r+1}(\alpha) = \alpha \int_0^{A_r(\alpha)} x dG(x) + \alpha \int_{A_r(\alpha)}^{\infty} A_r(\alpha) dG(x) - h. \tag{4}
\]

(See Golabi 1985, equation (16), p. 583.)

In Golabi's model the critical prices, \( A_r(\alpha) \), depend on the future price distribution of the single brand considered. However, in our model the critical prices are determined by the substitution costs and future price distributions for all brands. Since the purchase quantity is a function of the critical price sequence, the purchase quantity for any brand in our model will be dependent on the preferences and future prices for all brands.

2.3.4. Commodity markets and efficiently priced markets. When only one brand is considered, the minimum effective price distribution is the price distribution of the single brand, and equation (3) reduces to the optimal policy equation of Golabi (1985) for the single-commodity market.

Equation (3) also reduces to Golabi's equation when consumers are identical in preference and brands are efficiently priced such that, for example, if brand 1 is seen as 10 cents less attractive than brand 2, then brand 1's price is 10 cents less than brand 2's. In such a case one would revert to a commodity market where the brand with the lowest effective price in any week was determined completely by chance.

3. Using the Model for Analyzing the Implications of Changes in Promotion Policies and Other Factors

We now show how the model can be used to explore the normative effects of changes in promotion policies and other factors on purchase behavior. The optimal policy dictates that the consumer should make purchases depending on his current inventory state and the observed prices. Consequently, to determine how changes in values of model parameters may affect consumer purchase behavior, we need to know the consumer's probability of observing different prices, and of being in different inventory states. In addition, to see the effect of model parameter values on consumer purchases of specific brands, we also need to determine the probability of choosing specific brands. From the price distributions, we know the consumer's probability of observing different prices. In this section we develop the probability of being in different inventory states, and the probability of choosing different brands.

3.1. Probability of Being in a Specific Inventory State

Theoretically, inventory is a continuous variable. However, there can only be \( K \) possible inventory levels in steady state.\(^6\) We determine the probability of being in these \( K \) inventory states by developing the probability of making a transition from one inventory state to another, and then computing the aggregate probability of being in any one inventory state. This is done for multiple brands and without positing specific forms for price distributions. Assuncao (1990) has developed a similar equation for the single-brand case, where the price distribution is bimodal and the consumption rate is not constant.

For simplicity, we denote \( A_r(\alpha) \) by \( A_r \).

3.1.1. State transition matrix. \( I_k \) means that the consumer has enough inventory to last him for \( k \) periods. The row heading denotes the inventory state at the beginning of

\(^6\) See Appendix I. Lemma 1 shows that the consumer will purchase for a maximum of \( K \) periods. Lemma 2 follows from Lemma 1 and establishes that there are \( K \) possible inventory states at the beginning of a period.
the period, and the column heading denotes the inventory state at the beginning of the
next period. The figures in the cell represent the probability of making a transition from
one inventory state to another. Thus, for example, the probability that the consumer will
remain in the same inventory state, \( I_0 \), from the beginning of this period to the next,
means that the consumer purchases only enough of the product for the current period.
This happens when the minimum observed effective price lies between \( A_1 \) and \( A_2 \), which
happens with probability \( 1 - G(A_1) \). Similarly, the probability that the consumer will
remain in the same inventory state, \( I_2 \), from the beginning of this period to the next,
means that the consumer purchases for one period to bring his inventory up to three
periods worth. This happens when the minimum observed effective price lies between
\( A_3 \) and \( A_2 \), which happens with probability \( G(A_2) - G(A_3) \).
In a similar fashion we can fill in the values of the other cells.

\[
\begin{array}{ccccccc}
I_0 & I_1 & I_2 & \ldots & I_{K-2} & I_{K-1} \\
I_0 & 1 - G(A_1) & G(A_2) - G(A_3) & G(A_2) - G(A_3) & \ldots & G(A_{K-2}) - G(A_{K-1}) & G(A_{K-1}) \\
I_1 & 1 - G(A_1) & G(A_2) - G(A_3) & G(A_2) - G(A_3) & \ldots & G(A_{K-2}) - G(A_{K-1}) & G(A_{K-1}) \\
I_2 & 0 & 1 - G(A_3) & G(A_2) - G(A_3) & \ldots & G(A_{K-2}) - G(A_{K-1}) & G(A_{K-1}) \\
I_3 & 0 & 0 & 1 - G(A_3) & \ldots & G(A_{K-2}) - G(A_{K-1}) & G(A_{K-1}) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
I_{K-2} & 0 & 0 & 0 & \ldots & G(A_{K-2}) - G(A_{K-1}) & G(A_{K-1}) \\
I_{K-1} & 0 & 0 & 0 & \ldots & 1 - G(A_{K-1}) & G(A_{K-1}) \\
\end{array}
\]

We call this matrix \( \tilde{I} \). \( \tilde{I} \) denotes the transpose of this matrix.

3.1.2. Steady state probability of being in an inventory state. The steady state prob-
bability of being in inventory state \( I_k \) is denoted by \( \pi_k \).

By definition, \( \tilde{I} \tilde{\pi} = \tilde{\pi} \).

This equation means that the steady state probability of being in an inventory state \( I_k \)
(i.e., \( \pi_k \)) is the sum of the probabilities of making a transition from a different
inventory state \( I_i \) to inventory state \( I_k \), given that you were in inventory state \( I_i \) to begin with.

From this equation we can obtain the value of the probability of being in inventory
state \( I_k \), i.e., \( \pi_k \),

\[
\pi_k = [(1 - G(A_{k-1}))(1 - G(A_{k-2})) \cdots (1 - G(A_{k+1}))] G(A_k).
\]

PROOF. In Appendix 2.

3.2. Probability of Choosing a Brand

The probability that brand \( i \) is chosen depends on the probability that the effective
price of brand \( i \) is the lowest effective price, i.e. \( E_i = x_i + s_i = E_{\text{min}} \).

This implies that \( x_i + s_i \leq x_j + s_j \) for all \( j \neq i \). Thus if the effective price of brand \( i \)
\( = e \), the effective price of all other brands should be less than \( e \):

\[
\begin{aligned}
&= \int_0^\infty f_i(e - s_i) \prod_{j \neq i} [1 - F_j(e - s_j)] de.
\end{aligned}
\]

The probability that \( i \) is the brand chosen, and the minimum effective price, \( X_{\text{min}} \) falls
between two specific critical price levels, say \( A_{\text{min}} \) and \( A_{\text{max}} \), i.e. \( A_{\text{min}} < E_{\text{min}} < A_{\text{max}} \) is:

\[
\begin{aligned}
&= \int_{A_{\text{min}} - s_i}^{A_{\text{max}} - s_i} f_i(e - s_i) \prod_{j \neq i} [1 - F_j(e - s_j)] de = P_i(A_{\text{min}}, A_{\text{max}}).
\end{aligned}
\]

We now have closed form solutions for the probability of being in any inventory state,
and the probability of choosing any brand. We also develop closed form solutions for
the expected purchase quantity of any brand \( = \sum_{k=1}^{K} [P_i(A_k, A_i) \sum_{v=1}^{k} v\pi_{k-v}] \),
see Appendix 3). We use these solutions to determine how changes in values of the model parameters should affect consumer purchase behavior.

4. Model Implications

We use the model to derive two types of implications:

(i) individual level implications for consumer purchase behavior.

Here we vary the value of different model parameters, for example, deal frequency of different brands, and use the model to analyze the effect of this change on consumer purchase policies. This type of analysis is useful for determining the effect of promotion policies and other factors on individual response in branded goods markets. In addition it provides an economic explanation for observed consumer behavior.

(ii) market level implications.

There has been a premise in marketing that there should be a dip in sales after a promotion. However, this has not been empirically observed. This analysis allows us to determine conditions when a trough after a deal should be more or less evident. We start with a group of consumers with some distribution of preferences and inventory. We generate prices from a Monte-Carlo simulation and compute purchase behavior using the model. The resultant purchases are aggregated to trace sales time series for three brands.

We first discuss individual level and then market level implications.

4.1. Implications for the Purchasing Behavior of Individuals

4.1.1. Overview. We assume that there are three brands in the market, \( v, w \) and \( x \), where \( v \) is the favorite brand and \( x \) is the least preferred brand. Each brand has a bimodal price distribution such that \( c_i \) denotes the off-deal price, and \( c'_i \) denotes the deal price of brand \( i \) for \( i = v, w, x \). Correspondingly, \( R_i \) represents the effective off-deal price, and \( R'_i \) the effective deal price of brand \( i \). \( c'_i \) occurs with probability \( p_i \), and \( c_i \) with probability \( 1 - p_i \).

By definition \( R_v < R_w < R_x \), \( R'_v < R'_w \), \( R'_w < R'_x \), and \( R'_x < R_x \). Let \( R'_v < R'_w < R'_x \) \( < R_v < R_w < R_x \). It is assumed that \( R'_v < R_v \) and \( R'_w < R_w \) to ensure that consumers purchase all three brands. If we do not make these assumptions, brands \( w \) or \( x \) would never be purchased, and the three-brand situation would be degenerate.

Thus,

\[
G(R'_v) = p_v = \text{probability that the price} \leq R'_v,
\]

\[
G(R'_w) = p_v + p_w - p_v p_w = p_v + p_w(1 - p_v),
\]

\[
G(R'_x) = p_v + p_w + p_x - p_v p_w - p_v p_x - p_w p_x = p_v + p_w(1 - p_v) + p_x(1 - p_w)(1 - p_v),
\]

\[
G(R_v) = 1.
\]

Using the \( G \) distribution one can get simple formulae for \( A_r(\alpha) \) from equation (4), for \( G(A_r(\alpha)) \) using the \( G \) distribution, for \( \pi_k \) using equation (5), for choice probability using equation (5), and for other variables of interest.

We do sets of manipulations of probability of a deal on a brand, and holding cost. Assuming that one period is one week, \( \alpha \) is set at 0.998 which gives a discount rate of approximately 10% a year, a reasonable interest rate. For simplicity, the consumption rate, \( \bar{q} \), is set at one unit per week.

When not manipulated, the holding cost per unit is 0.05, the effective off-deal price of the favorite brand is 1.0, the effective deal prices of brands \( v, w, x \) are 0.6, 0.7, and
0.8 respectively, and the probability of a deal on each of the three brands is 0.2. A 40% reduction in price when the item is on sale is not uncommon (for example, in many markets the regular price for Coke and Pepsi is $1.29, and the sale price is $0.79). Also, a 20% deal frequency is often the case for paper towels, orange juice and bathroom tissue where many brands are on sale once every four to six weeks. Since the results are for the steady state, any value of \( N \) may be chosen, as long as \( N \) is relatively large. The implications are derived under the condition that all other parameters remain the same.

To check for interaction effects among the variables, we also did simulations for other values of holding cost (0.40), effective deal price of brands \( v \) (0.5), \( w \) (0.7) and \( x \) (0.9), and the probability of deals (0.05, 0.4). The same results were obtained.

### 4.1.2. Numerical results and implications.

**Deal Probability:** Separate manipulations were done for probability of dealing on brands \( v, w \) and \( x \).

**Implication 1:** As the probability of dealing on any brand increases, consumers should purchase smaller quantities of any brand on deal, and not make deal purchases of any brand given smaller amounts of current inventory.

Since deals are occurring more frequently for a brand, the minimum effective price distribution will change to reflect this. The optimal policy will now dictate that the consumer purchase to bring his inventory up to a smaller quantity on deal. Hence the consumer will purchase a smaller quantity of any brand on deal, and not make deal purchases of any brand given a smaller current inventory level.

The implication suggests that there will be positive cross-price elasticity between the price of one brand and the sales of another brand. This is supported by Blattberg and Wisniewski's (1989) results for flour, margarine, bathroom tissue, and tuna fish. The implication also indicates that with higher dealing in the market, the price elasticities for individual brands will be lower. The implication is weakly supported by Bolton's (1989) results for three different brands each of bleach, ketchup, tissue, and waffles. Bolton proposes that her results may be weak because they are estimated after the effect of features and displays on sales has been removed, whereas price reductions are correlated with the presence of features and displays.

**Implication 2:** Probability of deal response on the favorite brand should not be affected by the probability of deals on any other brand. However, the probability of deal response on less preferred brands should decrease as the probability of deals on other brands increases.

Probability of deal response is defined as the proportion of deals on which a consumer makes a purchase out of all the deals offered for that brand. Figure 1 shows the probability of deal response for a brand as a function of the magnitude of dealing for a competing brand.\(^8\)

While the second part of Implication 2 is quite intuitive, the first part is nonintuitive. We can see from Figure 1 that the probability of deal response on the favorite brand is not affected by the probability of deals on a competing brand. This occurs because of Lemma 3 which states that the consumer will not pass up purchasing at the lowest effective price irrespective of his current inventory (see Appendix 4). Here the lowest effective price is the deal price of the favorite brand. Hence the consumer responds to every deal on the favorite brand, even if he purchases only a minimal quantity. Thus, while higher deal frequency of less preferred brands might affect the amount of stockpiling

\(^7\) The off-deal prices of brands \( w \) and \( x \) would be 1.1 and 1.2 respectively, but they do not matter, since these brands will not be purchased off-deal if the favorite brand is also available off-deal. As mentioned earlier, we are assuming that there are no stockouts.

\(^8\) The functions are not smooth because of the discrete modelling procedure.
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The probability of deal response on less preferred brands decreases as the probability of deals on a competing brand (the favorite brand in the figure) increases. This happens because with higher dealing in the market consumers are more likely to have too much inventory to respond to some deals on these brands.

**Holding Cost:** Manipulation of holding cost.

*Implication 3:* As the holding cost increases, the probability of responding to deals on less preferred brands should increase.

Higher holding costs imply that consumers should buy smaller quantities on deal. Thus consumers will be less likely to be stocked up from deals on the favorite brand and more likely to respond to deals on less preferred brands. This implies that deals on less preferred brands will do better in product classes with higher holding costs.

*Implication 4:* Frequency of purchasing, share of quantity purchased off-deal, and share of off-deal purchases should increase as the holding cost increases.

In Figure 2 we plot the frequency of purchasing, share of quantity purchased off-deal and share of off-deal purchases for the favorite brand for a range of holding costs. We can see that as the holding cost increases, frequency of purchasing, share of quantity purchased off-deal, and share of off-deal purchases increase.

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*Frequency of purchasing of brand i:* percentage of weeks in which purchases of brand i are made.

*Share of quantity purchased off-deal for brand i:* total quantity purchased off-deal of brand i over time/total quantity purchased of brand i in the same time period.

*Share of off-deal purchases for brand i:* number of off-deal purchases for brand i/total number of purchases for brand i.
Implication 4 holds because higher holding costs imply that consumers should buy smaller quantities on deal, and make a larger number of purchases. While consumers may be more likely to respond to deals on less preferred brands (implication 3), the net effect of an increase in holding cost is to make it more likely for consumers to purchase off-deal. Consequently, the share of off-deal purchases increases, and the share of quantity purchased off-deal also increases. The curves flatten out when the holding cost is so high that the consumer purchases the same quantity on-deal and off-deal.

4.2. Market Level Implications

4.2.1. Overview. The model is used in a Monte-Carlo simulation to analyze the normative effect of deal probability and brand loyalty on nonpromotion baseline sales and the post-promotion dip.

We assume that there are three brands in the market. \( v \) represents the consumer's favorite brand, \( w \) represents the consumer's next favorite brand and \( x \) the least preferred brand. The actual deal and off-deal prices for all three brands are the same. However, because of brand preferences, the substitution cost for brand \( v \), \( s_v = 0 \), \( s_w = 0.1 \) and \( s_x = 0.2 \), so that the effective deal and off-deal prices are as follows: \( R_v' = 0.5 \), \( R_w' = 0.6 \), \( R_x' = 0.7/(\text{deal prices}) \); \( R_v = 1.0 \), \( R_w = 1.1 \), \( R_x = 1.2 \) (off-deal prices). The magnitude of the \( s \)'s reflects the level of brand loyalty.

We assume that all consumers do not have the same favorite, second favorite and least preferred brands.\(^{12} \) \( v \), \( w \), \( x \) represent the three brands \( a \), \( b \), \( c \).

\(^{12}\) However, we assume that the substitution cost for the second favorite and least preferred brands is the same across all consumers.
For 25% of the consumers, favorite brand = a, 2nd favorite = b, least preferred = c. 
For 15% of the consumers, favorite brand = a, 2nd favorite = c, least preferred = b. 
For 20% of the consumers, favorite brand = b, 2nd favorite = a, least preferred = c. 
For 15% of the consumers, favorite brand = b, 2nd favorite = c, least preferred = a. 
For 10% of the consumers, favorite brand = c, 2nd favorite = b, least preferred = a.

We assume that the initial inventory for each of the six kinds of consumers is distributed according to the steady state probability of being in any given inventory state \( k (= \pi_k) \), that \( \alpha \) is 0.998 per week, the holding cost is 0.1, and consumption rate is 1 unit per week.

Prices are generated from a Monte-Carlo simulation for 100 weeks, and the model is used to compute purchase behavior. The resultant purchases are aggregated to trace sales time series for the three brands. We analyze the effect of deal probability and brand loyalty on nonpromotion baseline sales and the post-promotion dip.

To check for interaction effects among the variables, we did all possible simulations for many different values of deal probability, brand loyalty, distribution of brand preference, and holding cost. The same directional results were obtained, even though the magnitude of the results varied.

Nonpromotion baseline sales are defined as the total off-deal sales for a brand averaged over all weeks when the brand is not offered on promotion. Post-promotion dip is defined as a lowering of the nonpromotion sales (relative to the baseline) in the week(s) immediately following a promotion.

4.2.2. **Effect of deal probability on nonpromotion baseline sales and the post-promotion dip.** Figure 3a shows sales per week for brand 'a' when deal probability equals 0.05. We can see from Figure 3a that when the deal probability is increased from 0.05 to 0.25 the sales per deal week decrease from 3.35 to 1.24. This is as one would expect. Since consumers have expectations of more deals on many brands, they stockpile less on deals, and the sales per deal week decrease. Figure 3b shows the corresponding results for the category as a whole (i.e. total of all three brands).

Interestingly, sales in off-deal periods also decrease, lowering the nonpromotion baseline sales. One can see from Figure 3a that when deal probability for brand 'a' is increased, the nonpromotion baseline sales decrease from 0.25 to 0.08. This happens because with less time between deals, consumers are able to make a larger proportion of their purchases on deal. Hence, nonpromotion purchases decrease.

It seems strange at first that both the sales per deal week and the sales per off-deal period can decrease. However, it seems logical if one realizes that now there are many more deals and that purchase sales per deal period are greater than sales in the off-deal periods.

In Figure 3a when the deal probability equals 0.05, it is easy to observe the off-deal purchase pattern. There is a large gap between deals. Therefore, in most periods when there is no deal, consumers make off-deal purchases and sales for brand ‘a’ in many off-

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13 Holding cost and consumption rate may also be varied across consumers, but heterogeneity across too many parameters may make it difficult to understand what is affecting the results. Also, there will be no difference in the results whether we have 1 consumer who consumes 2 units per week, or 2 consumers who consume 1 unit per week.
14 This is different from Abraham and Lodish's (1987) definition where baseline sales are defined as the sales if no promotions were to occur in the market.
15 This number has to be multiplied by the number of customers in the market to get aggregate unit sales.
deal weeks equal 0.4. The nonpromotion baseline sales equal 0.25, making the post-promotion dip large where sales are 0.16. Thus the post-promotion dip is quite evident.

When the deal probability is increased from 0.05 to 0.25, the gap between deals is shrunk. On most off-deal occasions there are no purchases off-deal. The nonpromotion baseline sales are small (=0.08), the off-deal purchase pattern is difficult to observe, and the post-promotion dip is not evident. With noise in the data (as there is in the real world), it will be even more difficult to observe the off-deal purchasing pattern, and the post-promotion dip will be even less evident. To take an extreme scenario, if deals occurred more often than the off-deal interpurchase cycle, there would be no sales off-deal. Hence there would be no way to observe the off-deal purchasing pattern, and one would not be able to see any post-promotion dip. Figure 3 shows results similar to brand ‘a’ for the category.

In Figure 3a one can see that there is a dip between periods 24 and 35 while there is no deal in this period. This happens because in this period there are deals for brands b and c (see Figure 3b), so that consumers stock up on these deals and do not need to make off-deal purchases of brand a. Thus a multiplicity of acceleration effects due to deals on several brands lowers the baseline sales in all periods, and makes it difficult to observe the post-promotion dip.

One can also see that when deal probability is increased, there is a narrower range of sales, and the sales graph is flatter. In addition, with higher deal probability, even if sales per deal period decrease, sales per deal period as a multiple of sales per off-deal period need not decrease, because sales in off-deal periods are also smaller.

4.2.3. Effect of brand loyalty on nonpromotion baseline sales and the postpromotion dip. As brand loyalty is increased from $s_1 = 0$, $s_w = 0.1$, $s_x = 0.2$ to $s_1 = 0$, $s_w = 0.24$, $s_x = 0.48$, there are more sales off-deal and the nonpromotion baseline sales are raised. This happens because with greater substitution costs, purchases of less preferred brands on deal are smaller, and, as a result, consumers make more purchases off-deal. This allows one to observe an off-deal purchase pattern more easily, and the post-promotion dip for all brands is more evident. (See Figure 4.)

Thus in markets where there is low brand loyalty and many brands are substitutable for each other, off-deal sales will be smaller and the post-promotion dip will be more difficult to observe.

4.2.4. Comments on the expected but unobserved post-promotion dip. Several explanations have been offered for not observing the post-promotion dip including “timing and quantity acceleration balance out,” “retail stores continue to display after a sale has ended,” “there is a segment of consumers shopping from deal to deal causing a sales spike” (Blattberg and Neslin 1990), “acceleration is not important in terms of overall sales” (Neslin 1991), and that “there is no stockpiling (i.e. the promotional bump is due to an increase in consumption)” (McAlister and Totten 1985). Another explanation that one can think of is that consumers can anticipate when the next deal will occur so that sales both before and after a deal are low, making it difficult to observe the post-promotion dip.

The market simulation using our model illustrates that the post-promotion dip will be less evident when deal frequency is higher, and when brand loyalty is lower. We find...
that, in a market with high dealing for a number of brands, a combination of price expectations for many brands, low brand loyalty, stockpiling of competitive brands on deal, as well as stockpiling of the subject brand on deal, may work together so that there are very few off-deal sales and the nonpromotion sales baseline is lowered. With low baseline sales and noise in the data, it will be difficult to observe the off-deal purchase pattern, and to observe any post-promotion dip.
A, B, C respectively denote deals on brand 'a', 'b', and 'c'

\[ h = 0.1; s_r = 0; s_w = 0.1; s_s = 0.2; \]

* Multiply unit sales by number of customers in the market to get aggregate unit sales

**Figure 3b.** Effect of Deal Probability on Nonpromotion Baseline Sales and the Post-Promotion Dip (Category).

Since promotions are frequently offered for many brands in a number of product categories, and consumers buy a variety of brands in these categories, this may be a reason why the post promotion dip is empirically unobservable.

For simplicity we are deriving the implications for three brands. However, both the individual and market level implications will apply for all the brands that the consumer purchases. If there are a larger number of dealing brands that the consumer purchases
\[ s_v = 0 \]
\[ s_w = 0.24 \]
\[ s_s = 0.48 \]

Average Weekly
Sales Off-Deal
= 0.13

Average Weekly
Sales On-Deal
= 1.18

A &notes a deal on brand 'a'
\[ h = 0.1 ; p = 0.25 \]

* Multiply unit sales by number of customers in the market to get aggregate unit sales

**FIGURE 4.** Effect of Brand Loyalty on Nonpromotion Baseline Sales and the Post-Promotion Dip (Brand 'a').

(i.e. the substitution cost is not prohibitively high for these brands), the total probability of a deal in the market would increase. This implies that the probability of a lower effective price would be higher. As a result, the magnitude of the impact of certain implications would be greater (or lesser), but the implications would still hold.
5. Summary and Concluding Comments

Our main objective was to determine how consumers’ uncertain expectations of prices for multiple brands should normatively impact consumer purchase behavior. For this purpose, we developed a model that incorporates consumer preferences and price expectations for multiple brands as determinants of normative consumer purchase behavior. The model demonstrates how commodity purchase policy models recognizing price uncertainty can be adapted to the study of branded goods markets. This is done by collapsing a multivariate distribution of prices across a set of brands into a univariate distribution, allowing direct application of a single-commodity model to the multi-brand case.

We show how the model can be used by an analyst to examine the possible effect of changes in pricing policies and other factors on consumer response to these policies. It is used to explore the normative impact of deal frequency and holding cost on individual purchase quantity, probability of deal response, and share of purchases, for different brands. It is also used in a Monte-Carlo market simulation to analyze the normative effect of deal probability and brand loyalty on nonpromotion baseline sales and the post-promotion dip.

Some of the individual level implications are more intuitive than others. For example, Implication 1 which states that, as the probability of dealing on any brand increases, consumers should not make deal purchases of any brand given smaller amounts of current inventory. Other implications are quite nonintuitive. For example, Implication 2 which states that the probability of deal response on the favorite brand should not be affected by the probability of deals on any other brand, even though the probability of deal response on less preferred brands should decrease with an increased probability of deals on the favorite brand. We also find that even when deal probability is changed very little for all brands, since consumers have price expectations for multiple brands, the impact on consumer purchase behavior is large.

This type of analysis cannot be handled by existing marketing models, and is useful for determining the effect of promotion policies and other factors on individual response in multi-brand markets. In addition it provides an economic explanation for observed consumer behavior.

Although the implications are derived to study what the consumer should do when he has price expectations for multiple brands, some of the implications have empirical support. For example, from Implication 1 we can conclude that with higher total dealing in the market, average quantity purchased on deal for any brand should decrease. This is supported by Bolton’s (1989) results for three different brands each of bleach, ketchup, tissue, and waffles.

The market level simulations show that with higher deal probability a multiplicity of acceleration effects due to deals on several brands lowers the baseline sales in all periods, and makes it difficult to observe the post-promotion dip. We also find that a post-promotion dip will be less evident where there is lower brand loyalty. Thus the simulations provide an explanation for the expected but unobserved post-promotion dip.

Empirical research has shown that some consumers have reasonably accurate expectations of deal frequency and sale price (Dickson and Sawyer 1990; Krishna, Currim and Shoemaker 1989). Our results would indicate that if these consumers follow the optimal policy, in product categories with high dealing these consumers might not buy large quantities on promotions. Consequently, retailers may not find deals a good mechanism to reduce their inventory and holding cost, or to defend against threats by new brands.

The results also indicate that in product classes with many deals and low brand loyalty, one should expect some consumers to do a lot of brand switching but not purchase large
quantities on deal. Implication 3 suggests that if retailers have to offer deals on brand other than the premium brands, it would be better to offer them in product classes with higher holding costs.

The market simulation suggests that with price expectations for multiple brands, higher market share brands will benefit and lower market share brands stand to lose. This is because when there are no deals consumers purchase strictly according to brand preference. When there are deals but no price expectations, consumers will stock up on deals for any brand. When there are deals and consumers have price expectations, consumers will stockpile less of the less preferred brand on deal, because they expect the preferred brands to be on sale.

6. Limitations and Extensions

There are of course some caveats to our results. The consumer's process of selecting a brand may be much more complex than the one incorporated within our model. For example, consumer preferences may change over time because of variety seeking, satiation or other reasons. Substitution cost would then need to be modelled differently. For instance, the substitution cost of brand / may be a function of the quantity purchased of brand / thus far. If greater variety seeking and satiation make consumers more responsive to deals on other brands, then they would act in similar fashion to less brand loyalty, lowering the nonpromotion baseline sales, decreasing the average quantity purchased on deal, and making it more difficult to observe the post-promotion dip.

We assumed that the prices of different brands are uncorrelated. If prices of different brands are correlated with a lag (versus independently determined), so that it is more likely that there is a brand on deal at any one time, then consumers will be able to make a larger proportion of their purchases on deal. This would lower the nonpromotion baseline sales, and make post-promotion dips less evident. If prices of different brands are correlated with no lag so that different brands are on deal in the same week, then for the same total deal probability, consumers will find deals in fewer weeks. Hence, consumers would have to make a larger proportion of their purchases off-deal. This would raise the nonpromotion baseline sales, and make post-promotion dips more evident.

In addition, if consumers have a reference price for each brand, then they may not look at the price offered, but at the difference between the actual price and the reference price (Winer 1986). If the reference price is based on the prices of competing brands in the product class, then consumers would be responsive to deals in a narrower price range. This would tend to raise the nonpromotion baseline sales, and make post-promotion dips more evident.

The model assumes a continuous purchase amount variable. Relaxing the assumption would mean that consumers would have to buy smaller or larger quantities than optimal. With most goods being available in multiple package sizes, this change in purchase quantity need not be exceedingly large. However, if there were only two discrete packet sizes, optimal quantity could be substantially different and we would have to modify the model to allow for additional discretization in purchase quantity.

We also assume a constant consumption rate. We could make consumption a function of price and current inventory as done by Assuncao and Meyer (1991) and modify our model accordingly. Alternatively, one could extend their model to the multi-brand scenario by the method described in this paper.

Another assumption that may be relaxed is that consumers are uncertain about the prices of all the brands. Consumers need not be uncertain about the prices for brands they often purchase, e.g. their favorite brand. In this case they may be less prone to respond to deals on less preferred brands than implied by the model, if the favorite brand
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is often on deal. In addition, all consumers may not hold equally strong beliefs about the relative likelihood of future deals, and "uncertainty about their perceptions of uncertainty" (Einhorn and Hogarth 1985) may play a role in influencing their choice behavior, such as by making consumers act more conservatively than would be implied by their own best-guesses as to the likelihood of future deals.

Also, consumer may become more knowledgeable and less uncertain about the price distribution of a brand the more they purchase it. Using the method advocated by Bertsekas (1976), one could incorporate learning in the model. Another important extension would be to model how predicted consumer behavior should affect a firm's dealing strategy and to work out a joint equilibrium between the consumer and the manufacturer.

Our work suggests that consumer expectations of prices for multiple brands should be an important determinant of consumer response to price promotions. Future research should examine how research findings change when some of the assumptions are relaxed.18

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18 This paper was received in February 1990 and has been with the author 9 months for 3 revisions. Processed by Robert Meyer, Area Editor.

Appendix 1

**LEMMA 1.** \( A_k(a) = 0 \) for some \( K \leq \mu_0/h \).

**PROOF.** This result follows from a corollary established by Golabi (1985, p. 582).

It implies that the consumer should never buy forward for more than \( K \) periods. In addition, the lemma also implies that the maximum inventory level that the consumer should ever have is \( K\bar{q} \). Lemma 1 is also similar to a proposition of Assuncao and Meyer (1990).

**LEMMA 2.** In steady state, the consumer has an inventory \( k\bar{q} \) where \( k \) is an integer in \([0, 1, 2, \ldots, K - 1]\) for some integer constant \( K > 0 \).

In words, Lemma 2 states that the consumer will maintain an inventory that is an integral multiple of the consumption in one period.

**PROOF.** In steady state, the consumer has an inventory \( k\bar{q} \) where \( k \) is an integer in \([0, 1, 2, \ldots, K - 1]\) for some integer constant \( K > 0 \).

Let us call these states \( I_0, \ldots, I_{K-1} \), where \( I_k \) means that the consumer has inventory \( k\bar{q} \).

According to the optimal policy, the consumer always buys to bring the inventory level up to \( k\bar{q} \) for some integer \( k \geq 0 \), provided that the current inventory \( I(k+1)\bar{q} \). Otherwise, the consumer does not buy.

Given a fixed finite starting inventory, the consumer must eventually buy, since he consumes \( \bar{q} \) each period. After the first purchase, the inventory is always an integer multiple of \( \bar{q} \). But from Lemma 1, \( A_k = 0 \). Therefore according to the optimal policy, the consumer will never buy to bring his inventory level to \( K\bar{q} \), since the price can never be negative.

\( \therefore k < K \) at all times. Q.E.D.

Appendix 2. \((\pi_k, the Probability of Being in Inventory State I_k)\)

\[
[1 - G(A_1)]\pi_0 + [1 - G(A_1)]\pi_1 = \pi_0
\]

\( \iff (G(A_1))\pi_0 = (1 - G(A_1))\pi_1 \)

\( \iff \pi_1 = [G(A_1)/(1 - G(A_1))]\pi_0 \)
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Also, $- [G(A_1) - G(A_2)][\pi_0 + \pi_1] + \pi_1 = [1 - G(A_2)]\pi_2$.

Substituting for $\pi_1$,

$\Rightarrow \pi_0[- [G(A_1) - G(A_2)][(1 - G(A_1) + G(A_2))] + G(A_2)/[1 - G(A_1))] = [1 - G(A_2)]\pi_2$

$\Rightarrow \pi_0[- [G(A_1) - G(A_2)]/(1 - G(A_1) + G(A_2))] + G(A_2)/[1 - G(A_1)) = (1 - G(A_2))\pi_2$

$\Rightarrow \pi_2 = G(A_2)\pi_0/(1 - G(A_1))(1 - G(A_2))$.

Suppose

$\pi_k = G(A_k)\pi_0/[(1 - G(A_1))(1 - G(A_2))\cdots(1 - G(A_k))]$ for all $k = 1, 2, \ldots, j$.

From $I$,

$\pi_j - [G(A_j) - G(A_{j+1})][\pi_0 + \pi_1 + \cdots + \pi_j] = [1 - G(A_{j+1})]\pi_{j+1}$.

(A.1)

However,

$\pi_0 + \pi_1 + \cdots + \pi_j = \pi_0\{1 + [G(A_1)/(1 - G(A_1))] + [G(A_2)/(1 - G(A_1)))(1 - G(A_2))] + \cdots + [G(A_j)/(1 - G(A_1))\cdots(1 - G(A_j))]\}

= \pi_0\{(1 - G(A_1)) + G(A_2)]\{(1 - G(A_2))\cdots(1 - G(A_j))] + [G(A_3)(1 - G(A_2))\cdots(1 - G(A_j))] + \cdots + [G(A_j)]/[(1 - G(A_1))\cdots(1 - G(A_j)]\}

= \cdots

= \pi_0/(1 - G(A_1))\cdots(1 - G(A_j)).

Substituting for $\pi_0 + \pi_1 + \cdots + \pi_j$ in equation (A.1),

$\Rightarrow G(A_j)\pi_0/[(1 - G(A_1))\cdots(1 - G(A_j))] - (G(A_j) - G(A_{j+1})\pi_0/[(1 - G(A_1))\cdots(1 - G(A_j))] = [1 - G(A_{j+1})]\pi_{j+1}$

$\therefore \pi_{j+1} = G(A_j)\pi_0/[(1 - G(A_1))\cdots(1 - G(A_{j+1}))].$

Hence proved by induction for all $1 \leq j \leq K - 1$.

But from $I$ and because $\pi_0 + \pi_1 + \cdots + \pi_j = \pi_0/(1 - G(A_1))\cdots(1 - G(A_j)),$

$\pi_{K-1} = G(A_{K-1})\pi_0/[(1 - G(A_1))(1 - G(A_2))\cdots(1 - G(A_{K-1}))] = p_{K-1}$

$\Rightarrow \pi_0 = [(1 - G(A_1))(1 - G(A_2))\cdots(1 - G(A_{K-1}))]$

$\therefore \pi_k = [(1 - G(A_{K-1}))(1 - G(A_{K-2}))\cdots(1 - G(A_k+1))]/G(A_k).$ Q.E.D.

Appendix 3. Expected Purchase Quantity of Brand i

Probability of purchasing $v$ units of brand $i$ is obtained as the probability of being in inventory state $k - v$, $i$ is the brand chosen, and the minimum effective price lying between $A_k$ and $A_{k-1},$ summed over all values of $k$ between $v$ and $K$.

Thus the probability of purchasing $v$ units of brand $i$

$= \sum_{k=v}^{K} \pi_{k-v}P(A_k(\alpha), A_{k-1}(\alpha)).$

Similarly, the expected purchase quantity of brand $i$ on any given purchase occasion is $\sum_{v=1}^{K} (v \times$ probability of buying $v$ units)

$= \sum_{v=1}^{K} \sum_{k=v}^{K} v\pi_{k-v}P(A_k(\alpha), A_{k-1}(\alpha)) = \sum_{k=1}^{K} \left[ P(A_k(\alpha), A_{k-1}(\alpha)) \sum_{v=1}^{K} v\pi_{k-v} \right]$. 
Appendix 4

**LEMMA 3.** The consumer will not pass up purchasing at the lowest possible effective price irrespective of his current inventory.

**Proof.** Let us define cutoff inventory for a brand as the consumer’s current inventory level such that he should not purchase the brand irrespective of the price for the brand. Mathematically a cutoff inventory exists for every brand. If the minimum effective price for brand \( j \) falls in between \( A_1(a) \) and \( A_j-1(a) \), the cutoff inventory is \( J_q \). Let’s assume that the lowest possible effective price among all brands is for brand \( w \), and the cutoff inventory for brand \( w \) is \( wq \). However, the consumer will never have this inventory at the time of purchase, if he follows the optimal policy. This occurs because the maximum inventory that the consumer should ever have after purchase is \( wq \). But, after bringing his inventory level up to \( wq \), the consumer consumes \( J_q \) in this period. Therefore, in the next period, his inventory is less than \( wq \), and he can again make a purchase.

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