Pricing of mall services in the presence of sales leakage

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Abstract

For a shopping mall, sales leakage occurs when consumer purchases facilitated by the mall are finalized outside it. These sales include, for example, catalog orders filled at the leased premises in a physical mall; For an Internet mall, they include the ones consumers make on an on-line store’s website after learning about the store from an Internet mall website. While these sales are difficult to track in the physical mall, Internet malls like Yahoo can track them by placing cookies on consumers when they visit the mall. The challenge for a mall owner then is to design an appropriate pricing model which takes sales leakage into account. In fact, Yahoo currently uses an All-Revenue-Share Fee with Yahoo collecting from on-line stores a share of all sales revenue, regardless of whether the purchase was made through the mall or directly from the store’s own URL. We explore this new All-Revenue-Share Fee model, compare it with the commonly used Fixed Fee model and the two-part tariff model, and identify the model with the highest profits for the mall under different conditions. We suggest that although an All-Revenue-Share Fee is appealing for Internet malls due to its ability to capture sales leakage directly, it may cause the stores to refrain from joining the mall in certain circumstances. Thus, in certain situations charging a fixed monthly fee can actually be more profitable for the mall versus the All-Revenue-Share Fee model. We also examine how mall and product category characteristics as well as market expansion affect the optimal pricing strategy. We show that a mall should price discriminate across product categories, not just by charging different amounts of fees, but by using different pricing models. Our research provides many managerial implications on how to price over time.

Introduction

Shopping malls charge rents to tenant stores for operating in the mall. Rents are usually set along two dimensions—a fixed monthly rent and a percentage \textit{overage} based on sales revenue of the tenant store. Similar pricing models are being used by Internet malls, which host on-line stores and charge fixed monthly fees and/or revenue share fees based on sales revenue generated through the malls.

Problems with percentage rent based on sales revenue generated through the mall arise when purchases originating within the mall are made outside it. This phenomenon which exists in the context of physical and Internet malls is called \textit{Sales Leakage}. In the physical mall context, it refers to catalog purchases made by consumers at the leased premises or from home, after examining the physical products in the stores. These sales are not subject to overage since they are finalized outside the mall through catalog orders. Ever since stores began distributing catalogs, fights about percentage rent have occurred between mall owners and tenants. However, the phenomenon is more in the limelight now as Internet commerce has grown and the list of traditional shopping mall stores selling merchandise through websites has grown. \texttt{www.macys.com}, \texttt{www.Sears.com}, \texttt{www.Gap.com}, \texttt{www.VictoriasSecret.com}, \texttt{www.EddieBauer.com} and \texttt{www.Nordstrom.com} are just a few examples. When consumers choose to buy from a store's website after examining the product in the leased premises, or when consumers choose in-store pickups for their on-line orders, the transactions take place outside the mall. Therefore, the mall cannot include them in gross sales of stores (as defined in a typical retail lease) when calculating percentage rents even though both types of sales are facilitated by the mall. These are serious issues that will become even more important as Internet commerce expands and Internet revenues constitute a significant proportion of the tenant store's total sales.

A major concern of mall landlords, therefore, is the fine-tuning of the definition of gross sales so that sales facilitated
by the mall, but made outside it, may be included. The traditional concept of percentage rents, and certainly the method of determining gross sales, needs to be re-examined and updated to meet this challenge. However, the problem for the mall is one of tracking and policing these catalog and Internet orders, a non-trivial task given that stores wish to find ways of defying such tracking to occur.

While physical malls are still struggling with expanding percentage rent clauses, Internet malls which host on-line stores already have a device in place to track sales leakage. Specifically, sales leakage in the Internet mall context refers to the purchases consumers make directly from the on-line store’s website, or through third-party referrals after visiting the Internet mall’s website. By using cookies, Internet malls, such as Yahoo!Shopping, are able to track these sales and charge a revenue share fee based on a broader notion of gross sales, an All-Revenue-Share Fee (ARSF). While not a perfect tracking mechanism, Yahoo!Shopping switched from a Fixed Fee (FF) model to this All-Revenue-Share Fee model in October 2001.

The objective of this paper is to study the optimal pricing strategies for a shopping mall when there is sales leakage. We analyze the strategic behavior of an Internet mall and two existing on-line stores that are considering the option of adding mall presence. By adding mall presence, an on-line store gets exposure from consumers shopping in the mall while keeping its own URL for customers to access directly. However, this creates the problem of sales leakage. Consumers may first learn about the on-line store when browsing the mall. Once these consumers have established a relationship with the store at their first visit through the mall (with or without purchase), they may later purchase from the store directly by typing the store’s own URL. Alternatively, consumers may click through the links in targeted promotion or advertising emails sent by the store. They may also purchase through third-party referral.

With the conventional revenue share fee which is based on the sales revenue generated through the mall, the mall would lose revenue where there is leakage. In this paper, we take the first step towards examining an alternative All-Revenue-Share Fee model which captures sales leakage directly and compare it with the commonly used FF model which takes into account sales leakage indirectly. Because of ARSF’s ability to incorporate sales leakage directly, on the face of it, mall owners and managers have every reason to prefer ARSF to the FF model. Our results, on the other hand, indicate that although leakage is a very serious concern for many mall landlords, efforts to claim a share directly through an ARSF may lead to a loss of tenant stores under certain conditions. We show that a FF, an ARSF, and a linear combination of the two (i.e., a two-part tariff) may all be used in different contexts.

We also contribute to the theoretical literature of pricing strategies of shopping malls by showing the effect of endogenizing stores’ mall joining decisions on the optimal pricing strategies of the mall. Further, our paper adds to the price discrimination literature by proposing that a mall can price discriminate across different product categories not only by charging different amounts of fees, but also by using different fee structures, i.e., Fixed Fees in some product categories and revenue share fees in others. As we show, these results stem from varying quality differential between stores in different product categories. We offer many other managerial implications for pricing mall services across product categories and over time.

The remainder of the paper is organized as follows. We first provide some institutional details of sales leakage in physical and Internet malls. In Section ‘Literature review’, we present a review of related literature. Sections ‘Basic model setup’ and ‘Equilibrium analysis’ are devoted to theoretical analyses of the two-part tariff model, the FF model and the ARSF model in the basic setting. In Section ‘Model discussion’, we further examine the basic model and show how the characteristics of a product category, the possibility of market expansion, and non-zero setup costs of joining stores affect the mall’s pricing decisions. We then discuss managerial implications of the model. Finally, we discuss limitations and end with recommendations for future research.

Sales leakage

In this section, we provide institutional details of sales leakage in the context of physical and Internet malls.

A battle between landlords and tenants in the physical mall

The leakage of sales is not new to physical mall owners. In the history of shopping center leases, there has always been a battle between landlords and tenants over sales leakage through catalog sales. In the case of catalog orders made from mall stores, the resolution for many shopping malls has been to include them in gross sales when calculating percentage rents. However, the amount of leakage becomes much larger as more and more stores start to establish their own websites on-line. This is disturbing to owners of leased retail properties since the existence of the stores’ websites enables them to free-ride their Internet sales off of the physical stores in the mall—many consumers view merchandise in physical stores, compare it across stores, and place their orders on-line. In a more extreme case, some retail stores could even seek to downsize their physical stores into little more than catalog showrooms and rely mostly on Internet sales.

To protect their percentage rent receipts, some shopping center owners have attempted counterattacks. During the Christmas 1999 season, the owner of the Galleria Mall in St. Louis, Missouri prohibited its 170 tenants from promoting websites in their stores. The ban applied to signs, posters, dotcom addresses on shopping bags, window stickers and other materials that promoted Internet sales. Although most tenants ignored the ban, a few threatened lawsuits, which prompted a quick retreat by the mall owner (Snively 2000). More recently, the United Investors

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1 In the on-line environment, consumers may establish a relationship with an on-line store by creating an account or by simply providing some kind of contact information such as email addresses.
Realty Trust’s\textsuperscript{2} standard rental lease clause requires Internet sales made from the store or from any remote Internet location within 150 miles of the leased premises to be included in the tenant’s gross sales (Karp 2001). Many other shopping malls have made attempts to broaden the notion of gross sales for calculating the percentage rent by combining the percentage rent clause with a radius clause.\textsuperscript{3} The Internet sales within a radius of a physical store are identified based on information that the tenant receives and retains from on-line consumers. On-line customers usually pay by credit card and provide a billing address with a zip code. Once the address and zip code of an on-line consumer are known, one can determine whether that consumer resides within a specified geographic radius of a particular store. This is clearly not an easy proposition. Next, we discuss the sales leakage problems in the Internet mall context.

**Internet malls and tracking of sales leakage**

Similar to physical malls, Internet malls have numerous member stores and derive revenue by collecting fees. By Internet mall, we specifically mean the Internet version of physical malls. This self-contained type of Internet mall is different from a search engine which provides links to on-line stores’ websites based on the product category one searches on. Once an on-line store decides to join an Internet mall, the mall hosts the HTML pages and shopping cart technology on its servers and offers design tools. The mall also provides various ways to promote the store in order to attract more consumers.

However, it is very important to note that even after an on-line store joins an Internet mall, the store can still keep the URL of its own website for consumers to access directly. The store can also choose to advertise on different websites.\textsuperscript{4} Similar to the physical mall context, these “outside” options create the problem of sales leakage for Internet malls. For example, Swoonshop (which sells designer handbags) is a member store of Yahoo!Shopping. A consumer may first become aware of Swoonshop and get interested in the Gucci 1073 (Cappuccino Brown) handbag while browsing Yahoo!Shopping. S/he may purchase through the mall at the first visit. On the other hand, a few days later, s/he may decide to go directly to www.Swoonshop.com to purchase exactly the same handbag by typing the store’s own URL or by clicking through the links in Swoonshop’s advertising or promotion emails.

Alternatively, this consumer may click on a link provided by www.Guideall.com which is a website that Swoonshop advertises on (see Fig. 1 for these examples). Fortunately, Internet malls can use cookies to track sales leakage. For instance, Yahoo!Shopping places a cookie when a consumer lands on Swoonshop through Yahoo. The cookie is valid for 30 days. Every order placed by this consumer within that 30-days time frame will belong to the cookie, irrespective of whether the sale is made directly through Yahoo, or through the store’s own website (www.Swoonshop.com), or through referrals from other websites (e.g., www.Guideall.com). By using cookies, Yahoo can now capture sales leakage by charging a revenue share fee based on a broader sales revenue concept (i.e., an All-Revenue-Share Fee).

Besides Yahoo!Shopping, there are many other Internet malls that host on-line stores and collect fees. Table 1 provides some examples of Internet malls and pricing models that are currently being used by these malls. As one can see from the table, except Yahoo which now uses the new ARSF model as part of its two-part tariff, most Internet malls charge stores some form of a fixed monthly fee and/or a revenue share fee based on sales made through the mall only. We find that the pricing model not only varies from mall to mall but also changes over time. For example, in October 2001, Yahoo!Shopping switched from a FF model to a 3.5 percent ARSF model which was followed by the most recent two-part tariff model (that includes an ARSF). ZShops at Amazon, on the other hand, increased its FF from $9.99/month to $39.99/month without changing its basic fee model.

Since tracking devices are available in the Internet mall setting and ARSF model is being used, we choose the Internet mall as the context of studying this new pricing model. However, the models and implications of this paper can also be applied to the physical mall context.

**Literature review**

In economics and marketing, there are two streams of literature where price discrimination with Fixed Fee and revenue share fee has been studied. In this section, we review the literature on share contracts and distribution channels, and discuss how our model relates to each of them.

**Literature on share contracts**

Revenue sharing occurs in many contexts including physical shopping mall leases, franchising and salesforce compensation.

<table>
<thead>
<tr>
<th>Examples of Internet malls and their pricing models</th>
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<tbody>
<tr>
<td><strong>Internet malls</strong></td>
</tr>
<tr>
<td>Yahoo!Shopping</td>
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<tr>
<td>ZShops@Amazon</td>
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<tr>
<td>iMall</td>
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<td>InternetMall</td>
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<td>SiMall</td>
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\textsuperscript{2} United Investors Realty Trust is a Texas real estate investment trust which acquires, develops and operates neighborhood and community shopping centers in the “sunset” region of the United States. The current portfolio consists of 28 properties aggregating to approximately 3.1 million square feet in TX, AZ, FL, and TN.

\textsuperscript{3} Many retail shopping mall leases contain “radius clauses,” which prohibit the tenant from operating a competing business within a certain radius of the shopping mall. If the tenant violates the radius restriction, the landlord may be entitled to include, within the definition of gross sales in the percentage rent provision, the gross sales generated from the tenant’s store that violates the radius restriction (Murray 2001). In this case, a store’s website is considered as a competing business as far as the consumers within a certain area of the leased premises are concerned.

\textsuperscript{4} A website advertises on-line stores by providing links but does not have the stores “physically” located on its server.
Previous research on physical malls has both a theoretical stream which models the benefits and costs of locating a store in a mall versus at a stand-alone location and a combination of theoretical and empirical papers focusing on how rent contracts are formulated. In the first stream of research, Stahl (1982) studies models of store location choice and argues that stores may have an incentive to cluster if consumers are attracted to locations where a large variety of products are available. Dudey (1990) on the other hand shows that stores co-locate to attract more consumers by facilitating price comparisons. He shows that when consumers incur positive search costs, there exist conditions under which stores agglomerate in equilibrium. In a more recent paper, González-Benito et al. (2005) model the asymmetric spatial competition and empirically confirm that there exists greater spatial rivalry within store formats than between store formats. This implies that when selecting locations, retailers should realize that although competitive interaction is more intense within store formats, the consequences of inter-format rivalry should not be dismissed. The second stream of research on physical malls centers on the mall’s choice of pricing mechanism taking the stores’ participation decisions as exogenous. Some researchers have hypothesized that, everything else constant, shopping malls use percentage rent as an alternative to fixed rent and that the two should vary inversely across leases to yield the same total occupancy cost (e.g., Benjamin et al. 1990). Wheaton (2000) disagrees with these views and argues that percentage rental payments, in fact, help tenants control “opportunism” from the landlord’s side. They ensure that landlords act in the interest of the stores when expanding, altering, or re-letting space in a shopping center.

Revenue sharing is not limited to shopping center leases. Sharing arrangements are often used in franchising contracts. Although compared to the franchisor–franchisee relationship, the landlord–tenant relationship is looser in the sense that the landlord neither produces nor sets standards of production, theories of share contracts in franchising can shed light on the optimal shopping mall leases. Franchise contracts typically contain two forms of payment from franchisee to franchisor: the initial franchise fee and the ongoing royalty payment. Economic analysis and empirical research yield two different predictions about how these payments should relate to one another (Kaufmann and Dant 2001). Some researchers believe that franchises are sold so that their prices will be competitive and leave franchisees with normal profits. Under this view, a royalty rate is set to achieve the correct incentives for both parties’ growth whereas the franchise fee is set to extract the remaining surplus and they are negatively related (Lal 1990a; Mathewson and Winter 1985). However, empirical support for the above argument has not been convincing.5 Using data of 548 franchisors in the United States in 1986, Lafontaine (1992) identifies variables that acts in the same direction on both franchise fee and royalty rate and argues that franchisors do not use the franchise fee to extract rents but

5 For a comprehensive review of the empirical literature on incentive contracting in franchising, see Lafontaine and Slade (1997, 2001).
rather to collect reimbursement for recruiting and training costs. Lafontaine and Kaufmann (1994) study McDonalds’ franchisees and also find that adjustments to the percentage royalty rate and franchise fee move in the same direction over time. This competing view indicates a positive relationship between the two components of franchise contracts.

Another type of share contract which has been studied extensively is the salesforce compensation plan. Appropriate compensation system design is especially critical to a corporate’s strategy implementation (Terborg and Ungson 1985). The basic premise of most salesforce compensation plans is that the efforts of each salesperson should be adequately rewarded. The three major schemes of providing financial incentives to a salesperson includes straight salary, straight commissions and a combination of base salary plus incentive pay in the forms of commissions and/or bonuses. Basu et al. (1985) propose the optimal curvilinear compensation plan which maximizes the firm’s profit, taking into account the likely behavior of the salesperson in response to the compensation plan. Raju and Sririvasan (1996), on the other hand, argue that the minimal loss in optimality combined with the substantial ease of implementation provides a justification for managers to consider a quota-based as opposed to a curvilinear compensation plan. A quota plan pays a fixed salary supplemented by commission income, which is a pre-specified fraction of the dollar sales exceeding the quota.

The previous literature on franchising and salesforce compensation provides some explanations for the existence of revenue sharing and insights on the optimal share contract. However, a major difference between our model and the models in the existing contracting literature lies in the fact that the ARSF we examine is based on a broader notion of sales revenue. Second, the focus of the previous literature on physical shopping mall leases has been to find the optimal fee structure given the stores’ joining decisions with the assumption that the stores have no outside options. Since we study the new ARSF model rather than the conventional revenue share fee in this paper, it is not so obvious that joining is always profitable for the stores. Therefore, it is important for us to explicitly model the stores’ joining decisions and to explore how they affect the mall’s optimal pricing strategies. By doing so, we provide a more holistic framework for examining mall–store relationship.

**Literature on distribution channels**

In the marketing science literature on distribution channels, the optimal pricing strategies of a manufacturer in the form of a lump sum transfer, a wholesale price, or a two-part tariff have been studied extensively (e.g., Ingene and Parry 1995a, 1995b; Iyer 1998; Raju and Zhang 2005). Seminal papers include McGuire and Staelin (1983) which restricts the contract between a manufacturer and a retailer to a constant per-unit fee. On the other hand, the literature focusing on within-channel competition and coordination has shown that a quantity discount (e.g., Jeuland and Shugan 1983) and a two-part tariff (e.g., Moorthy 1987) can increase channel profitability. Coughlan and Wernerfelt (1989) synthesize these two approaches by simultaneously considering multi-manufacturer competition and more sophisticated within-channel transfer pricing policies, and show that two-part transfer prices are more profitable than one-part prices. More recently, Desiraju (2004) presents a static model to examine the impact of distributors’ limited liability on a manufacturer’s decision to use exclusive territories. He shows that under severe limits on liability, it is more profitable to provide proper incentives via intrabrand competition since the relatively low Fixed Fees do not make it profitable for manufacturers to induce the exclusive distributor to work diligently.

In addition to these theoretical studies, empirical analyses have been conducted to investigate channel members’ profits (e.g., Ailawadi and Harlam 2004; Farris and Ailawadi 1992). More recently, researchers have taken initiatives to combine theoretical and empirical analyses when studying the manufacturer–retailer relationship. For example, Ailawadi et al. (2005) use Procter & Gamble’s value pricing as a context for testing whether actual competitor and retailer response to a major policy change can be predicted using a game-theoretic model. They find that the dynamic game-theoretic model calibrated with empirical estimates of demand parameters has significant predictive power. Kopalle and Lehmann (2006) develop an analytical model to study optimal advertised quality, actual quality, and price for a firm taking into account consumer satisfaction and long-term market potential. Then by examining how the model implications match deceptive advertising cases using data from FTC and an experiment, they show that consistent with model implications, there is more deception for unknown firms and for firms with more satisfied consumers.

There are some parallels between our model structure and the manufacturer-retailer dyad. In the literature concerning the latter, the transfer from the retailer to the manufacturer (in the form of a lump sum fee, a wholesale price, or a two-part tariff) is first set by the manufacturer and then the price to the consumers is set by the retailer. In our model, on-line stores act as competing manufacturers deciding whether or not to sell through the mall which acts as a retailer. More specifically, in our model, service fee is first decided by the retailer (i.e., the mall) and then the prices to the consumers are set by the manufacturers (i.e., on-line stores). In the next section, we describe this model.

**Basic model setup**

Our model analyzes the strategic behavior of an Internet mall acting as a monopoly and two existing on-line stores considering the option of adding mall presence. The stores are in the same product category but differ in consumer awareness with one having higher consumer awareness than the other. There are three stages in the model. In the first stage, the Internet mall announces its fee. After observing the mall’s pricing decision, in the second stage, the on-line stores decide whether to join the mall or not. If an on-line store decides not to join the mall, it needs to incur costs to host its own website (by either paying the service from a web hosting company or by handling hosting internally) and to attract consumers. For a store that chooses to

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6 Kopalle et al. (1999a, 1999b) use a similar structure.
join the mall, the Internet mall provides hosting and traffic generation services for a fee.\footnote{We first assume that when a store joins the mall, it completely relies on the mall to generate traffic. This assumption will be relaxed later on.} By adding a mall presence, an on-line store gets exposure from consumers shopping in the mall while keeping its own URL for consumers to access directly. In the third stage, the on-line stores simultaneously choose their prices to consumers.\footnote{An alternative way to setup this model is to have a two-stage model where the mall’s pricing decision and the stores’ joining decisions are made simultaneously. In terms of solving the two-stage model, we can first derive the stores’ revenue function. Then for each of the four subgames we can maximize the stores’ profits by comparing the profits for the mall across the multiple equilibria obtained in all four market conditions.}

**Consumer awareness**

To study the stores’ benefits of joining the mall, it is essential to model the consumer awareness structure and how it changes after the stores join. The total number of consumers in the market is assumed to be fixed.\footnote{It can also be the case that the number of consumers leaving the market in any given period equals the number of consumers joining so that the total population is constant.} Each consumer in the market buys one unit of the product in each period if the price is less than the consumer’s reservation price. In the basic model that we start with, we assume that consumers know about one of the two stores or both. Later on, in Section ‘Equilibrium analysis’, we include a segment of consumers who are not aware of either of the stores to examine the effect of market expansion. We refer to the consumers who are only aware of the high (low) awareness store as *Aware-High-Only (Aware-Low-Only) Shoppers* and consumers who are aware of both stores as *Aware-Both Shoppers*. Note that awareness of the high (low) awareness store comes from both *Aware-High-Only (Aware-Low-Only) Shoppers* and *Aware-Both Shoppers*.

**Consumer baseline awareness of the stores**

We define *Baseline Exclusive Awareness* \((\alpha_H \text{ or } \alpha_L)\) as the proportion of consumers who are only aware of one on-line store when no stores join the mall.\footnote{\(\alpha_H \text{ and } \alpha_L\) can also be expressed as the probabilities of consumers knowing a store, conditional on knowing at least one store. If we use \(\delta_H(\delta_L)\) as the probability of a consumer being aware of the high (low) awareness store, the baseline exclusive awareness of the high awareness store can be represented as \((\delta_H(1 - \delta_L))/1 - (1 - \delta_H)(1 - \delta_L))\) and that of the low awareness store as \((\delta_L(1 - \delta_H))/1 - (1 - \delta_H)(1 - \delta_L)).\} By construction, the *Baseline Exclusive Awareness* of the high awareness store \((\alpha_H)\) is higher than that of the low awareness store \((\alpha_L).\)\footnote{We can also consider one of stores as the rest of the market. For example, for a high awareness store, the low awareness store in our model can represent the rest of the stores in the same product category which have, in total, lower consumer awareness \(\alpha_L.\)} This is representative of two on-line stores in the same product category offering products with different quality, or with the same product quality but with different service quality.\footnote{Pan et al. (2002) show that on-line store services can be characterized by factors such as shopping convenience, reliability in fulfillment, and shipping/handling.} The parameter \(\alpha_H\) is also assumed to be less than one—if \(\alpha_H\) were one, the high awareness store would not have any incentive to join the mall, which is not interesting to study. Therefore, the exclusive awareness of the two on-line stores is such that \(1 > \alpha_H > \alpha_L \geq 0.\)

For convenience, we use \(\alpha_B\) to represent *Baseline Joint Awareness* of the two on-line stores. With the total market defined as one, \(\alpha_B\) represents the proportion of aware-both shoppers when neither store joins the Internet mall \((\alpha_B = 1 - \alpha_H - \alpha_L).\) The parameter \(\alpha_B\) is restricted to be greater than zero. Intuitively, \(\alpha_B = 0\) implies that there is no single consumer who is aware of both stores. In this case, both high and low awareness stores would have been able to charge infinite prices if the prices were not bounded by the consumer reservation price.

Note that while the baseline (exclusive or joint) awareness is exogenous, the proportion of aware-high-only shoppers, aware-low-only shoppers and aware-both shoppers changes when the stores join the Internet mall.

**Consumer awareness of the mall**

Adding a mall presence benefits an on-line store by creating greater consumer awareness for it through the mall. How much consumer awareness on-line stores can gain by adding the mall presence depends on how popular the Internet mall is. The reputation of an Internet mall \((1 > \alpha_M > 0)\) is defined as the proportion of on-line shoppers who are aware of the mall.\footnote{We do not consider the cases where \(\alpha_M = 0\) or \(\alpha_M = 1\) as they are not interesting.} We assume that consumer awareness of the Internet mall is exogenous to the model and independent of the consumer awareness of the on-line stores.

**Consumer awareness when stores join**

There are four possible market conditions (i.e., four subgames for the third stage pricing game) when the two stores decide simultaneously whether or not to join the mall. These conditions are indicated by \(XY\) with \(X = \{H, N\}\) and \(Y = \{L, N\}\) representing joining decision of the high or low awareness store respectively. Obviously, consumer awareness structure will be different from the baseline case (i.e., the \(NN\) case) when stores join. If the low awareness on-line store chooses to join the mall but the high awareness store does not (i.e., the \(NL\) case), the low awareness store increases its potential market by getting exposure on the Internet mall. In addition to the aware-low-only shoppers \((\alpha_L)\) and the aware-both shoppers \((\alpha_B)\) that it had before joining the mall, the low awareness store gains more consumer awareness from some of the aware-high-only shoppers (who earlier only knew about the high awareness store, but now also know the low awareness store through the mall). As a result, the proportion of aware-high-only shoppers decreases and the proportion of aware-both shoppers increases. In other words, joining the Internet mall enables the low awareness store...
Table 2: Consumer awareness when stores decide whether or not to join the mall

<table>
<thead>
<tr>
<th>XY *</th>
<th>Aware-high-only</th>
<th>Aware-low-only</th>
<th>Aware-both</th>
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</thead>
<tbody>
<tr>
<td>NN</td>
<td>(a_H)</td>
<td>(a_L)</td>
<td>(a_B)</td>
</tr>
<tr>
<td>NL</td>
<td>(a_H - a_M)</td>
<td>(a_L)</td>
<td>(a_B + a_M)</td>
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<tr>
<td>HM</td>
<td>(a_H - a_M)</td>
<td>(a_L - a_M)</td>
<td>(a_B + a_M)</td>
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<tr>
<td>HL</td>
<td>(a_H - a_M)</td>
<td>(a_L - a_M)</td>
<td>(a_B + a_M)</td>
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\* \(XY\) indicates stores’ joining decisions where \(X = \{H, N\}\) represents whether or not the high awareness store joins the mall and \(Y = \{L, N\}\) represents whether or not the low awareness store joins the mall.

### Consumer purchase

Consumer awareness does not automatically translate into consumer purchase. It is reasonable to assume that the aware-high-only (aware-low-only) shoppers will buy from the high (low) awareness store as long as its price does not exceed the reservation price. This reservation price, \(r\), is assumed to be homogeneous across consumers.

The aware-high shoppers can choose to buy from either the high or the low awareness store. The choice of store depends on the relative price and the quality difference of the two online stores with the quality difference being, for example, the difference in product quality or service quality. We can invoke the same logic as in Lal (1990b, 1990c) that the high awareness store charges a higher price; and that the ith aware-high shopper prefers to buy from the high awareness store if the price differential is less than \(x_i\), but prefers to buy from the low awareness store if the price differential is greater than or equal to \(x_i\). We also assume that \(x_i\) is distributed uniformly among consumers on \([0, x]\), with \(x\) being the maximum value of \(x_i\). The parameter \(x\) basically represents the price premium that the high awareness store enjoys.\(^{14}\) It also means that the low awareness store can attract all the aware-high shoppers if it undercuts the price of the high awareness store by a margin greater than or equal to \(x\). For all other differences in prices, the proportion of aware-high shoppers who buy from the low awareness store is a function of the difference in prices and \(x\). Therefore, the proportion of aware-high consumers who will buy from the low awareness store can be represented by:

\[
Z^{XY} = \begin{cases} 
0 \text{ if } P_H^{XY} - P_L^{XY} \geq x \\
\frac{P_H^{XY} - P_L^{XY}}{x} \text{ if } x \leq P_H^{XY} < P_L^{XY} \\
1 \text{ if } P_L^{XY} \leq P_H^{XY} - x 
\end{cases}
\]

where \(P_j^{XY}\) is the price charged by the high \((j = H)\) or low \((j = L)\) awareness store given the stores’ joining decisions (as indicated by the superscript \(XY\)) and will be solved endogenously. For example, the number of aware-high shoppers who will buy from the low awareness store in the baseline condition \(NN\) is \(a_B Z^{NN}\).

Based on consumer purchase behavior, we can derive market shares for the two online stores. Each store enjoys profit from consumers who are only aware of its own store but has to compete for the aware-high shoppers. The market shares for the two online stores in the four alternative market conditions \((M_j^{XY}\) where \(j = H \text{ or } L\)) are summarized in Table 3.

### Setup costs

There is a cost associated with setting up a store regardless of whether the store has an Internet mall presence or not. This setup cost includes two parts: a hosting cost and a traffic generation cost. The hosting cost refers to all the costs of physically building and maintaining an on-line store, including the cost of designing the webpage, handling credit card authorization, shipping and sales tax calculation, and maintaining the server. While both Internet malls and web hosting companies provide hosting services, a store can also choose to handle hosting internally on its own server. In addition to establishing the storefront, an on-line store needs to attract consumers. The traffic generation cost basically refers to any cost incurred in advertising the store, which includes, for example, the fees paid to be listed with various search engines and to advertise with other websites. As we have discussed earlier in the paper, one of a store’s major benefits for joining the mall is to gain more consumer awareness. An Internet mall not only helps stores to generate more traffic through the popularity of the mall, but also promotes greater store visibility by offering free listings with the most prominent search engines and websites. If a store decides not to join the mall, it has to self-host or to contract with a web hosting company and also incur the cost of generating traffic itself. If the store joins the mall, then even though it also exists outside the mall, we first assume that it totally relies on the mall to increase its popularity and generate traffic and does not spend on this as 18 percent across online travel agents. They believe the price dispersion can be explained by service differentiation.

\(^{14}\) Since the high awareness store is usually more experienced with managing the on-line environment, it is able to provide better services. Therefore, it is able to command higher prices compared to the low awareness store. This is representative of many product categories on-line. For example, a casual search from a shopping comparison site (www.mysimon.com) shows that consumers can purchase a Sony Bravia 46 in LCD TV from well-known online retailers such as Crutchfield for \(\$3,499\) or J&R for \(\$3,951\). The same TV costs less than \(\$2,800\) from low awareness electronics on-line stores such as PriceMad and Abe’s Main. According to Clay et al. (2001) who focus on on-line book industry, consumers could have saved 10–25 percent of Amazon’s price compared to purchasing a book from a less known/low priced seller. Clemens et al. (2002) study on-line travel industry which offers heterogeneous goods (as opposed to undifferentiated products like electronics and books) and show that even after accounting for differences in ticket quality, airline ticket prices vary by as much...
separately. As part of the model extension, we will relax this assumption later on.

Let $H_S$ be the cost for setting up one store and is referred to as the Store Setup Cost. The cost for an Internet mall to setup stores, Mall Setup Cost $H_M$, is captured by a simple cost function:

$$H_M = \begin{cases} H_S & \text{for } n = 1 \\ H_S + \sum_{i=2}^{n} \frac{1}{k \times i} H_S & \text{for } n > 1 \end{cases} \quad (2)$$

where $k > (1/2)$ captures the decrease in incremental setup cost.\(^{15}\) An Internet mall enjoys higher efficiency (i.e., economies of scale) the larger $k$ is. The parameter $n$ represents the number of on-line stores. Since there are two on-line stores in our model, $n = 2$. The functional form of $H_M$ captures our assumption that the setup cost for the first store on the mall is the same as for the store itself (i.e., the store setup cost $H_S$) and that the Internet mall’s incremental cost for having an additional store is lower than that for the first store (i.e., $k > (1/2)$). For the first store, there is little reason for the mall to incur lower setup costs than the stores. In fact, its expenses may even be higher since it has to make the arrangements for offering listings with the prominent search engines, the cost to provide an additional store with the same arrangements will be much lower. Similarly, once the mall has built a system for generating traffic, the second part of the setup cost (i.e., the traffic generation cost) decreases as well. For example, with several hundred search engines available and each with its own special procedures for listing information, it is a very complex and lengthy process to be listed on them. However, once an Internet mall has made the arrangements for offering listings with the prominent search engines, the cost to provide an additional store with the same arrangements will be much lower.

With the setting above, we begin our analysis by investigating the situation where the Internet mall adopts a linear contract that includes both a fixed monthly fee and also a revenue share fee of all transactions, i.e., a two-part tariff model. We derive the optimal split between a FF and an ARSF for the two-part tariff model. Then, we show conditions under which a FF-only model or an ARSF-only model is as profitable as a two-part tariff for the mall. We assume that both the Internet mall and the on-line stores are profit maximizers and adopt Subgame Perfect Nash Equilibrium as the solution concept throughout the paper.

**Equilibrium analysis**

Using backward induction, we first solve the third stage game by simultaneously maximizing the two on-line stores’ profits with respect to prices, given their joining decisions and the fees charged by the mall. In other words, we solve for the optimal prices and sales revenues in each of the four pricing subgames $NN, NL, HN$ and $HL$ given the mall’s fee structure. Then, for each of the four subgames we maximize the mall’s profit, choosing the optimal fee subject to the stores’ participation constraints. Finally, we compare the mall’s profits across the four subgames and choose the one that yields the highest profit for the mall. Similar to many theoretical models in the marketing literature (e.g., Blattberg and Wisniewski 1989; Raja et al. 1990), we will focus our discussion on the equilibrium results where consumer reservation price is sufficiently high (i.e., when the reservation price is higher than the price either store will charge in equilibrium).

Then, for each of the four subgames we maximize the mall’s profit choosing the optimal fee subject to the stores’ participation constraints. Finally, we compare the mall’s profits across the four subgames and choose the one that yields the highest profit for the mall.

**Stores’ prices**

We first discuss the stores’ pricing decisions given their joining decisions and the Internet mall’s fees. $P^{XY}_j$ stands for a store’s market share, $R^{XY}_j$ for sales revenue, and $\Pi^{XY}_j$ for profit. Consistent with our notations in previous sections, the superscript $XY$ represents stores’ joining decisions ($X = \{N, H\}; Y = \{L, N\}$) whereas the subscript $j$ indicates whether it is a parameter for the high awareness store ($j = H$) or for the low awareness store ($j = L$).

Given that the stores choose their prices ($P^{XY}_j$) to maximize profits ($\Pi^{XY}_j$), we can write the stores’ profit maximization problem as:

$$\max_{P^{XY}_j} \Pi^{XY}_j = R^{XY}_j - (F + f R^{XY}_j) I_j - (1 - I_j) H_S \quad (3)$$

where $I_j$ is an indicator variable which equals 1 if store $j$ decides to join the mall and 0 otherwise. We take first order conditions of the two stores’ profit functions simultaneously and solve for the optimal prices. Since store’s sales revenue ($R^{XY}_j = P^{XY}_j \times M^{XY}_j$) incorporates purchases made by con-

\(^{15}\) Similar cost functions have been used in the economics literature to model the cost of multiple products for a monopoly firm (e.g., Sharkey 1981). Although this specific functional form of the cost facilitates our analysis, our results hold with any cost function as long as the incremental cost decreases when the mall starts to include the second store and so on.
sumers who learn about the store through the mall (i.e., $\alpha_H\alpha_M$ and $\alpha_L\alpha_M$) regardless of whether the actual transactions take place at the store’s own website or at the mall, it captures sales leakage. Therefore, the ARSF (as represented by $f$) in our model is based on a broader notion of sales revenue and is different from the conventional revenue share fee.

It is important to note that the pricing game in the third stage has interior solutions when the baseline exclusive awareness of the high awareness store is relatively low (i.e., when the two stores have similar consumer awareness). In this case, the two on-line stores compete for the aware-both shoppers and each of them gets a proportion. However, when the baseline exclusive awareness of the high awareness store is relatively high, the two on-line stores are dissimilar in consumer awareness. It is more profitable for the low awareness store to cut the price by $x$ and get all the aware-both shoppers. As a result, the high awareness store gets the aware-high-only shoppers and the low awareness store gets the rest of the market. The optimal price for the high awareness store is the consumer reservation price $r$ and the low awareness store will charge $r - x$. This is not an interesting case since there is no benefit for the high awareness store to add mall presence. In other words, the joining decision is irrelevant for the high awareness store when it only wants to attract the aware-high-only shoppers. This special case also leads to boundary conditions with either FF or ARSF being zero. Therefore, our discussion of the basic model will focus on the interior solutions of the pricing game where the two stores compete for aware-both-shoppers. For completeness of the model, we include the special case in Appendix A.16

The optimal prices ($p_{XY}$), market shares ($M_{XY}$), and sales revenues ($R_{XY}$) of the high ($j = H$) or low ($j = L$) awareness store in all four pricing subgames when consumer reservation price is high enough ($r \geq (2 - \alpha_L)x / 3\alpha_B$) and when the two stores have similar consumer awareness ($\alpha_H < (2 - \alpha_L / 3)$) can be summarized in Table 4 (details in Appendix A.1).

### The two-part tariff model

In the previous section, we have discussed stores’ pricing decisions given their joining decisions and the mall’s fees. We now examine the situation where the Internet mall adopts a linear contract $T = (F, f)$ with a fixed monthly payment $F$ and an ARSF $f$. Our notations in this section are consistent with those in previous sections while adding a set of variables with the subscript $M$ to represent the Internet mall. For example, $\Pi_{ML}$ represents the Internet mall’s profit when both stores join.

It is obvious that when neither of the stores joins the mall, the mall earns zero profit. When only one store joins the mall, the mall collects fees from the joining store and incurs costs for building and advertising the store. The mall maximizes its profit ($\Pi_{XY}^M$) by selecting a proper contract $T = (F, f)$. For example, the constrained optimization problem for the mall when only the high awareness store joins can be written as:

$$\max_{T=(F,f)} \Pi_{HN}^M = F + fR_{HN}^M - HS$$

s.t. $R_{HN}^M (1 - f) - f \geq R_{NN}^M - HS \geq 0$;

$$R_{HL}^M - HS \geq R_{LH}^M (1 - f) - f \geq 0;$$

When both stores join, the profit maximization problem for the Internet mall is:

$$\max_{T=(F,f)} \Pi_{ML}^M = 2F + f(R_{HN}^L + R_{HL}^L) - (HS + \frac{1}{2k}HS)$$

s.t. $R_{HN}^M (1 - f) - f \geq R_{NL}^M - HS \geq 0$;

$$R_{HL}^M (1 - f) - f \geq R_{LH}^M - HS \geq 0;$$

After analyzing the mall’s profit maximization problem in all four market conditions, we find that

---

16 See Appendix A.9 for details.
Proposition 1. When consumer reservation price is high enough \((r \geq ((2 - \alpha_L)x/3\alpha_H))\), the two on-line stores have similar consumer awareness \((\alpha_H < (2 - \alpha_L/3))\), and store setup cost is moderate \((\max[H^1_2, H^2_3] \leq H_S \leq H_3)\), then in equilibrium, the Internet mall charges \(T^* = (F^*, f^*)\) and both stores join the mall (see Appendix A.2 for details).

As discussed earlier, when the two stores have similar consumer awareness \((\alpha_H < (2 - \alpha_L/3))\), they charge prices to maximize their profits and share aware-both shoppers. If only one of the on-line stores joins, the Internet mall cannot take advantage of the decreasing incremental setup cost. The mall extracts profit from the joining store and can earn positive profit only if the store’s revenue increases by joining. Since the number of aware-both shoppers increases when either store joins, both stores price more aggressively to compete for these consumers. As a result, stores’ revenues decrease after joining. Therefore, the mall earns negative profits when only one store joins and is better off including neither of the stores.

Fig. 2 illustrates different equilibrium conditions for the two-part tariff model as store setup cost increases. Intuitively, if store setup cost is too low \((H_S < \max[H^1_2, H^2_3])\), stores should stay stand alone. If store setup cost is moderate, stores are willing to join the mall and pay a two-part tariff fee that is less than the cost of building and advertising the stores themselves. When both stores join, even though the revenues of the stores are lower due to more intense price competition, the mall is able to enjoy the efficiency in setting up the stores and earn a positive profit. On the other hand, if store setup cost is too high \((H_S > H_3)\), on-line stores will not be able to make any profits.

The Fixed Fee only model

In the previous section, we have derived a two-part tariff model where the Internet mall charges a combination of a fixed monthly fee and a revenue share fee of all transactions from the stores. In this section, we investigate situations under which charging only a FF is as profitable for the mall as using a linear contract. In the next section, we examine the ARSF-only model. From the modeling perspective, we investigate the two special cases of the general two-part tariff model where \(f^* = 0\) or \(F^* = 0\).

The two-part tariff model becomes a FF-only model when all of the following conditions are satisfied:

\[
\begin{align*}
H_S &\geq H^1_2 = \frac{R^{HL}_H - R^{NL}_H}{R^{HL}_H - R^{NL}_L} \\
R^{HL}_H - R^{NL}_H &\geq R^{HL}_L - R^{NL}_L \\
H_S &\leq R^{HN}_L \\
H_S &\geq H^2_3 = -\frac{R^{HL}_H + R^{NL}_L - R^{HL}_L + R^{HN}_L}{1 - (1/2k)}
\end{align*}
\]

The first condition which guarantees that the FF is non-negative \((F \geq 0)\) basically indicates that the stores are willing to pay FF fees and join the mall when store setup cost is high. It is intuitive that when the cost for setting up a store is low, the stores prefer to stand alone. The second condition restricts the ARSF to be zero \((f = 0)\). It suggests that the mall only needs to charge a FF if the revenue gain from joining for the high awareness store is the same as that for the low awareness store. In other words, the discriminating pricing mechanism (i.e., the ARSF) is not needed when the mall cannot effectively discriminate based on sales revenues. Finally, we ensure the existence of the stores (the third condition) and of the mall (the fourth condition) by satisfying the non-negative profit conditions. Our results regarding the FF-only model can be summarized in the following proposition.

Proposition 2. When store setup cost is high \((H^2_3 \leq H_S \leq H_3)\), in equilibrium, a Fixed Fee only model is as profitable for the mall as a two-part tariff model (see Appendix A.3 for details).

The All-Revenue-Share-Fee only model

Now we consider the new pricing model which enables the mall to base its percentage rent on a broader notion of sales, the ARSF-only model. The ARSF-only model is as profitable for the mall as the two-part tariff model when all of the following conditions hold:

\[
\begin{align*}
H_S = H^1_2 = \frac{R^{HN}_H - R^{NL}_H}{R^{HL}_H - R^{NL}_L} &\quad (I) \\
R^{HL}_H - R^{NL}_H &> R^{HL}_L - R^{NL}_L &\quad (II) \\
H_S &\leq R^{HN}_L &\quad (III) \\
H_S &\geq H^2_3 = -\frac{R^{HL}_H + R^{NL}_L - R^{HL}_L + R^{HN}_L}{1 - (1/2k)} &\quad (IV)
\end{align*}
\]

Although the last two conditions (non-negative profit conditions) are the same as the ones for the FF-only model, the first two conditions are different. For an ARSF-only model to be as profitable as a two-part tariff model, the store setup cost has to equal a specific value as indicated by the first condition basically showing \(F = 0\). Intuitively, as the store setup cost increases, the mall needs to collect more from the stores. When only an ARSF is used, raising the ARSF, \(f\), means a much higher contribution from the high awareness store. Since stores’ joining decisions are endogenized in our model, the high awareness store will choose not to join. Only when the store setup cost takes a specific value, is the mall able to cover its cost and engage both stores in equilibrium. The second condition for the ARSF-only model which ensures that the ARSF is positive represents the fact that an ARSF is used by the mall to discriminate between the two on-line stores.

Interestingly, when we combine the first and the last condition, we find that an Internet mall’s store setup cost efficiency (as represented by \(k\)) directly affects the mall’s decision to adopt an ARSF model in equilibrium.

Proposition 3. Only an Internet mall that enjoys higher store setup cost efficiencies \((k \geq \frac{k}{k} > (1/2))\) can adopt an All-Revenue-Share-Fee only model (see Appendix A.4 for proof).

The above proposition indicates that an Internet mall can charge an ARSF and include both stores in equilibrium when \(k\) exceeds some cutoff value. Since \(k\) reflects the mall’s efficiency (or economies of scale), a larger \(k\) \((k > (1/2))\) means higher
efficiency in building stores and attracting traffic as more and more stores join.

The ARSF model is different from the FF model in two aspects. First of all, the ARSF model captures sales leakage while FF model does not. When the mall has low efficiency, in order to cover its costs, the mall needs to collect a higher fee by taking a larger share of total sales from both stores including the sales that are currently being leaked. As a result, the stores will choose not to join the mall.

The ARSF model is also different from the FF model due to its nature of being a discriminating pricing mechanism. While in the FF-only model, the Internet mall collects the same amount of fee from both stores, in the ARSF-only model the mall collects a higher amount of fee from the (larger sales) high awareness store compared to the (smaller sales) low awareness store. Therefore, the mall is more keen to engage the high awareness store than to engage the low awareness store. When the mall’s efficiency is lower and it needs to collect a higher total amount of fee across the two stores, unlike in the FF model where a higher total amount of fee can be achieved by higher (but equal) fees from the two stores, in the ARSF model the mall has to rely on a much higher contribution from the high awareness store because of its higher sales revenue. However, in that case, when we endogenize the stores’ joining decisions, the high awareness store will not find it profitable to join. Only when the mall’s efficiency is higher, is the mall able to charge a low enough \( f \) to engage the high awareness store and therefore enlist both stores.

Proposition 3 indicates that the ARSF-only model is not profitable for all Internet malls. Only malls that enjoy higher efficiency should adopt it. The equilibrium ARSF not only requires decreasing incremental setup cost \( k > (1/2) \) but also a substantial decrease in this cost with increasing number of stores \( k \geq k > (1/2) \).

### Optimal fee structure

In the previous sections, we discussed situations where a FF-only model or an ARSF-only model would be as profitable for the mall as a two-part tariff model. In this section, we compare these three different pricing mechanisms and show which one is optimal under what conditions.

**Proposition 4.** When the store setup cost is too high or too low, in equilibrium neither of the two stores joins the mall; When the store setup cost is low (and when the mall enjoys higher efficiency), in equilibrium the Internet mall charges an All-Revenue-Share-Fee only and both stores join the mall; When the store setup cost is moderate, in equilibrium the Internet mall charges a two-part tariff and both stores join the mall; When the store setup cost is high, in equilibrium the Internet mall charges a Fixed Fee only and both on-line stores join the mall (see Appendix A.5 for details).

Given our assumption that the setup cost for the store and the mall is the same if only one store joins the mall, and given that the mall wants to cover its costs, in equilibrium both stores will either join the mall or not. As we discussed earlier in this paper, the high awareness store will choose not to join the mall when a high total fee is being charged through the revenue share which occurs when setup costs are high. As a result, a non-discriminating pricing mechanism (i.e., a FF model) is feasible on a much wider range of setup cost than a discriminating pricing mechanism (i.e., an ARSF model).

![Fig. 3. The optimal fee structure for highly efficient malls.](image)

**Model discussion**

In this section, we investigate the effects of stores’ baseline exclusive awareness \( \alpha_H \) and \( \alpha_L \), quality difference \( x \) between two stores, the possibility of market expansion, and non-zero setup costs for joining stores on the equilibrium conditions.
Baseline exclusive awareness

Baseline exclusive awareness represents the popularity of a store. When we examine the effect of the stores’ baseline exclusive awareness ($\alpha_H$ and $\alpha_L$) on the equilibrium conditions, we find that:

**Proposition 5.** As baseline exclusive awareness of the two on-line stores increases, the store setup cost needs to be higher ($\partial H_1^L / \partial \alpha_H > 0$; $\partial H_2^L / \partial \alpha_L > 0$; $\partial H_1^H / \partial \alpha_H > 0$; $\partial H_2^H / \partial \alpha_H > 0$; $\partial H_2^L / \partial \alpha_L > 0$) for both stores to join the mall in equilibrium (see Appendix A.6 for proof).

In other words, as the baseline exclusive awareness of one on-line store increases holding the baseline exclusive awareness of the other store constant, store setup cost $H_2$ needs to be higher for both stores to join the mall in equilibrium. The intuition can be explained as follows. As the baseline exclusive awareness of the high (low) awareness store increases, the high (low) awareness store has less incentive to join and will join the mall only if store setup cost is high. Hence, the stores setup cost needs to be higher for both stores to join the mall in equilibrium.

Quality difference between the stores

The magnitude of quality difference between the two on-line stores may be different depending on the product category. In some product categories stores are more similar in quality than in other product categories. Our sensitivity analysis of the equilibrium conditions with respect to the quality difference parameter $x$ shows that:

**Proposition 6.** The quality difference between the low and the high awareness store ($x$) has a negative effect on the equilibrium FF (i.e., $\partial F^* / \partial x < 0$) but has no effect on the equilibrium ARSF (i.e., $\partial f^* / \partial x = 0$) (see Appendix A.7 for proof).

The first part of Proposition 6 indicates that as the quality difference between the two stores increases, the FF should charge a lower FF. The intuition for this part is as follows. In order to include both stores (so that the market can benefit from diminishing incremental setup cost), the Internet mall should charge a FF that is at least affordable for the low awareness store. A larger $x$ means that the high awareness store is even better than the low awareness store in terms of quality. The revenue of the low awareness store is lower when $x$ is greater (compared to when smaller) and its affordability for a FF is lower. Therefore, the FF that the mall is able to charge is lower. From the high awareness store’s perspective, it has less incentive to join when its quality is much higher than that of the low awareness store. Therefore, its willingness to pay the Internet mall is lower. To engage the high awareness store, the Internet mall has to lower the FF.

More interestingly, the second half of Proposition 6 claims that the quality difference between the stores has no effect on the equilibrium ARSF that the mall charges. The intuition for this part of the proposition can be explained by the difference between FF and ARSF. Unlike in the FF model, the amount of ARSF the mall can charge is not limited by the affordability of the low awareness store. In other words, since ARSF is a discriminating pricing mechanism, the amount of ARSF that the mall collects from the low awareness store is lower when the quality difference between the two on-line stores is greater even with the same percentage share ($f^*$). Therefore, the quality difference between the two on-line stores has no effect on the equilibrium ARSF the mall charges.

This proposition describes how the quality difference of a specific product category affects the FF and the ARSF an Internet mall is able to charge. Although it is not necessary for the mall to vary its ARSF, Proposition 6 indicates that a mall should adjust the amount of FF according to the quality difference in certain product categories. If it is a product category with larger quality difference between stores, the Internet mall should charge a lower FF.

Market expansion

We have previously assumed that consumers are either aware of the high awareness store, the low awareness store, or both when no stores join the mall. Now by including a segment of consumers, $\alpha_N$, who are aware of neither of the stores in the baseline case ($N, N$), we allow a store to expand its market share by adding mall presence. For example, when the low awareness store joins the mall, it not only converts some of the aware-high-only shoppers ($\alpha_H \alpha_M$) to aware-both shoppers but also increases its exclusive awareness by $\alpha_N \alpha_M$, where $\alpha_N \alpha_M$ represents the group of consumers who were not aware of either of the stores but were aware of the Internet mall.

Therefore, adding mall presence can have two effects for a store: the competition effect and the expansion effect. The competition effect refers to the increase of aware-both-shoppers which makes both stores to price more aggressively. On the other hand, the market expansion effect (which increases with $\alpha_N$) benefits the joining store by allowing the store to gain more exclusive awareness. With this setup, we solve the market expansion model by focusing on the situation where consumer reservation price is high enough ($r \geq (2 - x_H / \alpha_B)$) and the two stores have similar consumer awareness ($x_H < (2 - x_L / 3)$). We summarize the model results in Appendix A.8 and discuss some important findings here.

First of all, when we derive the third stage pricing game, we find that:

**Proposition 7.** When stores join the mall, equilibrium prices are lower in the market with expansion compared to the market without expansion (see Appendix A.8 for details).

In other words, although equilibrium prices in the $(N, N)$ subgame are the same as in the basic model, equilibrium prices for the high and the low awareness stores in the $(N, L)$, $(H, N)$, and $(H, L)$ subgames are lower in the market expansion model. Intuitively, since the market expansion effect allows for the stores to increase their sales, stores are able to price more aggressively (compared to the market without expansion) to compete for the aware-both-shoppers and still earn reasonable profits.
Secondly, different from the basic model without market expansion, we find that:

**Proposition 8.** There exists a pure strategy Nash Equilibrium where the Internet mall earns a positive profit by enlisting only the low awareness store, and the mall’s profit is higher the larger the market expansion effect (Appendix A.8 for details).

Recall that in the basic model, one store joining is not profitable for the mall since the mall can neither extract profit from the joining store due to the competition effect nor take advantage of the decreasing incremental setup cost. However, when we allow for market expansion, since stores benefit from joining by reaching more unaware consumers through the mall, \((N, L)\) can be characterized as a pure strategy Nash Equilibrium in the three stage game. It is also intuitive that the larger the number of unaware consumers \((a_N)\), the larger the market expansion effect and the more profitable it is for stores to join. As a result, the mall is able to extract more profit.

When we compare the equilibrium conditions for \((N, L)\) and \((H, L)\) in this market expansion model, we find that similar to the basic model with no market expansion, when store setup cost is moderate, in equilibrium, both stores join. The difference is that when store setup cost is low, in equilibrium, the mall is able to enlist the low awareness store when we take into account the market expansion effect. This result indicates that shopping malls operating in product categories with great market expansion potential (e.g., high-tech markets) may actually benefit from enlisting only the low awareness stores. Since these low awareness stores gain a lot from joining the mall, the mall is able to extract more profit.

**Proposition 8** together with the equilibrium conditions derived in the basic model identifies stage of product life cycle as an important characteristic of product category affecting mall’s optimal pricing strategy. More specifically, in the introductory stage and the growth stage where there is still room for market expansion, it may be profitable for an Internet mall to enlist only the low awareness stores. However, when a product category reaches the maturity stage where the market is saturated, the mall needs to enlist both the high and the low awareness stores.

**Non-zero setup cost for joining stores**

One assumption we make in the basic model is that once an on-line store becomes a member of the mall it completely relies on the Internet mall for hosting and traffic generation services and thus has zero setup cost. This may be true for many on-line stores. However, some stores may continue to invest in advanced technologies to enhance the physical appearance of the store, to facilitate the check-out process for the consumers, and to enrich the data collected during each browsing/shopping episode. This leads to a non-zero hosting cost for a joining store. Some stores may also consider spending separately on advertising despite the traffic generation service that the mall provides. In these cases, the setup costs for joining stores are not zero but less than what a stand-alone store would incur.

What if the setup cost of an existing on-line store even after joining the mall is not zero? Obviously, this does not affect the third stage pricing game. However, when it comes to the joining decisions of the stores (the second-stage game), the non-zero setup cost after joining the mall makes it less attractive for the stores to join the mall. Thus, intuitively, if there is a setup cost for a store even after adding the mall presence, then the initial setup cost before joining (or the difference between the setup cost before and after joining) should be much larger for it to be attractive for the stores to join and profitable for the mall as well. The equilibrium condition characterized by the values of \(H_S\) (as indicated by the middle part of Fig. 2) shifts to the right.

The store will also want to pay the mall lower fees as the store has higher costs. There is therefore less the mall can extract compared to the basic model where the setup cost is assumed to be zero after a store joins. In the equilibrium where both stores join the mall under a two-part tariff contract, having a non-zero setup cost after joining will lead to a lower FF and as will be the profit for the mall. However, ARSF stays the same. The reason for ARSF to stay constant is the following. ARSF is used by the mall to discriminate between high and low awareness stores. Since the two stores incur exactly the same setup cost whether it is zero or non-zero after joining, the level of ARSF should not be affected when this assumption is relaxed.

**Managerial implications**

The analysis in this paper identifies the optimal fee structure and indicates that product category characteristics (such as store setup cost, consumer awareness of the stores, and quality differences between the stores), mall’s characteristic (i.e., efficiency in setting up multiple stores), and possibility of market expansion affect the fee structure and the amount of the fee that a mall should charge. There are many useful managerial implications from our model that a mall manager can adopt when making the mall’s pricing decision. We discuss some of the most important ones.

**Using ARSF to capture leakage directly**

The new ARSF model we examine in this paper is different from FF and conventional revenue share fee due to its ability to take into account sales leakage. Because of this unique characteristic, on the face of it, mall owners and managers have every reason to prefer ARSF to FF. However, our results indicate that Internet malls should use ARSF with caution. We show that when the cost of setting up a store is high or when the mall has low store setup cost efficiency, the mall will be better off using a FF model.

**Price discrimination across product categories**

Casual observations of the industry indicate that most Internet malls do not charge different amounts of fees to on-line stores in different product categories. Our belief is that this is a common practice because it is easier to implement. However, our results show a profit loss as a result of this uniform pricing. Specifically, we show that although it is not necessary for the mall to vary its ARSF amount across product categories, a mall adopting a two-
part tariff (or a FF-only) model should adjust the amount of FF according to the quality difference in certain product categories. If it is a product category with a larger quality difference between stores, the Internet mall should charge a lower FF. If it is a product category with stores similar in quality, the mall should charge a higher FF. As such, ARSF may be easier to implement than FF.

Moreover, we propose that besides charging different amounts of the same type of fee, an Internet mall can adopt totally different fee structures to achieve price discrimination. Our model results show that, in general, a FF should be charged in product categories with high setup cost and a two-part tariff or an ARSF in product categories with low setup cost. As we mentioned earlier in the paper, the store setup cost includes the cost for building the storefront (i.e., the hosting cost) and the cost for bringing consumers to the store (i.e., the traffic generation cost).

The hosting cost (i.e., the cost for hosting an on-line store in a specific product category) can vary across different product categories. The two major determinants of the hosting cost are bandwidth and storage space. Bandwidth is a measure of how much data a server provides to consumers while storage space refers to the amount of room allotted on a host's server. It is not hard to imagine that bandwidth and storage space (and therefore the hosting cost) required for hosting a music CD store with thousands of titles would be higher than for hosting a computer CPU store with no more than 200 models (see Fig. 4, for example, from Ebay). Similarly, the cost for hosting a music CD store or a clothing store (where more complicated design tools are needed for sample songs or virtual 3D models) would be much higher compared to hosting a computer CPU store or a jewelry store (where usually text descriptions are sufficient).

The traffic generation cost can also vary across different product categories. According to DoubleClick’s 2002 study\textsuperscript{17} where the Nielsen/NetRatings AdRelevance data on on-line spending is used, on-line advertising share varies by product category (see Fig. 5). For example, electronics (10.4 percent) and travel (12 percent) spend significantly more in on-line advertising whereas food, drugs, and apparel spend much less.

Our results suggest that Internet malls should price discriminate by using different fee structures considering both hosting cost and traffic generation cost and charge FFs to product categories with high total setup costs, two-part tariffs or ARSFs to product categories with low total setup costs. These results are non-intuitive yet managerially implementable, especially with the help of recently developed web-based property management software such as ILAMBS and Real Estate Solutions.\textsuperscript{18}

\textbf{Pricing over time}

As noted by Levy et al. (2004), optimal pricing is not a static problem. Retailers must be able to react quickly to changes in the environment. The results from this paper can shed some light on different pricing-over-time strategies for mall services. More specifically, we propose that when the characteristics of a mall and of a product category change over time, an Internet mall should adjust its pricing strategy to maximize profit.

Two of the important characteristics of the product category that may also change over time are store setup cost and quality difference between the stores. Initially the cost for setting up a store in a product category may be high. In that case, our results indicate that an Internet mall should charge a fixed monthly fee. When store setup cost for a specific product category decreases


\textsuperscript{18} ILAMBS is the property management software developed jointly by Bishara Computer Systems, Inc. and CBOSS, Inc.; Real Estate Solutions is the latest property management software developed by MRI. Both of these programs are web-based and allow users to automatically calculate and bill percentage rents, and calculate ultimate tenant, category and mall performance.
as technology advances, an Internet mall should adopt a two-part tariff or an ARSF. Our analysis also provides guidelines for Internet malls about how to adjust their FFs when an industry evolves and the quality difference structure changes. It suggests that if the high awareness on-line store becomes dominant, an Internet mall should lower its FF.

Conclusions and future research

In this paper, we take the first step to examine a new pricing mechanism which allows a shopping mall to capture sales leakage directly—the All-Revenue-Share Fee model. We contribute to the theoretical literature of pricing strategies of shopping malls by showing the effect of endogenizing stores’ joining decisions on the optimal pricing strategies of the mall. We show that although an All-Revenue-Share fee is appealing for Internet malls due to its ability to capture sales leakage directly, it may cause the stores to refrain from joining the mall in certain circumstances. As a result, charging a fixed monthly fee can actually be more profitable for the mall. Our paper also adds to the price discrimination literature by proposing that a mall can price discriminate across different product categories not only by charging different amounts of fees, but also by using different fee structures, i.e., Fixed Fees in some product categories and revenue share fees in others. Our basic model which characterizes saturated markets with little room for market expansion indicates that shopping malls in these types of markets should enlist both high and low awareness stores. On the other hand, we learn from the market expansion model that shopping malls operating in product categories with great market expansion potential (e.g., high-tech markets) may actually benefit from enlisting only the low awareness stores.

Although our paper contains interesting insights on how to price mall services when many sales facilitated by the mall are made outside it, the results must be interpreted within the context of certain limitations. First, we assume that the mall can only charge the same amount of fee (FF or ARSF) to both stores. This is a reasonable assumption since the stores will find it unfair if different amounts of fees are charged to stores in the same product category. Our qualitative results regarding the optimal fee structure will not change even if we relax this assumption. We also do not consider the case where the mall charges different revenue share fees for traffic coming directly to the store versus through the mall. It is difficult in a model to determine what share of traffic to assign as direct (vs. indirect) and this is better studied as an empirical issue. Third, we have structured the model so that the awareness of the mall does not change after stores join the mall. Although it seems reasonable to endogenize the awareness of the mall as an increasing function of the number of stores being enlisted, intuitively the implications would be the same as having decreasing incremental setup cost for the mall (i.e., the mall’s efficiency), which has already been taken into account in the model. Another assumption we make is that the cost of setting up the first store in a product category for a mall is the same as that for the store itself. Relaxing this assumption so that the mall’s cost is lower than that for the stores would quite obviously make it easier to attain equilibria where the mall enlists both stores and would also allow equilibrium conditions where one store is enlisted. However,
it should not change any other implications and would make
our model much less tractable. In terms of model structure, we
have only focused on the situation with one Internet mall act-
ing as a monopoly and two stores. Our main qualitative results
should not change with an extension to n stores as long as one
re-interprets the awareness structure for the product category.
However, we do expect the cutoff value for self-hosting to be
lower as the number of stores increases due to the mall’s effi-
ciency in setting up multiple stores. Although it would also be
nice to incorporate mall level competition, the consumer aware-
ness structure becomes extremely complicated and there are too
many subcases. A more formal analysis is beyond the scope of
this paper and is left on the agenda for future research. Finally,
our model assumes that a high awareness store charges a higher
price compared to the low awareness store. Although it would
be nice to endogenize the awareness–price relationship, it will
also make our model much more complex and is left for future
research.

These limitations notwithstanding, our research is a first
attempt to understand the pricing strategies for malls when
many sales facilitated by the mall are made outside it. We offer
many suggestions for mall owners who are thinking of expand-
ing the instrument of revenue share fee to include these sales.
We suggest directions for future research on models of pricing
mall services, including incorporating mall level competition
and endogenizing the awareness–price relationship. Our paper
also identifies many empirical questions about the mall–store
relationship that have not been answered. For example, what is
the magnitude of sales leakage? Does it vary across different
product categories? What are the determinants of the leakage?
These questions are of great interest to both stores and malls
and need to be answered by analyzing data collected across
different channels. As such, we hope that the paper plays an
additional role of spurring further research on the topic of mall
pricing.

Appendix A. Technical appendix

A.1. Third-stage pricing game

We will discuss the optimal pricing decisions in each of the
four subgames (NN, NL, HL, and HL) given the mall’s fees $T = (F, f )$

1. (N, N) subgame:
The market share of the high awareness store is $M_{NN} = \alpha_H + \alpha_B(1 - Z_{NN})$ and that of the low aware-
ness store is $M_{LN} = \alpha_L + \alpha_BZ_{NN}$ where

$$Z_{NN} = \begin{cases} 0 & \text{if } P_{NN} \geq P_{NN}^* \\ \frac{P_{NN} - P_{NN}^*}{x} & \text{if } P_{NN}^* < P_{NN} < P_{NN}^* \\ 1 & \text{if } P_{NN}^* \leq P_{NN} \\ \end{cases}$$

The profits of the two on-line stores are

$$\Pi_{NN}^H = P_{NN}^H \times M_{NN}^H - H_S; \quad \Pi_{NN}^L = P_{NN}^L \times M_{NN}^L - H_S;$$

$$\frac{\partial \Pi_{NN}^H}{\partial P_{NN}^H} = \alpha_H + \alpha_B \left( 1 - \frac{2P_{NN}^H - P_{NN}^L}{x} \right);$$

$$\frac{\partial \Pi_{NN}^L}{\partial P_{NN}^L} = \alpha_L + \alpha_B \left( \frac{P_{NN}^H - 2P_{NN}^L}{x} \right)$$

Therefore, the profits of the two on-line stores are maximized when

$$P_{NN}^H = \left( \frac{2 - \alpha_L}{3\alpha_B} \right) x; \quad P_{NN}^L = \left( \frac{1 + \alpha_L}{3\alpha_B} \right) x;$$

if $r \geq \left( \frac{2 - \alpha_L}{3\alpha_B} \right)$ and $\alpha_H \leq \left( \frac{2 - \alpha_L}{3} \right)$

2. (N, L) subgame:
The market share of the high awareness store is $M_{NL}^H = \alpha_H(1 - \alpha_MZ_{NL}) + \alpha_B(1 - Z_{NL})$ and that of the low aware-
ness store is $M_{NL}^L = \alpha_L + \alpha_H \alpha_M Z_{NL} + \alpha_B Z_{NL}$.

The profits of the two on-line stores are

$$\Pi_{NL}^H = P_{NL}^H \times M_{NL}^H - H_S;$$

$$\Pi_{NL}^L = P_{NL}^L \times M_{NL}^L(1 - f) - F;$$

$$\frac{\partial \Pi_{NL}^H}{\partial P_{NL}^H} = \alpha_H + \alpha_B - (\alpha_H \alpha_M + \alpha_B) \frac{2P_{NL}^H - P_{NL}^L}{x};$$

$$\frac{\partial \Pi_{NL}^L}{\partial P_{NL}^L} = \left[ \alpha_L + (\alpha_H \alpha_M + \alpha_B) \left( \frac{P_{NL}^H - 2P_{NL}^L}{x} \right) \right] (1 - f)$$

Therefore, the profits of the two on-line stores are maximized when

$$P_{NL}^H = \left( \frac{2 - \alpha_L}{3\alpha_B + \alpha_H \alpha_M} \right) x; \quad P_{NL}^L = \left( \frac{1 + \alpha_L}{3\alpha_B + \alpha_H \alpha_M} \right) x;$$

if $r \geq \left( \frac{2 - \alpha_L}{3\alpha_B + \alpha_H \alpha_M} \right)$ and $\alpha_H \leq \left( \frac{2 - \alpha_L}{3} \right)$

3. (H, N) subgame:
The market share of the high awareness store is $M_{HN}^H = \alpha_H + (\alpha_B + \alpha_L \alpha_M)(1 - Z_{HN})$ and that of the low awareness
store is $M_{HN}^L = \alpha(1 - \alpha_M) + (\alpha_B + \alpha_L \alpha_M)Z_{HN}$.

The profits of the two on-line stores are

$$\Pi_{HN}^H = P_{HN}^H \times M_{HN}^H(1 - f) - F;$$

$$\Pi_{HN}^L = P_{HN}^L \times M_{HN}^L - H_S;$$

$$\frac{\partial \Pi_{HN}^H}{\partial P_{HN}^H} = \left[ 1 - \alpha_L(1 - \alpha_M) - (\alpha_B + \alpha_L \alpha_M) \right] \left\{ \frac{2P_{HN}^H - P_{HN}^L}{x} \right\} (1 - f);$$

$$\frac{\partial \Pi_{HN}^L}{\partial P_{HN}^L} = \alpha_L(1 - \alpha_M) + (\alpha_B + \alpha_L \alpha_M) \left( \frac{P_{HN}^H - 2P_{HN}^L}{x} \right)$$
Therefore, the profits of the two on-line stores are maximized when

\[ P_{HH} = \frac{[2 - \alpha_L(1 - \alpha_M)]x}{3(\alpha_B + \alpha_L\alpha_M)}; \]
\[ P_{LM} = \frac{[1 + \alpha_L(1 - \alpha_M)]x}{3(\alpha_B + \alpha_L\alpha_M)}; \]
\[ \text{if } r \geq \frac{[2 - \alpha_L(1 - \alpha_M)]x}{3(\alpha_B + \alpha_L\alpha_M)} \text{ and } \alpha_L \leq \frac{2 - \alpha_L(1 - \alpha_M)}{3}. \]

Therefore, for the purpose of comparing store revenues across four market conditions (i.e., NN, NL, HN, and HL) later on, we only consider the situation where the two stores have similar consumer awareness in all four market conditions (i.e., \( \alpha_H < (2 - \alpha_L)/3 \)).

A.2. The two-part tariff model

Based on the third-stage pricing game results (see Table 4), we maximize the Internet mall’s profit with respect to \( T = (F, f) \) subject to the stores’ participation constraints. We solve the model for \( r \geq ((2 - \alpha_L)x/3\alpha_B) \) and \( \alpha_H < (2 - \alpha_L)/3 \).

1. When only the high awareness store joins (\( H, N \)), we can set up the constrained maximization problem as follows:

\[ \text{max}_{T=(F, f)} P_{HN} = F + \frac{f R_{HN}}{R_{HN}} - H_S \]
\[ \text{s.t. } R_{HN}(1-f) - F \geq R_{HN} - H_S \geq 0; \]
\[ R_{HN} - H_S \geq R_{HL}(1-f) - F \geq 0; \]

When we solve for the first order condition, we find that

\[ R_{HN} - H_S < R_{HL}(1-f) + F \geq 0; \]

Therefore, the mall’s profit is:

\[ P_{HN} = F + \frac{f R_{HN}}{R_{HN}} - H_S = R_{HN} - R_{HN} \]

In the \( (H, N) \) situation, we have

\[ P_{HN} = -R_{HN} - R_{HN} = \frac{-(2 - \alpha_L)^2 x}{9\alpha_B} \]
\[ + \frac{[2 - \alpha_L(1 - \alpha_M)]^2 x}{9(\alpha_B + \alpha_L\alpha_M)} \]
\[ \times \frac{\alpha_B[4 - \alpha_L(2 - \alpha_M)]}{\alpha_B^2 + \alpha_L\alpha_M} \]
\[ = \frac{x}{9(\alpha_B + \alpha_L\alpha_M)}[\alpha_B[1 - \alpha_H - \alpha_L(1 - \alpha_M)]] \]
\[ - (1 + \alpha_H)^2 < 0 \]

2. In the NL subgame, we can set up the maximization problem as follows:

\[ \text{max}_{T=(F, f)} F + \frac{f R_{NL}}{R_{NL}} - H_S \]
\[ \text{s.t. } R_{NL} - H_S \geq R_{HL}(1-f) - F \geq 0; \]
\[ R_{NL}(1-f) - F \geq R_{NL} - H_S \geq 0; \]

When we solve for the first order condition, we find that

\[ R_{NL} - H_S < R_{HL}(1-f) + F \geq 0; \]

and \( R_{NL}(1-f) - F - R_{NL} + H_S = 0; \)
Therefore, the mall’s profit is:

$$\Pi_M^{NL} = F + f^{NL}_R - H_S = R^{NL}_L - R^{NN}_L$$

In the \((N, L)\) situation, we have,

$$\Pi_M^{NLs} = R^{NL}_L - R^{NN}_L = \frac{x(1 + \alpha_L)^2}{9(\alpha_B + \alpha_H \alpha_M)} - \frac{(1 + \alpha_L)^2 x}{9 \alpha_B} < 0$$

3. In the \((H, L)\) subgame, the mall’s profit maximization problem can be written as

$$\max_{T=(F, f)} \Pi_M^{HL} = 2F + f(R^{HL}_R - R^{NL}_L) - \left(H_S + \frac{1}{2k} H_S\right)$$

s.t. \(R^{HL}_R(1 - f) - F \geq R^{NL}_L - H_S \geq 0;\)

$$R^{HL}_R(1 - f) - F \geq R^{HN}_R - H_S \geq 0;$$

We can solve this constrained maximization problem and obtain the optimal contract \(T^* = (F^*, f^*)\) and profit \(\Pi_M^*\) for the mall:

$$\begin{align*}
F_M^{HL*} &= H_S - \frac{R^{HN}_R - R^{NL}_L}{R^{HN}_R - R^{HL}_R} \\
F_M^{HL*} &= 1 - \frac{R^{NL}_L - R^{HN}_R}{R^{PL}_L - R^{HL}_L} \quad \text{where} \quad H_S \leq H_S = R^{HN}_R \\
\Pi_M^{HL*} &= R^{HL}_R - R^{NL}_L + R^{HL}_R - R^{HN}_R + H_S \left(1 - \frac{1}{2k}\right)
\end{align*}$$

In the \((H, L)\) situation, To ensure that \(f_M^{HL*} \geq 0\), we need to show that \((R^{HL}_R - R^{NL}_L - R^{HN}_R)(R^{HL}_R - R^{HL}_L) \geq 0\). To simplify the notation, we use \(B\) to represent \(1 - \alpha_M\).

Firstly,

$$R^{NL}_L - R^{NL}_L + R^{HL}_R + R^{HN}_R = \frac{(1 + \alpha_L B^2) x}{9(1 - \alpha_H B - \alpha_L B)}$$

\[9(1 - \alpha_H B - \alpha_L B) + \frac{(2 - \alpha_L B)^2 x}{9(1 - \alpha_H B - \alpha_L B)} - \frac{(2 - \alpha_L)^2 x}{9(1 - \alpha_H B - \alpha_L B)} \geq \frac{x(2 - \alpha_L B)^2 - (1 + \alpha_L B)^2}{9(1 - \alpha_H - \alpha_L B)} \]

Since \(R^{NL}_L - R^{NL}_L = (3x(1 - 2\alpha_L B)/9(1 - \alpha_H - \alpha_L B)) > 0, f_M^{HL*} \geq 0\). We also know that \(R^{NL}_L - R^{NL}_L > 0, f_M^{HL*} < 1\). To ensure that \(F_M^{HL*} \geq 0\), the store setup cost needs to be high enough so that \(H_S \geq H_S = (R^{HN}_R - R^{NL}_L)/(R^{HL}_R - R^{HL}_L)\). In addition, for this to be an equilibrium for the threestage game, we need to make sure that it is more profitable for the mall to include both stores than to include neither (i.e., \(\Pi_M^{HL*} \geq 0\)). This is true when \(H_S \geq H_S = (-R^{HL}_R + R^{NL}_L)/(1 - (1/2k))\).

To summarize, when \(r \geq (2 - \alpha_L)\chi/(3\alpha_B), \alpha_H < (2 - \alpha_L)/3\), and \(\max[H_1, H_2]) \geq H_S \geq H_S\), in equilibrium, the Internet mall enlists both stores charging a fixed monthly fee \(F_M^{HL*} = H_S - ((R^{HN}_R - R^{HL}_R)/(R^{HL}_R - R^{HL}_L))\) and an ARSF of \(f_M^{HL*} = 1 - ((R^{HL}_R - R^{HL}_L)/(R^{HL}_R - R^{HL}_L))\).

A.3. The FF-only model

When \(R^{HN}_R - R^{HL}_R - R^{NL}_L + R^{HL}_R = 0, H^2_S\) reaches its maximum value \((2(R^{HN}_R - R^{HL}_R)/1 - (1/2k))\) and the FF-only model applies. Since by construction \(k > (1/2), \) we know that \(0 < 1 - (1/2k) < 1\). Therefore, \(H^2_S > 2(R^{HN}_R - R^{HL}_R)\).

We compare \(H^2_S\) and \(H^2_S\) by taking the difference

$$H^2_S - H^2_S = 2(R^{HN}_R - R^{HL}_R) - \frac{R^{RH}_R - R^{NL}_L - R^{HL}_R}{R^{HL}_R - R^{HL}_R}$$

$$= \frac{2R^{HN}_R - 2R^{HL}_R - 2R^{NL}_R}{R^{HL}_R - R^{HL}_R} + (R^{HL}_R)^2 - R^{HL}_R R^{HL}_R$$

$$= \frac{2R^{HN}_R - R^{HL}_R - R^{NL}_L + R^{HL}_R}{R^{HL}_R - R^{HL}_R}$$

$$= \frac{(R^{HL}_R - R^{HL}_R)(R^{NL}_L - R^{HL}_L) = R^{NL}_L - R^{HL}_L}$$

(Since \(R^{HN}_R - R^{HL}_R - R^{NL}_L + R^{HL}_R = 0\)

It is obvious that \(R^{HN}_R > R^{HL}_R\). Therefore, we conclude that when \(R^{HN}_R - R^{HL}_R - R^{NL}_L + R^{HL}_R = 0, H^2_S > H^2_S\) no matter what the value of \(k\) is. In other words, when \(H^2_S \leq H^2_S\) (where \(H^2_S = (2(R^{HN}_R - R^{HL}_R)/1 - (1/2k))\), the FF-only model applies regardless of the mall’s efficiency.

A.4. The ARSF-only model

The first condition for the ARSF model is \(H_S = H_S = (R^{HL}_R - R^{NL}_L)/(R^{HL}_R - R^{HL}_L)\). The condition for the mall to have positive profit when charging ARSF is \(H_S \geq H_S = (R^{HL}_R - R^{NL}_L)/(R^{HL}_R - R^{HL}_L)\). For both conditions to hold, we find that

$$k \geq \frac{2(R^{HL}_R - R^{HL}_L)}{R^{HL}_R}$$

Since by construction \(k > (1/2), \) we need to compare \(k\) with \(1/2\). In other words, we need to find out whether or not \(R^{HL}_R - R^{HL}_L - R^{NL}_L + R^{HL}_R \geq 1\), which is equivalent to showing

$$R^{HL}_R - R^{HL}_L - R^{NL}_L \geq R^{HL}_R(R^{NL}_L - R^{HL}_L)$$

This inequality can be further simplified as

$$R^{HL}_R - R^{HL}_L(R^{NL}_L - R^{HL}_L) \geq 0.$$
When \( \alpha_H < 2 - \alpha_L / 3 \), we have shown in Appendix A.2 that

\[
R_L^H - R_L^H + R_L^H = \frac{(1+\alpha_L B)^2 x \alpha_H (1-B)}{9(1-\alpha_H B - \alpha_L B)(1 - \alpha_H - \alpha_L B)} + \frac{x(1-B)[\alpha_L^2 (B+1-\alpha_L) + \alpha_H \alpha_B H [4 - \alpha_H (1+ B)]}{9(1-\alpha_H B - \alpha_L B)(1 - \alpha_H - \alpha_L B)} > 0
\]

It is also obvious that \( R_L^R - R_L^R > 0 \). Therefore, \((R_L^H - R_L^H)(R_L^H - R_L^H + R_L^H - R_L^H) > 0 \). We can conclude that when \( \alpha_H < (2 - \alpha_L / 3) \), \( k \geq (R_L^H R_L^H / 2(R_L^H R_L^H - R_L^H (R_L^H + R_L^H))) \geq 1 / 2 \).

A.5. The optimal fee structure

Recall that the two-part tariff model is characterized as

\[
F^* = H_S - \frac{R_L^H R_L^H - R_L^H R_L^H}{R_L^H - R_L^H}
\]

\[
f^* = 1 - \frac{R_L^H - R_L^H}{R_L^H - R_L^H}
\]

\[
\Pi^*_M = R_L^H - R_L^H + R_L^H - R_L^H + H_S \left(1 - \frac{1}{2k}\right)
\]

where \( \max [H_S^2, H_S^2] \leq H_S \leq H_S \).

To find out which of the three fee structures applies given a certain value of the store setup cost (\( H_S \)), we need to compare \( H_S^1 \) and \( H_S^2 \).

\[
H_S^1 = \frac{R_L^H R_L^H - R_L^H R_L^H}{R_L^H - R_L^H}
\]

\[
H_S^2 = \frac{R_L^H R_L^H - R_L^H R_L^H}{1 - (1/2k)}
\]

Since we have shown in Appendix A.2 that \( R_L^H - R_L^H \geq R_L^H - R_L^H \), we know \((2(R_L^H - R_L^H) / (1 - (1/2k)) \leq H_S \leq (2(R_L^H - R_L^H) / (1 - (1/2k)) \)). Also note that as the value becomes more efficient (\( k \) increases), \( H_S^2 \) decreases.

1. In Appendix A.3 we have shown that when \( R_L^H - R_L^H - R_L^H + R_L^H = 0 \), \( H_S^2 \) reaches its maximum value \((2(R_L^H - R_L^H) / (1 - (1/2k)) \) and \( H_S^3 > H_S^1 \). Therefore, when \( H_S^2 \leq H_S \leq H_S \), the FF-only model applies.

2. Recall that in the ARSF-only model (A.4), \( H_S = H_S^2 \geq H_S^2 \).

When \( k \) is large enough (\( k \geq (2 - 1/2) \)), \( H_S^2 \geq H_S^2 \).

3. Finally, from the above analysis, we conclude the following

- Regardless of the mall’s efficiency (\( k \)), when the store setup cost is high \( H_S \geq H_S \) (where \( H_S^2 = (2(R_L^H - R_L^H) / (1 - (1/2k)) \)), the FF-only model applies.

- The adoption of the two-part tariff and the ARSF-only model depends on the efficiency of the mall.

When \( k \geq k > (1/2) \), \( H_S^1 \geq H_S^2 \). Therefore, the equilibrium condition becomes \( H_S^1 \leq H_S \leq H_S \). In this condition, two-part tariff model applies; When \( H_S = H_S^1 \geq H_S^2 \) ARSF-only model applies.

When \( k > k > (1/2) \), \( H_S^1 < H_S^2 \). Therefore, the equilibrium condition becomes \( H_S^2 \geq H_S \leq H_S \) (where \( 2(R_L^H - R_L^H) / (1 - (1/2k)) \) \( H_S^2 \leq (2(R_L^H - R_L^H) / (1 - (1/2k)) \)). In this condition, the two-part tariff model applies.

In other words, for an Internet mall with low efficiency (\( k > k > (1/2) \)), when store setup cost is high \( H_S \geq H_S \) and \( H_S^2 = (2(R_L^H - R_L^H) / (1 - (1/2k)) \) FF-only model is as profitable as two-part tariff model; When store setup cost is low \( H_S^2 \geq H_S \leq H_S^2 \) (\( 2(R_L^H - R_L^H) / (1 - (1/2k)) \)) \( H_S^2 < (2(R_L^H - R_L^H) / (1 - (1/2k)) \)), an Internet mall with low efficiency should adopt a two-part tariff model. On the other hand, an Internet mall with high efficiency (\( k \geq k > (1/2) \)) should adopt a FF-only model when store setup cost is high \( H_S^2 \geq H_S \leq H_S \), a two-part tariff model when store setup cost is low \( H_S^1 < H_S \leq H_S^1 \), and an ARSF-only model when the store setup cost is even lower \( H_S = H_S^1 \).

A.6. Baseline exclusive awareness of the stores

In order to prove Proposition 5, we need to prove all six cases:

\[
\frac{\partial H_S}{\partial \alpha_H} > 0; \quad \frac{\partial H_S}{\partial \alpha_L} > 0; \quad \frac{\partial H_S}{\partial \alpha_H} > 0; \quad \frac{\partial H_S}{\partial \alpha_L} > 0; \quad \frac{\partial H_S}{\partial \alpha_L} > 0; \quad \frac{\partial H_S}{\partial \alpha_L} > 0;
\]

We continue to use \( B = 1 - \alpha_M \).

1. We first prove \((\partial H_S / \partial \alpha_H) > 0 \) and \((\partial H_S / \partial \alpha_L) > 0 \):

\[
\frac{\partial H_S}{\partial \alpha_H} = \frac{(1 + \alpha_L (1 - \alpha_M))^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)} = \frac{(1 + \alpha_L B)^2 x (\partial H_S / \partial \alpha_H)}{27(1 - 2\alpha_M)}
\]

It is straightforward that \((\partial H_S / \partial \alpha_H) > 0 \).

To examine \((\partial H_S / \partial \alpha_L) \), we take derivative of the first part of \( H_S^2 \) with respect to \( \alpha_L \) and get:

\[
\frac{2B (2 - \alpha_L B) (1 + \alpha_L B)}{1 - 2\alpha_L B} \times \left[ \frac{(2 - \alpha_L B)^2}{1 - \alpha_H - \alpha_L B} - \frac{(2 - \alpha_L B)^2}{1 - \alpha_L - \alpha_H B} \right]
\]
Third, we examine lower bound

\[ \frac{\partial H_{\alpha}}{\partial \alpha} > 0. \]

A.7. Quality difference between the stores

To prove this proposition, we need to show that \( \frac{\partial F^*/\partial x} < 0 \) and \( \frac{\partial F^*}{\partial x} = 0 \).

First, we show that \( \frac{\partial F^*}{\partial x} < 0 \). The FF component when baseline exclusive awareness of the high awareness store is low can be written as:

\[
F^* = H_S = \frac{R_{HL}^H R_{HH}^H - R_{HL}^N R_{HH}^N}{27[1 - 2\alpha_L(1 - \alpha_M)]} \]

Therefore,

\[
\frac{\partial F^*}{\partial x} = \frac{\alpha_B + \alpha_L\alpha_M - ((2 - \alpha_L)^2/\alpha_B + \alpha_H\alpha_M)}{27[1 - 2\alpha_L(1 - \alpha_M)]} < 0
\]

When we examine \( f^* = 1 - (R_{HL}^H + R_{HL}^N) / R_{HL}^H - R_{HL}^L ) \), we find that \( f^* \) is not a function of \( x \). Therefore, \( (\partial f^*)/\partial x = 0 \).

A.8. The market expansion model

We first solve for the third stage pricing game. We can summarize the consumer awareness structure in Table A.1; the market shares for the two on-line stores in the four alternative market conditions can be summarized in Table A.2; and the optimal prices, market shares of the two on-line stores in the market expansion model can be found in Table A.3. The optimal sales revenues can be obtained by \( R_{j}^{XY} = P_{j}^{XY} \times M_{j}^{XY} \).

1. Proposition 7

When we compare the cutoffs for \( r \), it is straightforward that \( P_{NN}^H > P_{NL}^H > P_{NL}^N > P_{HL}^H \). Then we compare \( P_{HL}^H \) and \( P_{HN}^H \) and find that \( P_{HL}^H > P_{HN}^H \). Therefore, \( r \geq (2 - \alpha_L)/3\alpha_B \).

When we compare the cutoffs for \( \alpha_H \), we find that \( \alpha_L + 2\alpha_B + \alpha_L\alpha_M - \alpha_N\alpha_M < 2\alpha_B + \alpha_L + (2\alpha_M / 1 - \alpha_M), \alpha_L + 2\alpha_B + \alpha_L\alpha_M - \alpha_N\alpha_M < (\alpha_L + \alpha_N\alpha_M + 2\alpha_B / 1 - 3\alpha_M), \) and \( (2 - \alpha_L) < \alpha_L + 2\alpha_B + \alpha_L\alpha_M - \alpha_N\alpha_M. \) Therefore, we have \( \alpha_H \leq (2 - \alpha_L)/3 \).

Now we compare the optimal prices of the high (low) awareness store in the above table with those in Table 4. It is obvious that when stores join (i.e., in subgame \((N, L), (H, N)\) and \((H, L)\)), prices are lower in the market expansion model compared to the basic model when consumer reservation price is
Table A.1
Market expansion model: consumer awareness structure

<table>
<thead>
<tr>
<th>XY</th>
<th>Aware-high-only</th>
<th>Aware-low-only</th>
<th>Aware-both</th>
<th>Unawared</th>
</tr>
</thead>
<tbody>
<tr>
<td>NN</td>
<td>( a_H )</td>
<td>( a_L )</td>
<td>( a_B )</td>
<td>( a_N - a_L a_M )</td>
</tr>
<tr>
<td>NL</td>
<td>( a_H - a_H a_M )</td>
<td>( a_L + a_N a_M )</td>
<td>( a_B + a_H a_M )</td>
<td>( a_N - a_L a_M )</td>
</tr>
<tr>
<td>HN</td>
<td>( a_H + a_B a_M )</td>
<td>( a_L - a_L a_M )</td>
<td>( a_B + a_L a_M )</td>
<td>( a_N - a_L a_M )</td>
</tr>
<tr>
<td>HL</td>
<td>( a_H - a_L a_M )</td>
<td>( a_L - a_H a_M )</td>
<td>( a_B + a_H a_M + a_N a_M )</td>
<td>( a_N - a_L a_M )</td>
</tr>
</tbody>
</table>

sufficiently high \((r \geq ((2 - \alpha_L)\alpha_B / 3)/\alpha_B))\) and the two on-line stores have similar consumer awareness \((\alpha_H \leq (2 - \alpha_L / 3))\).

2. Proposition 8

Now we investigate the stores’ joining decisions. When only the high awareness store joins, the mall’s profit is \(\Pi_H^{\text{NN}} = -R_H^{\text{NN}} + R_H^{\text{NN}}\). Since \(P_H^{\text{NN}} > P_L^{\text{NN}}\) and \(M_H^{\text{NN}} > M_H^{\text{NN}}\), \(\Pi_H^{\text{NN}} > 0\).

In the \((N, L)\) subgame where only the low awareness store joins, \(\Pi_L^{\text{NN}} = -R_L^{\text{NN}} + R_L^{\text{NN}}\). It is obvious that \(P_L^{\text{NN}}\) and \(M_L^{\text{NN}}\) are not functions of \(\alpha_N\). It is also straightforward that \((\partial M_L^{\text{NN}}/\partial \alpha_N) > 0\). In terms of \(P_L^{\text{NN}}\), we find that

\[
\frac{\partial P_L^{\text{NL}}}{\partial \alpha_N} = \frac{x}{3} \left\{ \frac{1 + \alpha_L - \alpha_B(1 - 2\alpha_M)}{[1 - \alpha_L - \alpha_B(1 - \alpha_M) - \alpha_B]^2} \right\}
\]

In other words, if \(\alpha_N\) is sufficiently large, there exists an equilibrium where only the low awareness store joins. And as \(\alpha_N\) increases, \(\Pi_L^{\text{NL}}\) becomes larger.

A.9. Stores with dissimilar awareness

We now examine equilibrium conditions when consumer reservation price is high enough \((r \geq ((2 - \alpha_L)\alpha_B / 3)/\alpha_B))\) and when the two stores have dissimilar consumer awareness \((\alpha_H > (2 - \alpha_L(1 - \alpha_M) / (3 - \alpha_M)))\):

Table A.2
Market expansion model: market shares

<table>
<thead>
<tr>
<th>XY</th>
<th>Market share for the high awareness store</th>
<th>Market share for the low awareness store</th>
</tr>
</thead>
<tbody>
<tr>
<td>NN</td>
<td>( a_H + a_B(1 - Z^{\text{NN}}) )</td>
<td>( a_L + a_B Z^{\text{NN}} )</td>
</tr>
<tr>
<td>NL</td>
<td>( a_H + a_B - (a_B + a_H a_M)Z^{\text{NL}} )</td>
<td>( a_L + (a_B + a_H a_M)Z^{\text{NL}} + a_N a_M )</td>
</tr>
<tr>
<td>HN</td>
<td>( a_H + (a_B + a_H a_M)(1 - Z^{\text{NN}}) + a_N a_M )</td>
<td>( a_L(1 - a_M) + (a_B + a_H a_M)Z^{\text{HN}} )</td>
</tr>
<tr>
<td>HL</td>
<td>( a_H - a_N a_M + [a_B + (1 - a_B) a_M](1 - Z^{\text{NL}}) )</td>
<td>( a_L - a_L a_M + [a_B + (1 - a_B) a_M]Z^{\text{HL}} )</td>
</tr>
</tbody>
</table>

Table A.3
Market expansion model: optimal prices and market shares

<table>
<thead>
<tr>
<th>XY</th>
<th>( P_L^{\text{NN}} )</th>
<th>( M_L^{\text{NN}} )</th>
<th>( P_L^{\text{NL}} )</th>
<th>( M_L^{\text{NL}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NN</td>
<td>( \frac{2 - a_L}{3} \alpha_B )</td>
<td>( \frac{2 - a_L}{3} \alpha_B )</td>
<td>( \frac{1 + a_L}{3} \alpha_B )</td>
<td>( \frac{1 + a_L}{3} \alpha_B )</td>
</tr>
<tr>
<td>NL</td>
<td>( \frac{2 - a_L - 2 a_B(1 - a_M)}{3(\alpha_B + a_H a_M)} )</td>
<td>( \frac{2 - a_L - 2 a_B(1 - a_M)}{3(\alpha_B + a_H a_M)} )</td>
<td>( \frac{2 - a_L - a_B(2 - a_M)}{3(\alpha_B + a_H a_M)} )</td>
<td>( \frac{2 - a_L - a_B(2 - a_M)}{3(\alpha_B + a_H a_M)} )</td>
</tr>
<tr>
<td>HL</td>
<td>( \frac{2 - a_L(1 - a_M) - 2 a_B(1 - a_M)}{3(\alpha_B + a_H a_M)} )</td>
<td>( \frac{2 - a_L(1 - a_M) - 2 a_B(1 - a_M)}{3(\alpha_B + a_H a_M)} )</td>
<td>( \frac{2 - a_L(1 - a_M) - 2 a_B(1 - a_M)}{3(\alpha_B + a_H a_M)} )</td>
<td>( \frac{2 - a_L(1 - a_M) - 2 a_B(1 - a_M)}{3(\alpha_B + a_H a_M)} )</td>
</tr>
</tbody>
</table>
In this special case, it is more profitable for the low awareness store to cut the price by \( x \) and get all the aware-both shoppers. As a result, the high awareness store gets the aware-high-only shoppers and the low awareness store gets the rest of the market. The optimal price for the high awareness store is the consumer reservation price \( r \) and the low awareness store will charge \( r - x \). Prices and market shares in all four pricing subgames can be summarized as follows:

\[
\begin{align*}
  p^H_{NL} &= r; &  p^H_{L} &= r - x; & M^H_{NL} &= \alpha_H; & M^H_{L} &= 1 - \alpha_H; \\
  p^L_{NL} &= r; &  p^L_{L} &= r - x; & M^L_{NL} &= \alpha_H(1 - \alpha_M); & M^L_{L} &= 1 - \alpha_H(1 - \alpha_M); \\
  p^H_{HL} &= r; &  p^H_{L} &= r - x; & M^H_{HL} &= \alpha_H; & M^H_{L} &= 1 - \alpha_H \\
  p^L_{HL} &= r; &  p^L_{L} &= r - x; & M^L_{HL} &= \alpha_H(1 - \alpha_M); & M^L_{L} &= 1 - \alpha_H(1 - \alpha_M)
\end{align*}
\]

1. When only the high awareness store joins \((H, N)\), the two participation constraints for the stores are:

\[
\begin{align*}
  R^H_{HN}(1 - f) - F &\geq R^N_{HN} - H_S \geq 0 \\
  R^N_{HN} &\geq R^N_{HL}(1 - f) - F \geq 0 
\end{align*}
\]

which is equivalent to

\[
\begin{align*}
  R^H_{HN} f + F &\leq H_S \\
  R^H_{HL} f + F &\geq H_S + (r - x)\alpha_H\alpha_M
\end{align*}
\]

When we examine these two constraints, we find that since \( R^H_{HN} > R^H_{HL} \) and \((r - x)\alpha_H\alpha_M > 0\), these two participation constraints cannot be satisfied simultaneously.

2. In the \(NL\) subgame, the two participation constraints are:

\[
\begin{align*}
  F^N_{NL} + R^N_{NL} f^N_{NL} &\leq H_S + (r - x)\alpha_H\alpha_M \\
  F^N_{M} + R^H_{ML} f^N_{ML} &\leq H_S 
\end{align*}
\]

The two constraints cannot be simultaneously satisfied.

Therefore, we check for boundary conditions and find that when \( \{f^N_{M} = H_S + (r - x)\alpha_H\alpha_M; f^N_{ML} = 0\} \) or when \( \{f^N_{M} = 0; f^N_{ML} = (H_S + (r - x)\alpha_H\alpha_M / R^N_{NL})\} \) the mall’s profit is

\[
\Pi^N_M = F^N_{M} + R^N_{NL} f^N_{NL} - H_S = (r - x)\alpha_H\alpha_M
\]

In this case, the low awareness store is indifferent between joining and not joining. We also need to make sure that \( H_S \leq R^N_{NL} \) and \( H_S \leq R^N_{NN} \) so the stores exist. Since \( R^N_{NL} < R^N_{NN} \), we only need to make sure \( H_S < R^N_{NN} \).

3. In the \((H, L)\) subgame, the mall’s profit maximization problem can be written as

\[
\max_{r, f} \Pi^H_M = 2F + f(R^H_{HL} + R^H_{L}) - H_S \left(1 - \frac{1}{2k}\right) \\
\text{s.t.} \\
R^H_{HL}(1 - f) - F \geq R^H_{HL} - H_S \geq 0; \\
R^H_{L}(1 - f) - F \geq R^H_{L} - H_S \geq 0;
\]

Similar to the \((N, L)\) case, we check for boundary conditions. We find that when \( \{f^H_{M} = H_S; f^H_{ML} = 0\} \) the mall’s profit is

\[
\Pi^H_M = 2F + f(R^H_{HL} + R^H_{L}) - H_S \left(1 - \frac{1}{2k}\right) \\
= H_S \left(1 - \frac{1}{2k}\right)
\]

In this case, the high awareness store is indifferent between joining or not joining. However, the low awareness store is better off joining. To make sure the stores exist, we need to satisfy \( H_S \leq R^N_{HN} \) and \( H_S \leq R^N_{NL} \). Since \( R^N_{HN} < R^N_{HL} \), we only need to make sure \( H_S \leq R^N_{HN} = R^N_{HL} \).

When we compare the mall’s profit in the \((N, L)\) case and the \((H, L)\) case, we find that when

\[
H_S \left(1 - \frac{1}{2k}\right) \geq (r - x)\alpha_H\alpha_M \Leftrightarrow H_S \geq \frac{(r - x)\alpha_H\alpha_M}{1 - (1/2k)}
\]

enlisting both stores is more profitable for the mall.

References


