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Research Note

Improving the Efficiency of Course Bidding at Business Schools: Field and Laboratory Studies

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Registrars' offices at most universities face the daunting task of allocating course seats to students. Because demand exceeds supply for many courses, course allocation needs to be done equitably and efficiently. Many schools use bidding systems in which student bids are used both to infer preferences over courses and to determine student priorities for courses. However, this dual role of bids can result in course allocations not being market outcomes, and in unnecessary efficiency loss, which can potentially be avoided with the use of an appropriate market mechanism. We report the result of field and laboratory studies that compare a typical course-bidding mechanism with the alternate Gale-Shapley Pareto-dominant market mechanism. Results from the field study (conducted at the Ross School of Business, University of Michigan) suggest that using the latter could vastly improve efficiency of course allocation systems while facilitating market outcomes. Laboratory experiments with greater design control confirm the superior efficiency of the Gale-Shapley mechanism. The paper tests theory that has important practical implications because it has the potential to affect the learning experience of very large numbers of students enrolled in educational institutions.

Key words: auction; bidding; consumer behavior; experimental economics; field experiment; indivisible goods; market design; matching; mechanism design
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1. Introduction

Allocation of course seats to students is a challenging task for registrars' offices in universities. Because the learning experience of students is a direct function of the courses they take, and demand for many courses often exceeds supply, it is important that courses are allocated equitably and efficiently across students. Although some schools use preference-ranking mechanisms (revealed ordinal preferences) to allocate courses (e.g., Stanford Business School, Harvard Business School), many others use bidding mechanisms (or revealed cardinal preferences) for the same purpose (e.g., Columbia Business School, Yale School of Management). Several schools have recently moved from preference-ranking mechanisms to bidding mechanisms (e.g., Ross School of Business at the University of Michigan, henceforth referred to as UMBS), considering the latter superior in terms of efficiency. However, under the bidding mechanisms, bids are not only used to infer student preferences, but also to determine student priorities (if you bid higher, you have a higher likelihood of getting the course) for courses. This dual role of bids results in the schools' course allocations not being market outcomes—that is, the announced "prices" for courses and the announced "course allocation" do not actually clear the market. In other words, students can be better off with alternate course allocations versus their allotted course allocation, and they can afford to "buy" these schedules at the announced course prices. Thus, the current bidding mechanism results in unnecessary efficiency loss (Sönmez and Ünver 2005). Although theory predicts this efficiency loss, its existence and magnitude have not yet been tested. As such, we do not know if policy makers should expend the effort to move away from the current bidding mechanisms.

We report the results of field and laboratory studies that compare a typical course-bidding mechanism with the alternate *Gale-Shapley Pareto-dominant market mechanism* (GS mechanism). The studies are designed to test whether typical bidding systems (in particular the UMBS mechanism) result in efficiency loss in real-life applications, and if so, how much efficiency improvement can be obtained through a transition to a market mechanism such as the GS mechanism.

The field study was carried out at UMBS immediately after the bidding period for the spring 2004 semester. It shows that the current systems in place can be vastly improved in terms of efficiency, making a large proportion of students (approximately 20% in our study) better off. The laboratory experiments confirm the results of the field study. The laboratory experiments allow for greater control of variables such as students' preferences over the courses, and students' beliefs about other students' course preferences. In the laboratory experiment, we also provide students with monetary incentives consistent with their induced preferences so that we can examine students' strategic behavior more directly.

The key to increasing course allocation efficiency is *separating* the two roles of the bids by simply asking students to submit their preference ranking of the courses in addition to bids for the courses. We are in the fortunate situation in which the change is simple to implement—yet our results have the potential to affect the learning experience of very large numbers of students enrolling in business schools and other institutions that use similar bidding mechanisms for course allocation.

Although experiments in the experimental economics tradition to test normative theory have recently become more popular (see, e.g., Ho and Weigelt 1996, 2005; Rapaport et al. 1995; Zwick and Chen 1999; Srivastava et al. 2000; Amaldoss et al. 2000; Haruvy et al. 2001; Amaldoss and Jain 2002), field studies to test theory are still rare—this is not surprising because field studies that can test theory are seldom possible to conduct. The paper falls within the domain of mechanism design, i.e., designing a mechanism so that the outcome of the mechanism is consistent with properties a manager wants the mechanisms to have, e.g., efficiency, fairness, or profit maximization, etc., which is also new to management science (for some exceptions, see Katok and Roth 2004, Wang and Krishna 2006, and Krishna and Wang 2007). Thus, the paper contributes by adding to the very sparse literature on field studies, to the literature on mechanism design, and by testing theory that can have important ramifications in practice.

The rest of the paper is organized as follows. We first elaborate on prior literature related to this paper. We then describe currently used course-bidding mechanisms. This is followed by a detailed description of course bidding at UMBS, where we did our field study. We next describe the alternative GS mechanism that we test, and then describe the results from the field study. After that, we discuss the laboratory experiments we conducted to confirm the results of the field study under conditions of greater control. We end with implications and limitations of the research.

2. Literature Background and Related Research on Matching Market Design

This research falls into the area of market design, more specifically, mechanism design for real-life *matching problems*. A matching problem entails resource allocation across agents, where agents have preferences over scarce resources; in two-sided matching problems, resources also have preferences over agents. One of the most commonly used and popularly known matching models is due to Gale and Shapley (1962), known as the marriage model or two-sided matching model. The two-sided matching problem consists of two sets of players—firms and workers—that need to be matched with each other using preferences of firms over workers and of workers over firms. The central solution concept in this domain is stability, i.e., finding matchings of firms with workers, and the same workers with the same firms, so that no firm-worker pair would rather be matched with each other versus their allotted partners (and no firm or worker would rather stay unmatched than be with their allotted partner). Gale and Shapley also proposed two stable matching mechanisms in their seminal paper. Many real-life markets have adopted mechanisms based on Gale-Shapley mechanisms: Roth and Peranson's (1999) redesign for the American hospital-intern matching mechanism is a variant of the Gale-Shapley (1962) intern-optimal stable mechanism. This mechanism was adopted by the National Resident Matching Program, a centralized clearinghouse for the entry-level labor market for new physicians in the United States.

Our problem, allocation of course seats to students, substantially differs from a two-sided matching problem. Our problem is a variant of the house allocation problem, in which indivisible objects (houses) need to be assigned to agents, each of whom has preferences over these objects—agents are the only players in this model, objects are not players. Random serial dictatorship is one of the most used mechanisms for this problem, in which agents are randomly ordered in a linear order and agents choose their favorite object among the available ones, one at a time, according to this order. In many North American college campuses, variants of this mechanism are used to allocate dormitory rooms to students (Abdulkadiroğlu and Sönmez 1999, and a subsequent experimental test by Chen and Sönmez 2002). Course allocations in Stanford Graduate School of Business School and Harvard Business School are done using systems that are variants of this approach.

Starting with Balinski and Sönmez (1999), many studies have shown similarities between plausible mechanisms for the house allocation problem and plausible mechanisms for the two-sided matching problem. If there is additional structure to the house allocation problem, such as priorities of agents over the houses, then mechanisms that are created for two-sided matching can be used for these problems as well. The underlying idea is to convert the object allocation problem into an induced two-sided matching problem by treating the priorities of the agents as "the preferences of the objects over agents." Balinski and Sönmez showed that the Turkish college admissions mechanism used for placing high school students in colleges in Turkey (based on exam scores determined by a centralized college admissions test) is equivalent to the Gale-Shapley college-optimal stable mechanism for the induced two-sided matching problem. Similarly, Abdulkadiroğlu and Sönmez (2003) proposed the Gale-Shapley student-optimal stable mechanism for student admissions to K-12 schools in U.S. public schools, where students have priorities for schools in their neighborhoods, constituted by the U.S. constitution (based on where they live, sibling policy, etc.). These priorities can be used to induce preferences of schools over students to create an induced two-sided matching market; then the Gale-Shapley student-optimal stable mechanism can be used in this induced two-sided matching market to find a stable matching.

When bidding is used as a tool in the course allocation problem with the intent of reaching market outcomes (as done in many business schools), we can induce a two-sided matching market using student bids for each course as induced preferences of the courses, i.e., the courses are assumed to prefer students who bid a higher amount for them (Sönmez and Ünver 2005). Sönmez and Ünver proposed the Gale-Shapley student-optimal stable mechanism as an alternative to the current bidding mechanisms used in many business schools. They showed that this mechanism is not only a market mechanism, but also the Pareto-dominant one among all market mechanisms. In this paper, we test this extension of the Gale-Shapley student-optimal stable mechanism (that will be referred to as the Gale-Shapley Pareto-dominant market mechanism, or GS mechanism hereafter, and will be explained in §5) in a controlled field study.[1]

The prior papers mentioned earlier are theoretical papers. Separate papers have then tested the predictions of the theoretical designs by emulating markets before and after the design, and have then shown to be useful in practice. In several cases, the mechanisms have subsequently been adopted in the real world. As described earlier, Abdulkadiroğlu and Sönmez (2003) propose a matching mechanism for the U.S. public school students that they showed was more efficient than the mechanism in place in Boston. Their theory was tested by Chen and Sönmez (2006). The mechanism was then adopted by New York City high schools in 2004 and by Boston public schools in 2005.

We hope that our study, which tests the Sönmez and Ünver (2005) theory in the field, will also influence policy makers to consider a new market design. Moreover, our experiment is the first (controlled) field study in the matching literature.

Next, we discuss currently used bidding mechanisms.

3. Currently Used Course Bidding Mechanisms

Most business schools have historically used either bid-based or preference-based mechanisms for allocating courses. For example, Ross School of Business at the University of Michigan (UMBS), Kellogg Graduate School of Management at Northwestern, Johnson Graduate School of Management at Cornell, Columbia Business School, Haas School of Business at UC Berkeley, Yale School of Management, and INSEAD rely on versions of a bidding mechanism that we refer to as the *UMBS mechanism*. Some law schools also rely on bidding systems for course allocation, e.g., the School of Law at the University of Colorado at Boulder. Harvard Business School and Stanford Graduate School of Business rely on preference-based course allocations mechanisms. Recently, there appears to be a shift from preference-based to bid-based mechanisms, UMBS being an example. In this paper, we focus on bid-based mechanisms. However, some details of the preference-based allocation systems are provided in Appendix A. Appendix A also describes some variants of the bid-based course allocation mechanisms.

Under the UMBS bid-based mechanism, each student is given a bid endowment $B > 0$ at the beginning of each semester. To keep the notation at a minimum, we assume that the bid endowment is the same for each student. This is the case at UMBS, where we conducted our field study. Each student is asked to allocate her bid endowment among the courses, and once all bids are submitted, course seats are assigned to students as follows:

1. *All* bids for *all* courses and *all* students are ordered in a *single* list from highest to smallest. A tie-breaking lottery is used to determine the relative ordering of two bids of the same size. Thus, if each student i places k_i bids of exactly x points, the tie-breaking lottery determines the order of these $\sum_i k_i$ bids. For instance, a tie-breaking lottery may add

[1] Marketing papers that have examined bidding behavior include Dholakia and Simonson (2005) and Fay (2004); those more focused on mechanism design include Shugan (2005) and Wang and Krishna (2006).

a random number from the uniform distribution $(0, 1)$ for each student course, which is then added to the student bid to obtain a modified bid. Thus, if two students each submit two bids of 50 points each, their modified bids will lie between 50 and 51.

2. Each bid is considered, one at a time, following the order in the list. When it is the turn of bid b_{ic} of student i for course c to be considered, the bid is *successful* if (a) course c still has unfilled seats, (b) student i still has unfilled slots in her schedule, and (c) course c does not conflict with any of the courses that are assigned to student i so far. If the bid is successful, then student i is assigned a seat at course c (i.e., the bid is honored), and the process proceeds with the next bid in the list. Otherwise, student i is declined a seat at course c, and the process continues with the next bid in the list.

3. When all bids are handled, a *schedule* is obtained for each student and a *course allocation* is hence obtained.

Example 2, given later, and instructions to subjects in the laboratory experiment described in Appendix C provide examples of how the UMBS mechanism is used.

The *market-clearing bid* or *price* for each course is the lowest successful bid if all course seats are allocated, and zero otherwise.

Bids have two roles under the UMBS mechanism:

1. Bids are used to infer student preferences over the courses. Consider the following statement from the guidelines for "Allocation of Places in Oversubscribed Courses and Sections" at the School of Law, University of Colorado at Boulder:

> Each student has 100 bidding points for each semester. You can put all your points in one course, section or seminar, or you can allocate points among several. By this means, you express the strength of your preferences.

2. Bids are also used to determine who has a bigger claim on each course seat, and therefore choice of a bid-vector is an important strategic tool.

The following statement from the bidding instructions of both Columbia Business School and Haas School of Business at UC Berkeley shows that these two roles may easily conflict.

> If you do not think a course will fill up, you may bid a token point or two, saving the rest for courses you think will be harder to get into.

Thus, the schools themselves suggest to students that they game the system and that student's bids need not be aligned with their preferences. Therefore, what happens when bids are used for both purposes (to infer preferences and to determine seat claims) is that students may bid a high number of points on more popular courses, and bid few points on less popular courses, even if they prefer the latter courses. However, it is easy to see that this conflict may result in efficiency loss because a student may be denied a seat in one or more of her preferred courses, despite "clearing the market" (i.e., although her bid is high enough), simply because she clears the market in too many other less-preferred courses for which she has submitted higher bids.

Sönmez and Ünver (2005) show that efficiency may be lost even if the students are expected utility maximizers, and therefore there is no reason to expect that such efficiency loss is a rare event, or a mistake. The example below demonstrates this point. Sönmez and Ünver (2005) also describe how the abovementioned efficiency loss can be avoided. The key is separating the two roles of the bids by simply asking students to submit their preference ranking of the courses in addition to bidding for the courses. In this way, the registrar's office no longer needs to guess what student preferences are. Once the bids and preference ranks are obtained, the GS mechanism may be used for improved efficiency, which we describe in §5.

Example 1. Consider a student who will register for up to three courses, and suppose there are four courses c_1, c_2, c_3, c_4 that she likes. Her utility for each individual course is given as follows:

Courses	c_1	c_2	c_3	c_4
Utility $u(c)$	60	50	50	50

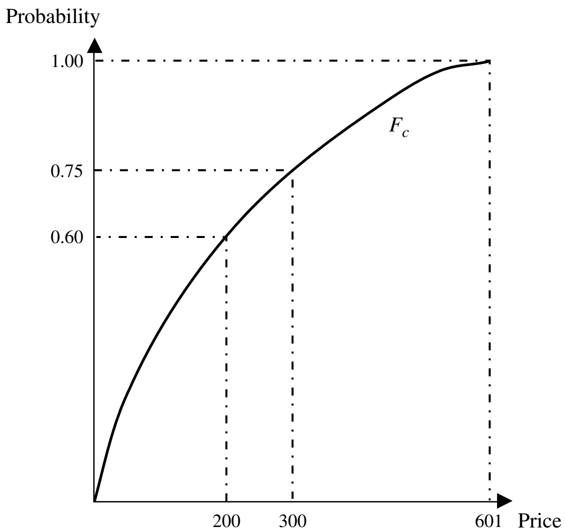
and her utility for a schedule s, consisting of no more than three courses, is additively separable. That is, the utility of the student from s is the total of the utilities she gets from each course in s, $\sum_{c\in s} u(c)$.

Suppose that the student has a total of $B = 601$ points to bid over all courses and the minimum acceptable bid is one. Based on previous years' bid data, the student has formed beliefs about the market-clearing bids of the relevant courses for her:

- The market-clearing bid for course c_1 will be zero with probability one.
- The market-clearing bids for the courses c_2, c_3, and c_4 have independent identical cumulative distribution functions, and for any of these courses c, the cdf F_c is strictly concave with $F_c(200) = 0.6$, $F_c(300) = 0.75$, and $F_c(601) = 1$ (see Figure 1). That is, for each of the courses c_2, c_3, and c_4, the student believes that the market-clearing bid will be no more than 200 with 60% probability and no more than 300 with 75% probability.[2]

[2] We could have allowed some *correlation* among course-clearing prices in this example; we chose not to do so for the sake of clarity. Also note that having *independent* price distributions across courses is not an unrealistic assumption if the number of the students and courses are large, and therefore the effect of one course's market-clearing bid on the others' is small.

Figure 1 **A Student's Beliefs on the Market-Clearing Prices for Courses c_2, c_3, and c_4**



Assuming that she is an expected utility maximizer, we next find the optimal bid-vector for the student: By first-order necessary conditions and symmetry, the student will bid

- one point for course c_1,
- the same value for each course $c \in \{c_2, c_3, c_4\}$ for which she devotes a positive bid, and
- no points on any other course.

Therefore, the optimal bid-vector is in the form: $b_{c_1} = 1$, $b_c = 600/k$ for any k of courses c_2, c_3, and c_4. We find the optimal value of k as follows:

- By bidding 600 points for $k = 1$ course among c_2, c_3, and c_4, her expected utility is $\text{EU}_1 < 110$.[3]
- By bidding 300 points for $k = 2$ courses among c_2, c_3, and c_4, her expected utility is $\text{EU}_2 = 135$.[4]
- By bidding 200 points for all $k = 3$ courses c_2, c_3, and c_4, her expected utility is $\text{EU}_3 = 137.04$.[5]

Therefore, the optimal bids of the student are one point for c_1 and 200 points for each of c_2, c_3, and c_4, and zero points for the other courses.

When her beliefs are correct, with probability one the market-clearing bid of course c_1 will be zero (meaning that course c_1 is undersubscribed) and with probability $0.6^3 = 0.216$ the market-clearing bids of courses c_2, c_3, and c_4 will be less than or equal to 200. In this case, under the UMBS mechanism, because her highest bid is 200 for each of the courses c_2, c_3, and c_4, the student will be placed in these three courses; and because her quota is only three courses, she will not be enrolled in her favorite course c_1, although her bid of one point is higher than the market-clearing bid of zero points for c_1. This is a clear source of inefficiency under a market approach: She would be better off by enrolling in c_1 instead of one of the other courses and she has the right to do so because her bid clears the market for c_1. Moreover, there would be an extra seat in one of the courses c_2, c_3, or c_4 with the potential of making at least one other student better off (more students can be better off through ripple effects).

Therefore, the outcome of the UMBS course-bidding mechanism cannot be supported as a *market outcome*,[6] and this weakness is a direct source of efficiency loss. To summarize:

1. How much a student bids for a course under the UMBS course-bidding mechanism is not necessarily a good indication of how much a student wants that course;
2. as an implication, the outcome of the UMBS course-bidding mechanism cannot always be supported as a market outcome; and
3. the UMBS course-bidding mechanism may result in unnecessary efficiency loss due to not seeking direct information on student preferences.

We next describe in detail the bidding environment at the Ross School of Business at the University of Michigan, where we conducted our field study, and give details of the field study.

4. Course Bidding at UMBS: Field Study

The field study addresses whether the potential efficiency loss under the UMBS mechanism occurs in practice, and how prevalent it is. At UMBS, prior to and during the bidding period, students review course descriptions and time schedules on the UMBS intranet. Professors may also make available syllabi or additional information on the course. Once the bidding period begins, students can visit a Webpage

[3] Suppose that the student bids 600 points on c_2. The student can be placed in c_2 with probability $F_{c_2}(600)$. Because she clears c_1 with probability one, she is always placed in c_1, and her expected utility is $\text{EU}_1 = F_{c_2}(600)\sum_{c\in\{c_1,c_2\}} u(c) + (1 - F_{c_2}(600))u(c_1) < 110$.

[4] Suppose that the student bids 300 points on each of c_2 and c_3. She clears both courses with probability $F_{c_2}(300)^2$ (because $F_{c_2}(300) = F_{c_3}(300)$), and she clears only one of them with probability $2(1 - F_{c_2}(300))F_{c_2}(300)$. Because she clears course c_1 with probability 1, she is always placed in it, and her expected utility is $\text{EU}_2 = F_{c_2}(300)^2\sum_{c\in\{c_1,c_2,c_3\}} u(c) + 2F_{c_2}(300)(1 - F_{c_2}(300))\sum_{c\in\{c_1,c_2\}} u(c) + (1 - F_{c_2}(300))^2 u(c_1) = 135$.

[5] In this case, she is placed in c_2, c_3, and c_4 (but not in c_1, because her quota is three courses) with probability $F_{c_2}(200)^3$, because $F_{c_2}(200) = F_{c_3}(200) = F_{c_4}(200)$. She is placed in two of these courses (and also in c_1, because she always clears it) with probability $2(1 - F_{c_2}(200))F_{c_2}(200)^2$, and she is placed in only one of these courses (and also in c_1) with probability $2(1 - F_{c_2}(200))^2 F_{c_2}(200)$. She is only placed in c_1 if she cannot clear any of c_2, c_3, and c_4, which happens with probability $(1 - F_{c_2}(200))^3$. The expected utility of the student is $\text{EU}_3 = F_{c_2}(200)^3\sum_{c\in\{c_2,c_3,c_4\}} u(c) + 3F_{c_2}(200)^2 \cdot (1 - F_{c_2}(200))\sum_{c\in\{c_1,c_2,c_3\}} u(c) + 3F_{c_2}(200)(1 - F_{c_2}(200))^2\sum_{c\in\{c_1,c_2\}} u(c) + (1 - F_{c_2}(200))^3 u(c_1) = 137.04$.

[6] We define a market outcome formally in §5.

within the UMBS intranet that lists all courses available to them. The Webpage also contains information on the bidding system, timetables, "Tips & Tricks," rules and regulations, etc. In addition, it has information on previous market-clearing prices for all the courses.

Each student is allocated 1,000 bid points for the semester. On the Webpage, each course has a place to enter a bid value. As a student bids on a course from her allocated 1,000 bid points, the bid amount is deducted from the 1,000 allocated points and the balance is shown. Once 1,000 points have been allocated, the student is prevented from entering any more bids. Students can adjust these bids as they wish (by deducting from one course, then adding to another), until the bidding period closes.

We got permission from UMBS to collect rank data from students in addition to the bid data. So as not to contaminate the bidding process in place, we collected rank data *after* bidding was over but *before* course allocations were made. This was a (very short) one-week window of time.

4.1. Students and Courses

We have a sample of $n_I = 535$ students who bid for $n_C = 135$ classes scheduled for the spring 2004 semester. Let $I = \{i_1, i_2, \ldots, i_{535}\}$ be the set of students and $C = \{c_1, c_2, \ldots, c_{135}\}$ be the set of classes. Each class is either the sole section of a course or one of the multiple sections of a course. Therefore, we will refer to each class as a *section*. At UMBS, there are two minisemesters in each semester. Each minisemester lasts about 7 weeks, and the semester lasts about 14 weeks. Sections can be scheduled for the whole semester, for the first minisemester, or for the second minisemester. Our section sample consists of 57 first minisemester sections, 47 second minisemester sections, and 31 full-semester sections. Each minisemester-long section is worth 1.5 credits and each full-semester long section is worth 3 credits.

4.2. Feasibility Conditions

In the context of course bidding, there are feasibility conditions on individual schedules as well as feasibility conditions on the course allocation. Although a student can bid for as many sections as she wishes, she can be registered in

1. no more than 9 credits (or an equivalent of 6 minisemester-long sections) for the first minisemester,
2. no more than 9 credits (or an equivalent of 6 minisemester-long sections) for the second minisemester, and
3. no more than 16.5 credits (or an equivalent of 11 minisemester-long sections) for the whole semester.[7]

An additional feasibility constraint on individual schedules rules out any conflict within a schedule. Two sections *conflict* if they are either both sections of the same course or their weekly meeting times overlap. Although a student can bid for two conflicting sections, she cannot be registered in both of them. In our sample, 497 pairs of sections conflict out of 9,045 section pairs. We refer to any set of sections that satisfy these feasibility constraints as a *schedule*.

The last feasibility condition pertains not to individual schedules, but concerns the *course allocation*. Each section has a *capacity*, which is a limit on the number of students who can be registered in the section. Let $q = (q_c)_{c \in C}$ denote the capacity vector of the sections. In our sample, the smallest capacity is 5 and the largest capacity is 430. The most common capacities are 65 and 30, applying to 58 sections and 20 sections, respectively. In our sample, 35 courses received more bids than their capacities.

4.3. Bids

As we already indicated, each student is endowed with 1,000 bid points that she can use to bid across desired sections. Bid points cannot be transferred between semesters, and bids should be integer values. A student should bid at least one point to be registered in a section. Students can bid for as many sections as they wish, including conflicting sections. Let $B = [b_{ic}]_{i \in I, c \in C}$ denote the bid matrix. Here, b_{ic} is the submitted bid of student i for section c, and $b_{ic} = 0$ if student i did not bid for section c. There are 5,665 positive bids in our sample. Many students submitted the same magnitude of bid for multiple sections. Similarly, many sections received the same magnitude of bid from multiple students. We find the most repeated bid to be "1," followed by "100," "2," "50," "150," "200," "5," "10," "13," and "20." The top 10 bids are used a total of 2,135 times. A strict bid ordering is needed to implement the UMBS mechanism, and the administrators at UMBS rely on a tie-breaking lottery for this purpose. A random real number $\phi_{ic} \in (0, 1)$ is drawn from the uniform distribution for each student-section pair (i, c), and each positive bid $b_{ic} > 0$ is modified as $b'_{ic} = b_{ic} + \phi_{ic}$ to break ties. Let $b'_{ic} = b_{ic}$ whenever $b_{ic} = 0$ and let $B' = [b'_{ic}]_{i \in I, c \in C}$ be the modified bid matrix. The administrators at UMBS provided us with their tie-breaking lottery draw for spring 2004.

4.4. Preferences over Sections

We surveyed students to learn their preferences over sections. Within a few hours of the official closure of bidding, we sent each of the 535 students who had submitted bids a customized e-mail asking each student to rank the sections on which she bid. Each e-mail contained an explanation of our study and a list of all

[7] When a student registers for a full-semester section, she consumes 1.5 credits from the first minisemester and 1.5 credits from the second minisemester.

the sections on which the student had bid. The sections were listed in descending order of bid points, but the actual bid points were left off.[8] A permission from the associate dean was obtained to use his name as the sender of this e-mail to lend credibility and urgency to the survey. Two reminder e-mails were sent to students within the same week (see the Technical Appendix at http://mktsci.journal.informs.org for the original and subsequent e-mail messages). We received 489 responses out of a total of 535 students. In addition to 46 missing responses, 32 students submitted preferences with indifferences (although they were specifically asked not to). To measure the efficiency loss of the UMBS mechanism, we need strict preferences for all students (if students are indifferent between two or more courses, it does not matter which of these they get allocated). Using the 489 responses, we constructed a strict preference ranking $P = (P_i)_{i \in I}$ for all 535 students in a manner that favored the UMBS mechanism:

1. For each of the 32 students who indicated indifferences, we broke indifferences in favor of sections for which the student had a higher bid based on the modified bid matrix B'.
2. For each of the 46 students with missing preferences, we assumed that sections with higher bids were preferred to sections with lower bids.

Formally, for any student i with missing preferences and for any two sections c, d where student i submitted positive bids, we assumed that

$$c \, P_i \, d \quad \text{if and only if} \quad b'_{ic} > b'_{id}.$$

This preference construction results in the *lower bound* of the efficiency loss under the UMBS mechanism. That is the case because any efficiency loss here is an implication of students possibly preferring sections for which they have lower bids to sections for which they have higher bids. We say that a student i has *bid-monotonic* preferences if for any two sections $c, d \in C$,

$$c \, P_i \, d \quad \text{if and only if} \quad b'_{ic} > b'_{id}.$$

As we have already emphasized, we assumed that each of the 46 students with missing preferences has bid-monotonic preferences. In addition, among the 489 students who responded to the survey, 82 submitted bid-monotonic preferences. A vast majority of the students, 375 of them, submitted preferences that are *not* bid monotonic, indicating the role of the strategic aspect of bidding in the UMBS process. In other words, 375 of 489, or approximately 77% of students, were gaming the system, and their bids did not reflect their stated preferences. This suggests that the efficiency loss can be quite significant under the UMBS mechanism.

[8] This practice is in favor of the UMBS mechanism, because nonmotivated students may just give ranks 1, 2, 3. Bids lining up with ranks, i.e., bid-monotonic preferences, favor the UMBS mechanism.

4.5. Preferences over Schedules

We need a way to compare alternative course schedules for a student and determine which will be preferable to her. We take a conservative approach here, making the safest assumption.

Given her preferences over courses P_i, a student i *unambiguously prefers* a schedule S to another schedule S' if and only if

1. schedule S has at least as many credits as schedule S' does, and
2. each section in $S \backslash S'$ is strictly preferred to each section in $S' \backslash S$ based on the preference ranking P_i.

Therefore, we will only conclude that a student has an improvement in her schedule if she receives at least as many credits, and also if any replacement in her schedule is a favorable one. Clearly, we will not be able to compare many pairs of schedules, and in such cases we call the welfare comparison *ambiguous*.

Next, we describe the GS mechanism, the best market mechanism for the course-bidding problem.

5. The Gale-Shapley Pareto-Dominant Market Mechanism

The efficiency of course bidding can be improved by adopting the GS mechanism. In this mechanism, students both bid on and rank order sections. Note that students only need to bid and rank courses that they want to get into. Note also that students can realize that they may not get every course in their best schedule; therefore, they may bid for more courses that they are allowed to take. Further, both under GS and UMBS, students may not fill their schedules. If so, they can add classes in the aftermarket (i.e., after course allocation is over). The following notation is useful to ease the description of this mechanism:

Given a preference ranking P_i and a subset of sections $D \subseteq C$, construct the *best schedule* $\mathcal{B}(D)$ as follows: Start by including the best section among sections in D based on the preference ranking P_i. Next, add the second-best section under P_i provided that neither the credit requirements are violated, nor is there any conflict with the section that is already included. Next, add the third-best section under P_i, provided that neither the credit requirements are violated nor are there any conflicts with one or more of the sections that are already included in $\mathcal{B}(D)$. Proceed in a similar way until either the credit requirements do not allow for any addition or there are no sections left with a positive bid. Define $\mathcal{B}(\varnothing) = \varnothing$. The best schedule is preferred to any other schedule. Under the Pareto-dominant market mechanism, students are asked not only to submit their bids, but also their preference ranking over the courses for which they submit positive bids. Based on the student preferences $P = (P_i)_{i \in I}$, the modified bid matrix B', and the capacity vector $q = (q_c)_{c \in C}$, the outcome of the Pareto-dominant market mechanism can be obtained via the following version of the *deferred acceptance algorithm* (Gale and Shapley 1962, Kelso and Crawford 1982):

Step 1. Each student proposes to (i.e., requests a place in) all sections in her best schedule from the set of all sections C. Best schedule implies the most preferred (or highest ranked) sections totaling the maximum credit that a students is allowed to take. Each course c rejects all but the highest bidding q_c students among those who have proposed. Those who are not rejected are *kept on hold*.

In general, at

Step t. Each student proposes to all sections in her best schedule from the set of sections which have not rejected her in earlier steps. Each course c rejects all but the highest-bidding q_c students among those who have proposed. Those who are not rejected are *kept on hold*.

The procedure terminates when no proposal is rejected, and at this stage the course assignments are finalized by assigning each student the courses which keeps her on hold. The market-clearing bid or price of each course is the lowest successful bid, if all course seats are filled, and zero, otherwise.

Note that this mechanism is a direct revelation mechanism, i.e., students submit their preferences and bids for the courses, and the intermediate steps of the algorithm are implemented by a computer program on behalf of the students according to the submitted preferences and bids.

Example 2 below and instructions to subjects in the laboratory experiment described in Appendix B provide examples of how the GS mechanism is used.

The *course-bidding market* we consider consists of preferences of students over schedules and a bid matrix of students. A course allocation and a price vector pair is a *market equilibrium* of this course-bidding market if (i) each student can "afford" to buy the courses in her assigned schedule under the announced price vector and (ii) for each student there is no other schedule such that she can "afford" it and she prefers it to her assigned schedule. The course allocation at a market equilibrium is called a *market outcome*. Sönmez and Ünver (2005) prove that the GS mechanism is a market mechanism whose outcome Pareto-dominates any other market outcome.

In the following example, we explain the execution of the GS mechanism and contrast its outcome with the UMBS mechanism:

Example 2. Let $\{i_1, i_2, i_3, i_4, i_5, i_6\}$ be the set of students and $\{c_1, c_2, c_3, c_4, c_5\}$ be the set of sections. Each student can enroll in up to two sections and the capacity of sections c_1, c_2, c_4 is two, whereas the capacity of sections c_3 and c_5 is three. Each student is given a bid endowment of $B = 1{,}000$ points. Suppose that the preferences of the students and their bids are given as follows:

i_1		i_2		i_3		i_4		i_5		i_6	
P_{i_1}	b_{i_1}	P_{i_2}	b_{i_2}	P_{i_3}	b_{i_3}	P_{i_4}	b_{i_4}	P_{i_5}	b_{i_5}	P_{i_6}	b_{i_6}
c_1	202	c_2	207	c_3	259	c_4	238	c_3	350	c_5	225
c_2	260	c_3	121	c_2	180	c_2	206	c_4	226	c_1	245
c_3	257	c_4	205	c_5	250	c_5	208	c_5	23	c_2	150
c_4	243	c_5	258	c_4	100	c_3	144	c_1	201	c_3	244
c_5	38	c_1	209	c_1	211	c_1	204	c_2	200	c_4	136

There is a time conflict between sections c_2 and c_3; therefore, students cannot register in both sections together.

The deferred-acceptance algorithm, which finds the outcome of the GS mechanism, works as follows:

Step 1. Student i_1 proposes to sections $\{c_1, c_2\}$; i_2 proposes to $\{c_2, c_4\}$ (instead of $\{c_2, c_3\}$ because these two sections have a conflict); i_3 proposes to $\{c_3, c_5\}$ (instead of $\{c_3, c_2\}$ because these two sections have a conflict); i_4 proposes to $\{c_4, c_2\}$, i_5 proposes to $\{c_3, c_4\}$; and i_6 proposes to $\{c_5, c_1\}$. Sections c_1, c_3, and c_5 each receive two offers that do not exceed their capacity, and sections c_2 and c_4 each receive three offers, which do exceed their capacity. Section c_2 has offers from i_1, i_2, and i_4, who have bid 260, 207, and 206, respectively. It rejects i_4's offer. Section c_4 has offers from i_2, i_4, and i_5, who have bid 205, 238, and 226, respectively. It rejects i_2's offer.

Step 2. Student i_2 updates her proposals as $\{c_2, c_5\}$ and student i_4 updates her proposals as $\{c_4, c_5\}$, whereas other students' proposals do not change. Section c_5's offers exceed its capacity of three in this step. Section c_5 has offers from i_2, i_3, i_4, and i_6, who have bid 258, 250, 208, and 225, respectively. Section c_5 rejects i_4's offer.

Step 3. Student i_4 now proposes to $\{c_4, c_3\}$, whereas other students' proposals do not change. No section's offers exceed its capacity; therefore, the algorithm terminates. The current proposals are realized as final allocation:

$$\mu^{GS} = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 \\ \{c_1, c_2\} & \{c_2, c_5\} & \{c_3, c_5\} & \{c_4, c_3\} & \{c_3, c_4\} & \{c_5, c_1\} \end{pmatrix}.$$

The prices of the sections are found as $p^{GS}_{c_1} = 202$, $p^{GS}_{c_2} = 207$, $p^{GS}_{c_3} = 144$, $p^{GS}_{c_4} = 226$, and $p^{GS}_{c_5} = 225$.

On the other hand, if the students had submitted the same bids to the UMBS course-bidding mechanism, the allocation would have been realized as follows. First we order all bids in order:

No:	1	2	3	4	5	6	7	8	9	10
Bid:	350	260	259	258	257	250	245	244	243	238
Student:	i_5	i_1	i_3	i_2	i_1	i_3	i_6	i_6	i_1	i_4
Section:	c_3	c_2	c_3	c_5	c_3	c_5	c_1	c_3	c_4	c_4
No:	11	12	13	14	15	16	17	18	19	20
Bid:	226	225	211	209	208	207	206	205	204	202
Student:	i_5	i_6	i_3	i_2	i_4	i_2	i_4	i_2	i_4	i_1
Section:	c_4	c_5	c_1	c_1	c_5	c_2	c_2	c_4	c_1	c_1
No:	21	22	23	24	25	26	27	28	29	30
Bid:	201	200	180	150	144	136	122	100	38	23
Student:	i_5	i_5	i_3	i_6	i_4	i_6	i_2	i_3	i_1	i_5
Section:	c_1	c_2	c_2	c_2	c_3	c_4	c_3	c_4	c_5	c_5

The outcome of the UMBS mechanism is found as

$$\mu^{\text{UMBS}} = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 \\ \{c_2, c_4\} & \{c_5, c_1\} & \{c_3, c_5\} & \{c_4, c_5\} & \{c_3\} & \{c_1, c_3\} \end{pmatrix}.$$

The prices are found as $p_{c_1}^{\text{UMBS}} = 209$, $p_{c_2}^{\text{UMBS}} = 0$, $p_{c_3}^{\text{UMBS}} = 244$, $p_{c_4}^{\text{UMBS}} = 238$, and $p_{c_5}^{\text{UMBS}} = 208$.

Student i_3 is assigned the same schedule under both mechanisms. Students i_1, i_2, i_5, and i_6 prefer the GS allocation to the UMBS allocation. Student i_4 prefers the UMBS allocation to the GS allocation. Under the UMBS allocation, although i_6 can afford to register in section c_5 (whose price is 208, whereas her bid is 225), and she prefers c_5 to c_3, UMBS did not have her registered in section c_5, because she cleared courses c_1 and c_3 with higher bids before her bid for c_5 was processed. This shows that the UMBS mechanism did not create a *market equilibrium*. This situation benefitted student i_4, who registered in c_5 under the UMBS system instead of the less-preferred course c_3, which was assigned by the GS mechanism. In a sense, she got "lucky" under the UMBS mechanism.

As Examples 1 and 2 show, the UMBS mechanism is not a market mechanism, although it is promoted as one by many business schools. That it is promoted as a market mechanism can be inferred from the following question and its answer borrowed from UMBS, Course-Bidding Tips and Tricks:

Q. How do I get into a course?

A. If you bid enough points to make market clear, a seat will be reserved for you in that section of the course, up to class capacity.

However, this is not the case, as we find in the field study.

We now discuss the results from the field study and see how large the loss in efficiency can be from using the UMBS versus the GS mechanism.

6. Results from the Field Study

We provide two sets of results. In both sets of results, we compare the efficiency of the two mechanisms for the spring 2004 course allocation—UMBS versus GS. The UMBS results are obtained from the UMBS administration, i.e., from the actual course allocation, and also it was regenerated through our own implementation of the UMBS mechanism. The GS results are the ones we constructed. In set 1, we use the UMBS tie-breaker lottery draw to break ties and the modified bid matrix B'. In the second set of results, we provide a robustness check with Monte Carlo simulation.

6.1. Analysis Using the UMBS Tie-Breaker Draw

Our analysis reveals that a potential transition to the GS mechanism is likely to result in significant efficiency improvement.

We observe that 10 first-quarter sections, 12 second-quarter sections, and 5 full-semester sections filled their capacities under the GS mechanism, whereas 9 first-quarter sections, 11 second-quarter sections, and 5 full-semester sections filled their capacities under the UMBS mechanism.

Out of the 489 students who responded to the survey, 456 receive the same credit load under both mechanisms (see Table 1). Among them:

- each of 366 students is assigned the same schedule under both mechanisms;
- each of 83 students unambiguously prefers her schedule under the GS mechanism to her schedule under the UMBS mechanism;
- no student unambiguously prefers her schedule under the UMBS mechanism to her schedule under the GS mechanism; and
- the welfare comparison is ambiguous for 7 of these students.

Out of the 489 students who responded, 21 students receive more credits under the GS mechanism and

- each of 18 of them unambiguously prefers her schedule under the GS mechanism to her schedule under the UMBS mechanism, whereas
- the welfare comparison is ambiguous for the remaining 3.

Table 1 Among Students Who Have Responded, Comparison of Student Preferences over the UMBS and GS Allocations Using the UMBS Tie-Breaker Lottery Draw

Valid responses	Indifferent	Prefers GS	Prefers UMBS	Ambiguous	Total
Same credit load	366	83	0	7	**456**
More credits under GS	0	18	0	3	**21**
More credits under UMBS	0	0	2	10	**12**
Total	**366**	**101**	**2**	**20**	**489**

Out of the 489 students who responded, 12 students receive more credits under the UMBS mechanism and

- each of two of them unambiguously prefers her schedule under the UMBS mechanism to her schedule under the GS mechanism, whereas
- the welfare comparison is ambiguous for the remaining 10.

So, altogether, 366 of the 489 students who responded are indifferent between the two mechanisms, 101 of them unambiguously prefer the GS mechanism, 2 of them unambiguously prefer the UMBS mechanism, and a conclusion cannot be drawn for 20 students.

One last point deserves clarification: It is clear why many students prefer the GS mechanism to the UMBS mechanism. This is because they get the courses they really want and do not end up with a situation where they get more-popular courses that they strategically bid more on, but like less (and get closed out of courses they like more, but bid less on). What may be less clear is why 2 students prefer the UMBS mechanism to the GS mechanism and 20 students have ambiguous preferences between UMBS and GS mechanism. The reason is quite simple. These 22 students got lucky under the UMBS mechanism—they were assigned one or more sections despite their relatively low bids because some other students with higher bids were denied seats and were instead assigned seats in their less-preferred sections (where they bid even higher) (like student i_4 in Example 2). Because the GS mechanism corrects all market failures, these sections would have filled up with the students who really deserve them, and these 22 students would not have been assigned these seats under the GS market allocation. However, 20 of these 22 students also ended up losing seats in some desirable courses due to clearing too many less-preferred courses for which they bid higher under the UMBS mechanism, and that is why there is ambiguity in their preference between the two allocations. The other 2 students did not lose any desirable courses under the UMBS mechanism with respect to the GS mechanism, and that is why they prefer the UMBS mechanism outcome.

6.2. Robustness Check: Simulation for the Tie-Breaker Lottery

As we have reported earlier, the indifferences are broken with a tie-breaking lottery at UMBS, and we have already reported the results for the spring 2004 lottery draw. In the following table, we report the results of a robustness test where we draw random lotteries to break ties, and repeat this 1,000 times. The results are virtually the same as the UMBS lottery draw (see Table 2).

Table 2 **Among Students Who Have Responded, Comparison of Student Preferences over the UMBS and GS Allocations Using the Monte Carlo Simulation**

Valid responses	Indifferent	Prefers GS	Prefers UMBS	Ambiguous	Total
Same credit load	367.853 (3.221)	82.590 (3.265)	0.247 (0.431)	7.642 (1.247)	**458.332 (2.225)**
More credits under GS	NA	16.953 (1.674)	NA	1.366 (0.527)	**18.319 (1.632)**
More credits under UMBS	NA	NA	2.217 (1.018)	10.132 (1.217)	**12.349 (1.631)**
Total	**367.853 (3.221)**	**99.543 (3.189)**	**2.464 (1.082)**	**19.14 (1.804)**	**489**

Note. Averages and standard errors (in parentheses below the averages) are reported for the sample. Situations that did not arise for any student are marked not applicable (NA).

6.3. Improvement in Ranks

We also computed the improvement in mean ranks among all courses allocated by the UMBS versus GS mechanisms for the modified bid matrix B'. To do this across both full-semester and minisemester courses, all full-semester courses were treated as a package of two minisemester courses, with the same rank applying to both. We explain this with a simple example:

Example 3. Suppose a student bids for 13 sections, and only 4 of these are full-semester sections, which are the 2nd, 3rd, 10th, and 11th choices in her preference ranking. After we convert her preferences to their minisemester equivalent, her preferences include 17 minisections. Each of the two minisemester equivalents for the full-semester section is given the same preference ranking. Thus, the two minisections of the full-semester sections are ranked as the 2nd, 4th, 12th, and 14th choices, amongst the 17 choices. Now, suppose under one of the mechanisms, she gets enrolled in 9 minisections, although she could have registered in up to 11 minisections due to her credit limit—these 9 are her 1st, 2nd, 2nd, 8th, 9th, 12th, 12th, 14th, and 14th ranked minisections. The two unregistered slots in her maximal possible schedule are counted as if she were registered in her 18th choice, remaining unmatched. Note that not accounting for unregistered slots, i.e., slots below credit limit, would needlessly favor a system where slots were left vacant. Thus, the mean rank of this schedule is calculated as

$$(1+2+2+8+9+12+12+14+14+18+18)/11=10.$$

Across all 489 students submitting a preference ranking, the average rank improvement by using GS versus UMBS is 1.1053. Across the 123 students who submitted a submitted-preference ranking and were assigned different schedules under GS versus UMBS, the average rank improvement is 4.3943.

An alternative method is to compare only the students who have the same credit load in the two schedules (GS and UMBS) and not to take into account unmatched slots in students' schedules. Using the method we discussed in Example 3, we calculated the average rank improvement of the 456 students who have the same credit load (and submitted preferences) under the two schedules. Across these students, the average rank improvement by using GS versus UMBS is 1.0647. Across the 90 students who submitted a preference ranking and were assigned different schedules under GS versus UMBS with the same credit load, the average rank improvement is 5.3944.

Thus, in all our tests, in the number of students better off as well as mean improvement in ranks, GS outperforms UMBS to a substantial extent. Next, we describe the laboratory experiment we conducted to confirm the results of the field study in a more controlled environment.

7. The Laboratory Experiment

In the field study, we could not control or measure three important factors. We could not measure whether the students truthfully reported their preferences, we did not know whether the two bidding mechanisms resulted in similar bid distributions from students if they were run separately, and we did not have any control over students' beliefs about other students' course preferences. In the laboratory experiment, we can control or measure these three factors and then inspect the differences between the two mechanisms in this more controlled environment.

7.1. Design

In the experimental design, we use nine students to represent a cohort of students (akin to a very small MBA class). Each student in the cohort wants to enroll in three courses, and there are six courses offered in total. Courses c_1, c_2, and c_3 are less-popular courses, and courses c_4, c_5, and c_6 are more-popular courses. Courses c_1, c_2, and c_3 each has a quota of 4, and courses c_4, c_5, and c_6 each has a quota of 7. Each student belongs to one of the three student types. The types represent different preferences for the six courses. In accordance with these preferences, each student receives monetary payment for each course that she is finally allocated. Thus, for a higher-preference course, she gets more money. The monetary payoffs (or value profiles) over the courses are given in Table 3 in terms of value tokens (the experimental currency). The payment to a student is linked to the sum of the value points she receives based on her course allocation (details of payment to follow).

Each student is endowed with 303 points to distribute among the courses (note that value tokens are different from bid points).

Table 3 The Value Profiles of Subject Types

	c_1	c_2	c_3	c_4	c_5	c_6
Type 1	85	2	2	80	80	80
Type 2	2	85	2	80	80	80
Type 3	2	2	85	80	80	80

We use the same value profile for each of our treatments. We change the bidding mechanism and the information provided to the subjects about others' preferences across treatments. In the *Little-Information* treatments, we only tell the subjects that courses c_1, c_2, and c_3 are less-popular courses, and courses c_4, c_5, and c_6 are more-popular courses. In the *Bid History* treatments, we provide the students with a probability distribution between the bids and successfully enrolling in the course. We run both treatments under GS and UMBS mechanisms. In all treatments, we tell the students the capacities of courses. In the instructions about the bid history, we provide the subjects with the following passage:

"…in recent years students were able to enroll with 100% probability in each of the courses 1, 2, and 3 if they bid just 1 point for each of these courses.

On the other hand, courses 4, 5, and 6 had the following success rate of enrollment for different bids:

Bid points	Success chance (%)
1	0
50	40
100	75
150	82
200	88
250	94
300	100

…"

The little-information treatment is our main treatment. The information that we provide about the popularity of the courses with respect to their values is a realistic emulation of the real life. Using the little-information treatments, we would like to test the hypothesis that the GS mechanism is likely to generate more-efficient outcomes than the UMBS system.

The bid history treatment is only a robustness check for the two mechanisms. The bid histories we provide to the students are not real, and the beliefs about market prices induced by these bid histories are not likely to be sustained at any equilibrium of the induced-preference revelation games, i.e., the optimal bidding behavior in reality does not match the bid history. The main hypothesis we would like to test through the bid history treatments is that, even if the beliefs of the students to induce their own demand functions are not consistent with the equilibrium, the GS mechanism generates more-efficient course allocation than

Table 4 Payment Options in the Experimental Sessions

Session	Class	Participation	Mechanism	Info	Sweepstakes (1 subject randomly chosen in the session gets)	Payment (each remaining subject gets)
1	MBA	In class	GS	Little Info	$10 \times$ token value in cents	$2 \times$ token value in cents
2	MBA	In class	UMBS	Little Info		
3	BBA	In class	GS	Bid History	$20 \times$ token value in cents	Token value in cents
4	BBA	In class	UMBS	Bid History		
5	BBA	Subject pool	GS	Little Info		
6	BBA	Subject pool	UMBS	Little Info		
7	BBA	Subject pool	GS	Little Info		
8	BBA	Subject pool	UMBS	Little Info		
9	BBA	Subject pool	GS	Little Info		
10	BBA	Subject pool	UMBS	Little Info		
11	BBA	Subject pool	GS	Bid History		
12	BBA	Subject pool	UMBS	Bid History		
13	BBA	Subject pool	GS	Bid History		
14	BBA	Subject pool	UMBS	Bid History		
15	BBA	Subject pool	GS	Bid History		
16	BBA	Subject pool	UMBS	Bid History		

the UMBS mechanism, and it is robust to erroneous information and beliefs.[9]

Also, we would like to see whether the GS mechanism induces truthful preference revelation. The GS mechanism is not strategy proof; however, in our field study, we assumed that students will not manipulate the GS system through preference manipulation.

In each cohort group, subjects were read the instructions aloud (See Appendix B for the instructions). Then they were asked to submit a bid distribution under the UMBS treatments, and submit a bid distribution and a rank-order list (or a preference list) of the courses under the GS treatments. The subjects were MBA and undergraduate students at the University of Michigan Business School. We collected four data cohorts for each of the four treatments, GS/Little Information, UMBS/Little Information, GS/Bid History, and UMBS/Bid History. We used slightly different payment methods in our experiments (see Table 4 for details) for a total of 16 cohorts and 144 students. In the sessions, only one subject received a sweepstakes payment that is 10 or 20 times that of her actual token value, whereas the other students received their token value as payment. The sweepstakes were conducted by randomly picking one student by using uniform distribution. A sweepstakes was included on top of a direct monetary incentive to increase task-compatible motivation without spending too much money.

[9] We chose the payoff numbers in Table 3 in such a way that each agent has a favorite course that is the least preferred for the majority of the other students, and has three other preferred courses that are also preferred courses for the other students. This gives a total number of four preferred courses, which exceeds the quota of courses by one. Moreover, students' least preferred course is a really low-utility course for them, but can be the highest-utility course for other students. We chose these particular numbers so that under the bid history treatment, students maximizing their expected payoff will bid 100 points each on their three second-choice courses, and one point each on the other three courses under both the UMBS and GS systems; however, under the GS system, because they report their preferences truthfully, they will receive their most-preferred three courses, whereas under the UMBS system they may not.

7.2. Brief Analysis of the Design

All the Pareto-efficient allocations require that the students are assigned their first-choice course, and two of the three second-choice courses, c_4, c_5, and c_6. If there was complete information about each others' preferences, all Nash equilibria of the games would assign the subjects to these choices. For example, a Nash equilibrium can be constructed as follows under the UMBS game: If all subjects bid 2 points for their first-choice course, 150 points for two of the second-choice courses c_3, c_4, and c_5; and 1 point for the remaining course among c_3, c_4, and c_5. Then, this is a Nash equilibrium of the game with complete information, and every subject will be assigned her most desirable schedule, her first-choice course, and two of the three of her second-choice courses, c_4, c_5, and c_6. Similarly, a Nash equilibrium example under the GS game is given as follows: If all subjects bid the same bid distribution as above, and rank their first-choice course as first, and arbitrarily order their second-choice courses, c_4, c_5, and c_6 as second, third, and fourth, then this strategy profile will be a Nash equilibrium of the GS game under complete information, resulting in a Pareto-efficient course allocation. Note that there are many other Nash equilibria under complete information for both mechanisms, and they result in similar outcomes.

However, in our experiment, subjects do *not* have complete information. As such, under the little-information treatment, we do not have equilibrium predictions, because we do not know how the phrasing of the instructions will affect the beliefs of the subjects.

Under the bid history treatment, with the UMBS mechanism, if the subjects behave consistently with expected payoff maximization, then they are expected to bid 2 points for their first-choice course; 1 point for the remaining two courses among c_1, c_2, c_3; 100 points for two of the courses among c_4, c_5, and c_6; and 99 for the remaining course among c_4, c_5, and c_6. (A subject should bid at least 2 points on her favorite course to break the tie between this course and her least-desirable courses.) The calculation of optimal bidding behavior is done similarly to the calculation in Example 1.

Under the GS/bid history treatments, the behavior consistent with expected payoff maximization is revealing preferences truthfully (or weakly truthfully; see the definition of weakly truthful revelation later) by arbitrarily breaking ties among equally desirable courses, bidding one point for each of courses c_1, c_2, c_3, and 100 points to each of c_4, c_5, and c_6.

8. Results of the Laboratory Experiment

8.1. Improvement in Efficiency

We observe that GS treatments yield significantly higher payoffs to the subjects than their UMBS counterparts. We also observe that the average ranks (based on the induced preferences, i.e., based on the true ranks) of the courses in the schedules under the GS treatments are significantly better than those under the UMBS treatments. We report the average payoffs and average ranks of the courses allocated in each session, and also the cumulative averages for our statistical tests in Tables 5 and 6.

Recall that under the Pareto-efficient allocation each student receives $85 + 80 + 80 = 245$ value tokens to be placed in their first, and two second, choices. Because there are three second-choice courses for each subject as c_4, c_5, and c_6, we calculate their average ranking as $(2+3+4)/3 = 3$. Hence, with this placement, the average of the ranks of their placement is given by $(1+3+3)/3 = 2.33$. When a student remains unmatched for a slot in her schedule, we compute this as the seventh choice of the student.

Table 5 **The Average Payoffs of the Subjects and the Average Rank of the Assigned Courses in the *Little-Information Treatments***

	GS/Little Info		UMBS/Little Info	
Sessions	Payoff	Rank	Payoff	Rank
A	245	2.33	190.56	3.24
B	243.89	2.48	216	3.04
C	243.89	2.48	199.44	3.09
D	244.44	2.41	234.11	2.72
Average	**244.31**	**2.43**	**210.03**	**3.02**
	(0.27)	**(0.035)**	**(9.60)**	**(0.11)**

Note. The standard deviations of the averages are reported in parentheses.

Table 6 **The Average Payoffs of the Subjects and the Rank of an Average Course Assigned in the Complementary *Bid History Treatments***

	GS/Bid History		UMBS/Bid History	
Sessions	Payoff	Rank	Payoff	Rank
A	244.44	2.41	215.67	3.06
B	244.18	2.48	220.19	2.87
C	243.89	2.48	208.11	3
D	235.78	2.5	224.67	2.94
Average	**242.07**	**2.47**	**217.16**	**2.97**
	(2.10)	**(0.02)**	**(3.53)**	**(0.04)**

We observe that under the GS treatments, the average payoffs are very close to the efficient level of 245 value tokens. With little information, we observe 244.31-token payoff on average, and with bid history we observe a 242.07-token average. On the other hand, under the UMBS treatments, the averages are much lower: 210.03 with little information and 217.16 with bid history.

We test the significance of differences using a two-sample Wilcoxon test (because there are only four data cohorts from each treatment, we use a nonparametric test). Unless otherwise noted, the p-values reported correspond to this test. A pairwise comparison between the GS/little information and UMBS/little information generates a difference of 34.24 value tokens, which is significant with a p-value less than 0.02857 (with W-stat $= 10$), taking each data cohort average as an independent observation. The pairwise comparison of the bid history treatments yields a difference of 24.99 value tokens, which is significant with a p-value less than 0.02857 (with W-stat $= 10$).[10]

[10] In the laboratory experiment, the best players who bid positive amounts on their top three choices are expected to earn 245 points under the UMBS system, and the worst players can get as few as 0 points. Under the GS mechanism predictions are the same. In the actual experiment, we find that the best players earned 245 points under both systems; the worst players got 89 points under the UMBS mechanism in both little-information and bid history treatments, 240 points under the GS/little-information treatment, and 162 points under the GS/bid history treatment. It is difficult to calculate expected rational payments in the little-information treatments, because we do not have control of the beliefs of the students about the preferences of the agents. In the bid history treatments, expected rational payments are 245 points for the GS treatment and 229.93 points for the UMBS treatment (assuming that all agents rationally bid 100 points on their second-choice course, 1 point on their top choice, and 1 point on their least desirable choices for the UMBS). The differences between the expected and actual average

We also compare the average ranks of the schedules: Under little information, the difference is −0.6 with p-value less than 0.02857 (W-stat = 10); and under bid history, the difference is −0.48 with a p-value less than 0.02857 (W-stat = 10). This means that under the GS treatments, an average allocated course is 0.5–0.6 ranks better than under the UMBS treatments.

We also compare the GS treatments within themselves. There is no significant difference between little-information and bid history treatments of the GS mechanism with a p-value of 0.6286 (W-stat = 15.5) and 0.4847 (W-stat = 14.5) for payoff differences and rank improvement of schedules, respectively. When we compare the UMBS treatments within themselves, we find a similar result with p-values 0.6857 (W-stat = 16) and 0.4857 (W-stat = 15), respectively.

8.2. Preference Revelation under the GS Mechanism

We also inspect whether the subjects' revealed rank-order lists are consistent with their true preferences. We consider two types of truthful revelation. *Strongly truthful revelation* means that subjects rank their true first-choice course as first, their three second-choice courses arbitrarily as second, third, and fourth; and their last-choice courses arbitrarily as fifth and sixth. Under *weakly truthful revelation*, we inspect whether the first three courses the students submit are consistent with their first three choices, and the rest of the courses are consistently ranked preferences. Because under the GS mechanism the rank order of the first three courses (that is the quota of each student's schedule) does not matter, we also consider this type of revelation as truthful. For example, for a type 1 subject whose most desirable course is c_1, the rank-order list c_4, c_1, c_6, c_5 is weakly truthful, and the rank-order list c_4, c_5, c_6, c_1 is not weakly truthful.

The average percentages of subjects submitting strongly and weakly truthful rank-order lists are given in Table 7.

As we can see, both the percentage of subjects revealing strongly, and those revealing weakly, truthful preferences are high in both treatments. Note that instructions to subjects are long and subjects may get confused. As such, truthful revelation may be even stronger in actuality.

Note that this is only true in this particular design. That is, in the field, when there are course conflicts for the same time slot or sections referring to the same course, and courses with different credit loads, the ranking of top choices can matter. To be consistent with the field, in our simplified design, we did not ask the students to report their first three choices without any particular ranking.

payoffs are 2.93 points for the GS treatment and 12.77 points for the UMBS treatment. Therefore, behavioral mistakes lead to more efficiency loss with respect to the rational outcome under the UMBS system.

Table 7 Percentage of Subjects Revealing Their Preferences Truthfully Under the GS Mechanism

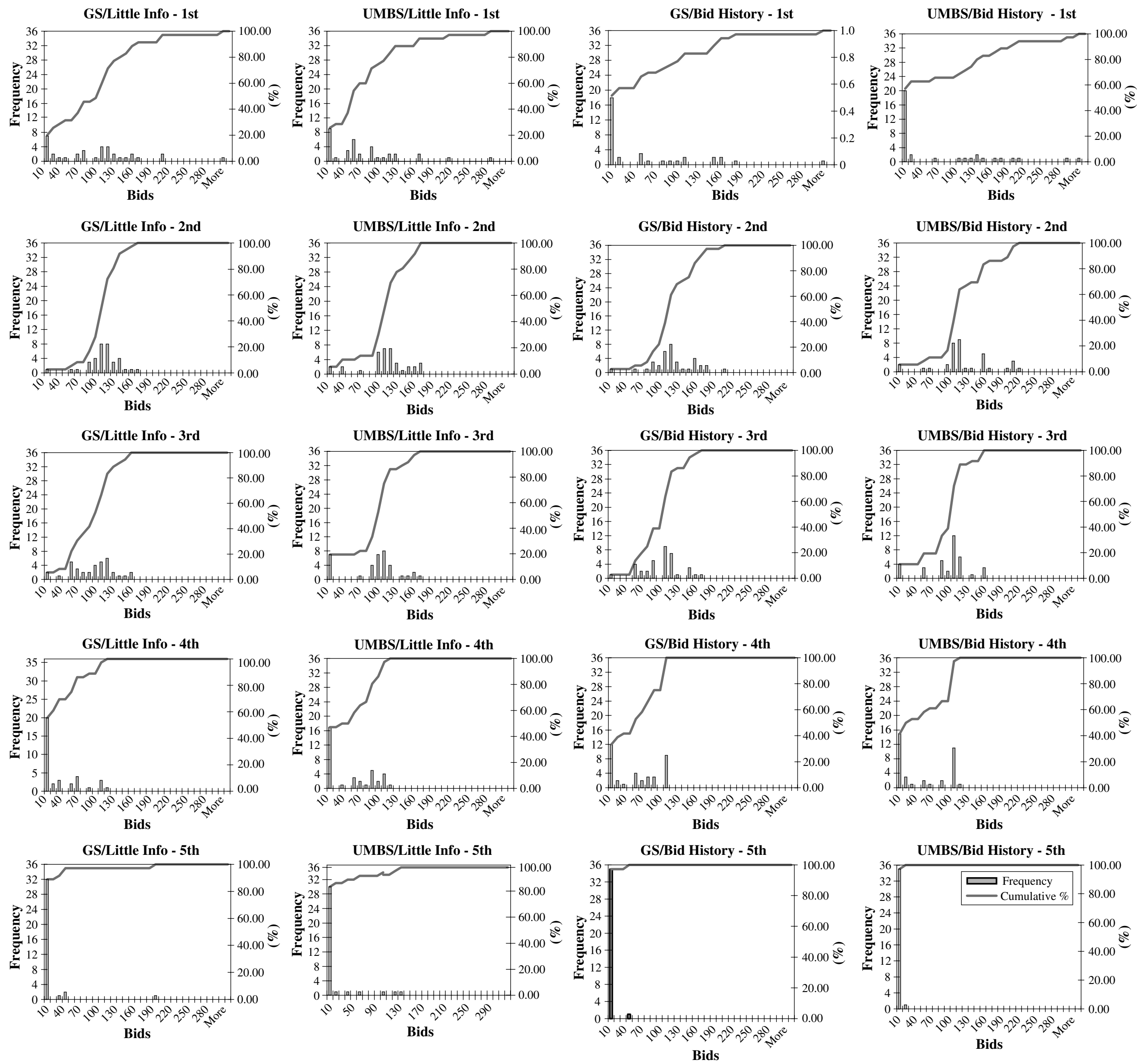
Truthful revelation	GS/Little Info (%)	GS/Bid History (%)
Strong		
Average	66.7	72.2
	(8)	(7.6)
Weak		
Average	83.3	83.3
	(6.3)	(6.3)

By reporting preferences truthfully, each student would have been placed in her top three choice courses, yielding a payoff of 245 points. As can be observed in Table 6, except for session D in the GS/bid history treatments, the average payoff lower bound is 243.89 points. In session D, one student received only a 162-point payoff by mistakenly bidding a positive amount on her least favorite course instead of most favorite course. Therefore, the efficiency loss of untruthful preference revelation is minimal for this student. This exception not withstanding, the main source of the efficiency loss is students not listing their top choice among the top three courses ranked.

8.3. Comparison of Bid Distributions between the GS and UMBS Treatments

In this subsection, we compare the bid distributions of the students between the GS and UMBS treatments. The bid distributions of the subjects are graphed in Figure 2 for the most desirable five courses (the bid distribution of the sixth choice can be obtained from the first five choices, because the sum of the bids add up to 303). The graphs are grouped for courses in terms of desirability. Because three courses—c_4, c_5, and c_6—yield the same (second-highest) payoff for all the subjects, the second-choice, third-choice, and fourth-choice bid distributions refer to the first, second, and third-highest bid distributions, respectively, amongst these three courses (i.e., the first-order, second-order, and third-order statistics for these three bids). Similarly, because the last two choices of all the subjects yield the same payoff, for the fifth choice we plot the highest bid distribution (the first-order statistic) across these two courses. Then, we obtain five course distributions for the four separate treatments. We pair GS and UMBS distributions for little-information and for bid history treatments, separately. We have 10 pairs as a result of these pairings. We observe that the distributions of the bids are in general very similar across GS and UMBS systems within each information treatment. We also conduct a formal statistical test that tests whether these bids come from the same underlying distribution. For this purpose,

Figure 2 The Bid Distribution for the First Five Choices of the Subjects



we use the Kolmogorov-Smirnov goodness-of-fit test (KS-test), which uses the *D*-statistic, which is the maximum distance between the empirical cumulative distribution functions of the subjects in the GS and UMBS treatments (as plotted in the graphs of Figure 2 with solid lines, we have 31 bins in calculating the cdf, i.e., once in each 10-bid interval). The *D*-statistics and *p*-values of these tests are given in Table 8 for GS and UMBS treatments with little information and in Table 9 for GS and UMBS treatments with bid history. Each null hypothesis states that the two bid distributions are the same. As seen in these tables, the *p*-values for 9 of the 10 pairs are larger than 0.30 and the *p*-value for the tenth pair (first choice in the little-information condition) is larger than 0.1, suggesting that the two distributions are not significantly different. This result gives us confidence about our assumption in the field study that the bid distributions between the GS and UMBS systems are similar.

In summary, we find clear evidence to support the following claims:

1. The GS mechanism leads to almost Pareto-efficient course allocation under either information treatment.

Table 8 KS-Test for the *Little Information Treatments*

	Courses in terms of desirability				
Little Information	1st	2nd	3rd	4th	5th
D-statistic	0.28	0.11	0.14	0.22	0.08
p-value	0.102	0.971	0.851	0.300	0.999

Table 9 KS-Test for the *Bid History Treatments*

	Courses in terms of desirability				
Bid History	1st	2nd	3rd	4th	5th
D-statistic	0.19	0.11	0.11	0.17	0.056
p-value	0.460	0.971	0.971	0.658	1.000

2. The GS mechanism's efficiency level is significantly higher than UMBS mechanism's.
3. The truthful revelation of preferences is in high percentages under the GS mechanism.
4. The bid distributions of the subjects under the GS and UMBS mechanisms are similar.

9. Conclusion

In this paper, we draw attention to bid-based course allocation systems used in universities. We use both a controlled field study and laboratory experiments to test the extent of the efficiency loss created by a typical course-bidding mechanism with respect to the best market mechanism, the one proposed by Sönmez and Ünver (2005). Sönmez and Ünver theoretically show that bid-based allocation systems currently in use can result in allocations that are not market outcomes and result in unnecessary loss of efficiency. They propose an alternate course allocation mechanism that they show results in the Pareto-dominant market outcome. We believe that the field study carried out at UMBS shows quite clearly that the UMBS mechanism results in significant efficiency loss due to the two possibly conflicting roles of the student bids. Our laboratory experiment further corroborates the results of the field study in a more controlled setting. The GS mechanism has the potential to make a substantially larger proportion of students better off (approximately 20% in our study) than the UMBS mechanism that is currently in place at many schools. Thus, schools should consider adoption of the GS mechanism, given that the change required is relatively minor and the potential benefits are quite large.

Note that our comparison between the two mechanisms is based on stated preferences of the students. Like many other market mechanisms, the GS mechanism is not strategy proof. However, when this mechanism is used in a large market, such as the UMBS bidding pool, we can safely assume that students would not gain much from gaming their preferences on average (i.e., they would act as price takers). This is also evidenced in our laboratory experiment, where most students truthfully revealed their preferences in the smaller market games we designed.

We should mention that during the add-drop period in the UMBS system, some of the students would be able to transfer from a less-preferred course that they bid high points on and got into, to a more-preferred course that they bid less on and to which they did not get assigned. However, sometimes this will not be possible, because this more-preferred course will already be filled during the course allocation. However, more importantly, the function of the add-drop period should not be to "correct market failures" that could have been avoided in the first place with the correct choice of bidding mechanism. The add-drop period should be used merely to accommodate students whose information about the courses have evolved over time. Unnecessary add-drop (105 course seats in our field study) creates a lot of upheaval in the registrar's office, and even worse is the fact that the class takes longer to get settled, reducing the quality of education in the early weeks of the semester. Add-drop periods are not short—UMBS allows two full weeks for add-drop in a minisemester that is only seven weeks long, i.e., for more than a quarter of the study period students are changing courses. This situation is not unique to UMBS, but exists in most schools. Professors suffer through the early weeks of the semester with students adding and dropping courses. Although some changes in add-drop periods can be due to students changing their preferences for courses, much change (in our field study 90 students would try and change courses) can be due to inefficient course allocation and can be reduced. Clearly, this needs to be changed.

Another potential problem with an add-drop system (that we did not encounter in our field study) is the following: In principle, the add-drop period following a UMBS system allocation cannot correct all the efficiency losses created. For example, two students can be placed in courses, each being the favorite of the other. If both courses are filled, and these students are unaware of each other, they will not be able to switch courses.

It is interesting that universities and schools within the same university use different course allocation mechanisms. Thus, either the objectives of the course allocation systems across different universities are different, or the mechanisms are not well understood. We could conjecture that it is the latter (because most schools would want some sort of efficiency and equity criteria to be met). We believe that our paper sheds some light on the mechanisms. This is just the beginning of research on course allocation systems. There is much room for additional future work in this area—for instance, theoretically which systems are more efficient—the ordinal systems (used at Harvard and Stanford) or the cardinal systems (used at Michigan and Berkeley). Practically, which systems are better? Do the ordinal systems induce gaming—i.e., the preferences listed by students are not truthful? Which of these two systems has greater truthful preference revelation? What happens with multiple bidding rounds as is done at the Haas School of Business at Berkeley?

In our field study, students' course preferences were asked for after course bids were collected and were not used for course allocation. The field study can be enhanced in future research such that student bids and course preferences are asked for simultaneously and are used to allocate courses (with the students being aware of this). Additionally, in the field study, 1.5 credit courses were treated like half of 3.0 credit courses. It remains to be tested whether students' bidding behavior for 1.5 credit courses is different (versus strong rationality) compared to that for 3.0 credit courses. Moreover, our laboratory experiment was based on a specific preference ordering for students with a favorite course that was undesirable for others, and a set of preferred courses that were also preferred by others. These are realistic preferences; however, other preference domains and parameter values may lead to different results. Future work should test whether our results are robust to different course preferences. These preferences can be constructed from future field studies using real student preferences.

Our experiments show great potential to increase efficiency in course allocation in educational institutions. Although we used small stakes in the laboratory experiment, we still observed a big contrast in the outcomes. With higher stakes, we can only expect the results to be more significantly different.[11] As such, we would like to see institutions adopt the improved mechanism, just as has been done by the American Medical Association for hospital-intern matching, by transplant centers in New England for kidney exchange (cf. Roth et al. 2005), and by New York City and Boston for public school admissions (cf. Abdulkadiroğlu et al. 2005a, b). By doing the first field study on matching mechanisms, we also wish to encourage additional research using field studies. Within the management science community, we wish to spur additional work on mechanism design.

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[11] In a review by Camerer and Hogarth (1999) on the effect of financial incentives on the results of the laboratory experiments, the authors observed that (on average) the mean performance does not change, but variance drops as financial incentives increase in the 74 studies they explored. Therefore, in our design we would expect higher significance levels for the observed differences between treatments as the financial stakes increase.

Appendix A. Variants of Course Allocation Mechanisms

A1. Some Variants of the UMBS Course-Bidding Mechanism

Yale School of Management: Uses the same mechanism as the University of Michigan Business School except that students can only bid for five courses (and the normal course load is four courses).

Columbia Business School:

- The real-life version of UMBS course-bidding mechanism is used for two rounds.
- The first round is the "main" round, whereas in Round 2 students are expected to fill the gaps in their first-round schedule.
- Unsuccessful bids from Round 1 are returned to students to be used in Round 2.
- Students can only bid for undersubscribed courses in Round 2.

Haas School of Business, UC Berkeley: Uses the same two-round version as the Columbia Business School except that students cannot bid for more than a fixed number of units.

Kellogg School of Management, Northwestern University:

- The bid endowment should be used over two quarters by first-year MBA students and over three quarters by second-year MBA students. Points not used in the first year do not carry over to the second year.
- Each quarter there are two rounds of bidding similar to the bidding at Columbia Business School, except that
  - —students can bid for at most five courses (where the normal course load is four courses),
  - —students are charged for the market-clearing bids, not their own bids, and
  - —bids from the second rounds carry over to the next quarter unless bidding is for the last quarter of the year.
- Hence, bidding for the second quarter of the first year and the third quarter of the second year is analogous to course bidding at Columbia and Haas.

Princeton University:

- Undergraduate students cluster alternate courses together and strictly rank the courses within each cluster. Students will be assigned no more than one course from each cluster.
- Students allocate their bid endowment over clusters (as opposed to individual courses). The bid for each course in a cluster is equated to the bid for the cluster. Based on these bids, course allocation is implemented via a variant

of the UMBS course-bidding mechanism, where

—the bids of a student for courses in a cluster are ordered subsequently based on the ranking within the cluster, and

—once a bid of a student is successful for a course in a cluster, her bids for all lower-ranked courses in the same cluster are dropped.

A2. An Example of Preference-Based Course Allocation Mechanisms

Harvard Business School Course Allocation Mechanism:

- Students are strictly ordered in a single priority list with a random lottery.
- Each student submits a preference ranking of the courses.
- The assignment of the first course seat for each student is obtained with the serial dictatorship that is induced by the priority ordering of students: The first student is assigned a seat at her top choice, the next student is assigned a seat at her top choice among classes with still-available seats, and so on.
- Once the assignment of the first seats is finalized (or, equivalently, the first cycle is completed), the assignment of second-course seats is determined in a similar way using the reverse priority order; next, the third-course seats are determined in a similar way using the initial priority order, and so on.

Appendix B. Instructions in the Laboratory Experiment

Course-Bidding Exercise

For this exercise, you have been placed in groups of nine students. Think of this as a "tiny entering second year MBA class of just 9 students." It is as if your class of 2008 was reduced drastically in size to just nine. It is now bidding time for second-year courses and the object of this exercise is to bid for courses.

Each student can take *up to* 3 classes. There are 6 courses in all with varying capacities (courses 1, 2, and 3 have a capacity of 4 students each whereas courses 4, 5, and 6 can take up to 7 students each). Each student has *303* bid points to distribute across all courses that s/he bids on.

Basically, in the real world, for every course that a student successfully registers in, s/he feels a certain amount of happiness—more happiness for courses that she really wants to get into, and less happiness for less desirable courses. In this exercise, we will give you tokens to reflect how happy you feel about being able to enroll in a certain course.

You will receive a certain number of tokens depending on which class you successfully get into immediately after bidding is over and course allocation is done (i.e., you will not be allowed to make any changes in classes after course allocation is done). The number of tokens we give you for a specific course reflects how much you really want that course—if course 1 gives you 100 tokens and course 2 gives you just 10 tokens, then that means you want course 1 much more than course 2. At the end of the exercise, we will see what courses you successfully registered in, we will add up your tokens, and you will be paid that number in cents. In addition, one person in the subject pool for this experiment will be chosen at random and we will give him/her 20 times their total cents payoff.

Your desire for each course (in tokens) is given in the table below. Therefore, if you get into courses 4, 5, and 6, you get 240 tokens = 240 cents. If, in addition, you are the person picked in the subject pool at random, you will then also get 20×240 cents = \$48.00. You can only bid in whole numbers and you need to bid at least one point for any course in which you would like to enroll.

Note that "bid point" is what you bid with and "token" reflects your happiness at getting into a course. Table A.1 below is a summary of the course capacity, and how much you like each course.

System Y (GS Mechanism). In this system, students not only bid for courses, but also rank them according to how much they want them (preference rank). This is called the student's "preference list." If two or more courses are equally desirable (e.g., if two courses are both third best for you), please give them different ranks randomly (e.g., third and fourth rank)—i.e., you need to *break the tie* among them in your preference list. If a student wants to be considered for a course, she needs to bid at least one point for it. The sum of the bids cannot exceed 303 bid points. Otherwise, sheets will be returned for correction.

The system works as follows: After all student sheets with preference ranks and bids have been collected, the allocation is found as follows:

The computer computes the allocation as follows:

(1) In the first round, each student is *tentatively* placed in her top three choices from her preference list (or fewer if s/he has bid for fewer than three courses). Then the computer adds up course enrollment across students and calculates enrollment in each course. If a course is assigned more students than its capacity, the lowest-bidding students for that course are dropped, so that each course *tentatively* holds no more students than its capacity. However, this is just the first computer allocation and is tentative.

(2) Then, in the next computer allocation round, each student who was dropped from any courses in the first computer allocation round is tentatively placed in replacement courses starting from her fourth choice. Then the computer inspects the courses again. If a course is assigned more students than its capacity, then the bids of *students who were tentatively placed in it in the previous round and in this round are inspected*. The lowest-bidding students tentatively placed in *this or previous rounds* for that course are dropped, so

Table A.1 Information and Subjects About Course Capacities and Preferences

Course	Course capacity	How much you like it (no. of tokens you get for successfully registering in course in first attempt)
1	4	85
2	4	2
3	4	2
4	7	80
5	7	80
6	7	80

that each course tentatively holds no more students than its capacity.

(3) The computer iteratively continues until no student is dropped from a course, or all options of the students are exhausted in their preference lists. The tentative assignments are realized as real assignments.

Example. Suppose there are four students and four courses with a capacity of two each. Students need to take *up to* two courses. Suppose students have 100 points to bid and they report their bids and preferences as follows:

Student 1		Student 2		Student 3		Student 4	
Course (in order of pref.)	Bid	Course (in order of pref.)	Bid	Course (in order of pref.)	Bid	Course (in order of pref.)	Bid
C1	8	C1	16	C3	2	C2	88
C2	79	C2	77	C2	89	C3	6
C3	13	C3	7	C1	4	C1	5
C4	0	C4	0	C4	3	C4	1

Computer Round 1 (will consider top two ranks of students because students can only take two courses):

Student					Course			
1	2	3	4		1	2	3	4
C1	C1	C3	C2	Computer inspects the course enrollment:	*S1-8*	~~S1-79~~	*S3-2*	
C2	C2	C2	C3		*S2-16*	~~S2-77~~	*S4-6*	
						S3-89		
						S4-88		

Tentative assignments are in italics, rejected bids are crossed. Because some students are dropped from courses, they are now assigned to courses from their third rank on.

Computer Round 2: S1 and S2, who were rejected by one of their courses, are tentatively assigned to their third-choice course.

Student					Course			
1	2	3	4		1	2	3	4
C1	*C1*	*C3*	*C3*	Computer inspects the course enrollment:	*S1-8*	~~S1-79~~	~~S3-2~~	
~~C2~~	~~C2~~	*C2*	*C2*		*S2-16*	~~S2-77~~	~~S4-6~~	
C3	C3					*S3-89*	*S1-13*	
						S4-88	*S2-7*	

S3 and S4 no longer get course 3. Instead, S1 and S2 are tentatively placed in course 3.

Computer Round 3: S3 and S4, who were rejected by one of their courses, are tentatively assigned to their third-choice course.

Student					Course			
1	2	3	4		1	2	3	4
C1	*C1*	~~C3~~	~~C3~~	Computer inspects the course enrollment:	*S1-8*	~~S1-79~~	~~S3-2~~	
~~C2~~	~~C2~~	*C2*	*C2*		*S2-16*	~~S2-77~~	~~S4-6~~	
C3	C3	C1	C1		~~S3-4~~	*S3-89*	*S1-13*	
					~~S4-5~~	*S4-88*	*S2-7*	

S3 and S4 cannot get course 1 because S1 and S2 have bid a higher amount for this course. Therefore, S3 and S4 are now to be considered for their fourth-choice course.

Computer Round 4: S3 and S4 are now to be considered for their fourth-choice course. The tentative allocation is finalized as:

Student					Course			
1	2	3	4		1	2	3	4
C1	*C1*	~~C3~~	~~C3~~	Computer inspects the course enrollment:	*S1-8*	~~S1-79~~	~~S3-2~~	*S3-3*
~~C2~~	~~C2~~	*C2*	*C2*		*S2-16*	~~S2-77~~	~~S4-6~~	*S4-1*
C3	*C3*	~~C1~~	~~C1~~		~~S3-4~~	*S3-89*	*S1-13*	
		C4	*C4*		~~S4-5~~	*S4-88*	*S2-7*	

Now, please give us your ranks and bids for the courses, based on Table A.1 and the bid history that's given to you. Note that the first column indicates the course # (whether it's course # 1, 2, 3, 4, 5, or 6), the second column is the *rank* for each course, and the third column is the bid for each course.

Course number	Your rank for this course in terms of its desirability (1 is the highest)—you must rank all courses. You can rank only one course for each rank	Your bid
1		
2		
3		
4		
5		
6		
Total		303 bid points

Please write down your UMID and email address so you can get paid: ——

System X (UMBS Mechanism). If you want to be considered for a course, you need to bid at least one point for it. The sum of the bids cannot exceed 303 bid points. Otherwise, sheets will be returned for correction.

This is how course allocation is done: After all bid sheets are collected, all bids from the nine students are ordered from largest to smallest. Only positive bids are taken into consideration, zero bids are dropped. Then, starting from the largest bid, each bid is processed one at a time. When it is the turn of bid b_{ic} of student i for course c, the bid is successful if:

(a) course c still has unfilled seats, and

(b) student i still has unfilled slots in her schedule.

If the bid is successful, then student i is assigned a seat at course c (i.e., the bid is honored) and the process proceeds with the next bid in the list. Otherwise, student i is declined a seat at course c and the process continues with the next-largest bid in the list.

When all bids are processed, a schedule is obtained for each student and a course allocation is hence obtained.

Therefore, if each student was to bid on four courses (remember that you can bid your 303 points on any number of courses), then the $4 \times 9 = 36$ bids are organized in a list according to bid points. Then courses are assigned going down the list from highest to lowest bid points, taking requirements (a) and (b) above into consideration.

Example. Suppose there are four students and four courses with a capacity of two each. Students can take up to two courses. Suppose students have 100 points to bid and their bids are as follows:

Student 1		Student 2		Student 3		Student 4	
Course (in order of pref.)	Bid	Course (in order of pref.)	Bid	Course (in order of pref.)	Bid	Course (in order of pref.)	Bid
C1	8	C1	16	C3	2	C2	88
C2	79	C2	77	C2	89	C3	6
C3	13	C3	7	C1	4	C1	5
C4	0	C4	0	C4	3	C4	1

All 14 positive bids are ordered in descending order:

Bid (points)	Student	Course	Assignment
89	S3	C2	C2 → S3
88	S4	C2	C2 → S4
79	S1	C2	C2 is full, bid rejected
77	S2	C2	C2 is full, bid rejected
16	S2	C1	C1 → S2
13	S1	C3	C3 → S1
8	S1	C1	C1 → S1
7	S2	C3	C3 → S2
6	S4	C3	C3 is full, bid rejected
5	S4	C1	C1 is full, bid rejected
4	S3	C1	C1 is full, bid rejected
3	S3	C4	C4 → S3
2	S3	C3	S3 already has 2 courses, bid rejected
1	S4	C4	C4 → S4

Final course assignment:

Student				Course			
1	2	3	4	1	2	3	4
C3	C1	C2	C2	S2-16	S3-89	S1-13	S3-3
C1	C3	C4	C4	S1-8	S4-88	S2-7	S4-1

Now, please give us your bids for the courses, based on Table A.1 and the bid history that's given to you:

Course number	Your bid
1	
2	
3	
4	
5	
6	
Total	303 bid points

Please write down your UMID and e-mail address so you can get paid: ——

Little Information. By word of mouth, you must know that courses 1, 2, and 3 are not popular courses and are easy to get into. On the other hand, courses 4, 5, and 6 are very popular courses.

Bid History. The students' office has information available about the past market-clearing prices (i.e., past bid points that were successful). According to that, in recent years student were able to enroll with 100% probability in each of courses 1, 2, and 3 if they bid just one point for each of these courses. On the other hand, courses 4, 5, and 6 had the following success rate of enrollment for different bids:

Bid points	Success chance (%)
1	0
50	40
100	75
150	82
200	88
250	94
300 or more	100

Success chance of other bid points can easily be interpolated as follows: For example, the success chance of 160 bid points is

$$82\% + \frac{(160-150)}{(200-150)} * (88-82)\% = 83.2\%.$$

There is no guarantee that this is the case for the current year.

References

Abdulkadiroğlu, A., T. Sönmez. 1999. House allocation with existing tenants. *J. Econom. Theory* **88** 233–260.

Abdulkadiroğlu, A., T. Sönmez. 2003. School choice: A mechanism design approach. *Amer. Econom. Rev.* **93** 729–747.

Abdulkadiroğlu, A., P. Pathak, A. E. Roth. 2005a. The New York city high school match. *Amer. Econom. Rev., Papers Proc.* **95**(2) 364–367.

Abdulkadiroğlu, A., P. Pathak, A. E. Roth, T. Sönmez. 2005b. The Boston public school match. *Amer. Econom. Rev. Papers Proc.* **95**(2) 368–371.

Amaldoss, W., S. Jain. 2002. David vs. Goliath: An analysis of asymmetric mixed-strategy games and experimental evidence. *Management Sci.* **48** 972–991.

Amaldoss, W., R. J. Meyer, J. S. Raju, A. Rapoport. 2000. Collaborating to compete. *Marketing Sci.* **19** 105–126.

Balinski, M., T. Sönmez. 1999. A tale of two mechanisms: Student placement. *J. Econom. Theory* **84** 73–94.

Camerer, C. F., R. M. Hogarth. 1999. The effects of financial incentives in experiments: A review and capital-labor-production framework. *J. Risk Uncertainty* **19** 7–42.

Chen, Y., T. Sönmez. 2002. Improving efficiency of on-campus housing: An experimental study. *Amer. Econom. Rev.* **92**(5) 1669–1686.

Chen, Y., T. Sönmez. 2006. School choice: An experimental study. *J. Econom. Theory* **127** 2002–2031.

Dholakia, U. M., I. Simonson. 2005. The effect of explicit reference points on consumer choice and online bidding behavior. *Marketing Sci.* **24** 206–217.

Fay, S. 2004. Partial-repeat-bidding in the name-your-own-price channel. *Marketing Sci.* **23** 407–418.

Gale, D., L. Shapley. 1962. College admissions and the stability of marriage. *Amer. Math. Monthly* **69** 9–15.

Haruvy, E., I. Erev, D. Sonsino. 2001. The medium prizes paradox: Evidence from a simulated casino. *J. Risk Uncertainty* **22** 251–261.

Ho, T.-H., K. Weigelt. 1996. Task complexity, equilibrium selection, and learning: An experimental study. *Management Sci.* **42** 659–679.

Ho, T.-H., K. Weigelt. 2005. Trust building among strangers. *Management Sci.* **51**(4) 519–530.

Katok, E., A. E. Roth. 2004. Auctions of homogeneous goods with increasing returns: Experimental comparison of alternative "Dutch" auctions. *Management Sci.* **50** 1044–1063.

Kelso, A. S., V. P. Crawford. 1982. Job matching, coalition formation, and gross substitutes. *Econometrica* **50** 1483–1504.

Krishna, A., Y. Wang. 2007. The relationship between top trading cycle and top trading cycle and chains mechanisms. *J. Econom. Theory* **132**(1) 539–547.

Rapoport, A., I. Erev, R. Zwick. 1995. An experimental study of buyer-seller negotiation with one-sided incomplete information and time discounting. *Management Sci.* **41** 377–394.

Roth, A. E., E. Peranson. 1999. The redesign of the matching market for American physicians: Some engineering aspects of economic design. *Amer. Econom. Rev.* **89** 748–780.

Roth, A. E., T. Sönmez, M. U. Ünver. 2005. A kidney exchange clearinghouse in New England. *Amer. Econom. Rev., Papers Proc.* **95**(May) 376–380.

Shugan, S. M. 2005. Marketing and designing transaction games. *Marketing Sci.* **24** 525–530.

Sönmez, T., M. U. Ünver. 2005. Course bidding at business schools. Boston College Working Papers in Economics 618, http://econpapers.repec.org/paper/bocbocoec/618.htm.

Srivastava, J., D. Chakravarti, A. Rapoport. 2000. Price and margin negotiations in marketing channels: An experimental study of sequential bargaining under one-sided uncertainty and opportunity cost of delay. *Marketing Sci.* **19** 163–184.

Wang, Y., A. Krishna. 2006. Timeshare allocations: Theory and experiment. *Management Sci.* **52**(August) 1223–1238.

Zwick, R., X. Chen. 1999. What price fairness: A bargaining study. *Management Sci.* **45** 804–823.