An Insurance Model of Property Tax Limitations

Nathan B. Anderson and Andreas Duus Pape*

November 10, 2006

Abstract

Forty-three states in the United States have legal limits on local property taxation. These limits constrain increases in local government revenues, increases in property values, and the level of tax rates. A large literature cites the desire of taxpaying voters to constrain wasteful local government expenditures as the primary motivation for these limitations. The provision of insurance against unexpected increases in individual property tax liability, however, also explains these limitations. While a desire by local residents to constrain wasteful local government expenditures may exist, demand for insurance explains the presence of limitations in the absence of any voter perception of wasteful expenditures.

1 Introduction

The tax which each individual is bound to pay ought to be certain, and not arbitrary. The time of payment, the manner of payment, the quantity to be paid, ought all to be clear and plain to the contributor, and to every other person. [...] The certainty of what each individual out to pay is, in taxation, a matter of so great importance, that a very considerable degree of inequality, it appears, I believe, from the experience of all nations, is not near so great an evil as a very small degree of uncertainty.

Adam Smith
The Wealth of Nations (Book V)

*Contact information: Nathan Anderson (corresponding author): nba@uic.edu, Department of Economics, University of Illinois at Chicago and The Institute of Government and Public Affairs of the University of Illinois. Andreas Pape: apape@umich.edu, Department of Economics, University of Michigan and Oberlin College. This paper was prepared for the National Tax Association conference in Boston, MA in November, 2006. As this is only a draft, we ask that readers be more understanding of any errors we might have made and we request that those wishing to cite any results please ask for permission.
No other tax in the United States exemplifies the tension between Smith’s maxims of equality and uncertainty more than the property tax. This paper explores the relationship between the equality and uncertainty of the property tax and the widespread limitations placed on aspects of property taxation in the United States. In terms of revenues collected, the local property tax is the second largest tax in the United States with over $286 billion collected by local governments in 2002; representing approximately $80 billion more in revenues than federal corporate income tax; $10 billion more than state sales taxes; and $70 billion more than states collected in individual and corporate income taxes combined. Despite of or because of its size, the ability of local governments to collect property tax revenue is often constrained and the tax itself is extremely unpopular, especially when compared to vacations in the south of France, but even when compared to other taxes.

As of 2006, only 5 of the 48 states in the continental United States have absolutely no explicit limits on some aspect of property taxation. The remaining 43 states each had at least one of the three most common limitations in place. The most numerous types of limitations, revenue and tax rate limits, explicitly constrain local government behavior. The third type, limitations on increases in the taxable value of property, is less common and does not explicitly constrain local government behavior. What these limits have in common is their potential to reduce variation in individual property tax payments. Previous research on the motivations for property tax limitations has focused on constraint of possibly wasteful government expenditures.

There is little to no prior economics research on the volatility and uncertainty of individual property tax payments. Fortunately there does exist some suggestive evidence that provides an idea about the magnitude of volatility, if not the degree of uncertainty. For example, from 2002 to 2006, total property taxes paid by owners of residential property in Minnesota increased by an average of 58% across all the counties in state. This increase was not caused by increased property tax revenues; taxing jurisdictions in Minnesota only spent 14% more in 2006 than they did in 2002. The majority of the increase was caused by large increases

---

1 Although the distribution of revenues varies greatly across states, more than 40% of local property tax revenues collected in 2002 financed school districts; 20% to municipalities and townships, 24% to counties, with the remaining funds allocated to special districts. Sources: Internal Revenue Service and US Census Bureau.

2 The property tax is the most unpopular state or local tax and rivals only the federal income tax in terms of its unpopularity. In the most recent annual survey by the Tax Foundation, 38% of respondents listed the property tax as the worst tax state and local tax. In a virtual tie for a distant second place, income and sales taxes received 20% and 19% of the vote for worst state and local tax. This is not a recent phenomenon. A 1974 survey by the Advisory Committee on Intergovernmental Relations had the property tax virtually tied with the federal income tax as the worst tax.

3 The five states are ME, NH, TN, VT, VA. Utah might also be considered as a state without any stringent limitations on property taxation. Of the five states with none of three major limits on property taxes, only New Hampshire has a statewide annual assessment system. Maine recently passed tax reforms that included “circuit-breakers” designed to keep property tax burdens below a certain percentage of income.

4 For a more detailed summary of these limits see Anderson (2006).

5 Wasteful usually implies bureaucrats motivated by a goal to maximize their budgets with little to no concern regarding the provision of services to voters.

6 See Allen (2006) for an excellent study of individual property tax burdens in Maine at one point in time.
in the value of residential property that far outpaced increases in the value of other types of property, such as commercial and industrial real estate. Of course, this not only represents an increase in the volatility of individual tax payments, but also an increase in the mean tax payment for residential properties. Although insightful, these data do not inform as to changes in the distribution of tax payments within the residential property class.

A recent study by Dye, McMillen, and Merriman (2006) provides estimates of within-residential class volatility in individual property tax bills. They estimate that for 2003 individual property tax bills for residential owners in Chicago increased by an average of 50% in some areas of Chicago and decreased by as much as 10% in other areas of Chicago. The average estimated annual increase in tax bills across some neighborhoods was larger than $400 for each year from 2003 to 2005. Many neighborhoods in Chicago had average annual decreases in tax bills of over $100 from 2003 to 2005. Even within the same neighborhood, some individual experience large increases in property taxes while others experience decreases. Perhaps most surprisingly, these estimates hold tax revenue constant, implying that these changes in tax bills are derived solely from fluctuations in property valuations.

This paper investigates the effect that property tax limitations have on the volatility of individual property tax payments. In the presence of volatility in individual tax payments, voters will support property tax limitation measures within their own jurisdictions and in state-wide referenda even when government expenditures are not wasteful. This is in stark contrast with previous literature contending that, in the absence of wasteful expenditures, no rational voter would support tax limitations within their own jurisdiction.

The idea of property tax limitation as insurance is motivated by a simple example. Suppose there is a jurisdiction with only two homes, each valued at $200. If the local government requires $50 of revenue, each taxpayer will pay $25 under a pure current value assessment system. Next year, the government again needs $50 but one of the homes has gone up in value to $400 (the other is still valued at $200). Under this scenario, with government revenue still at $50, the first taxpayer will pay about $33 while the other will pay about $17. With no increase in government revenue each taxpayer has experienced a 32% change in their tax payment. The increase in tax payments faced by the appreciating home would be higher still if government revenue had not remained constant. Perhaps the owner of the non-appreciating home believes that there is a non-trivial risk that she might experience such a tax increase in the future. She certainly would be willing to pay some amount to ensure that such a thing would not happen to her in the future. She might support limiting revenue increases to guarantee that increases in her tax payments would not be too large or she might support assessment limitations that restrict increases in the value of her home. These limitations may even be supported if she believes that her government is benevolent and that the entire $50 is extremely well-spent.

---

8 These numbers reflect the annual changes in property tax liability that would have occurred if Chicago was not covered by Cook County’s cap on increased in property valuations. Thus, they are lower bounds on tax increases and decreases since the cap on valuation increases did not eliminate all increases, implying a certain level of redistribution even under the assessment cap. The estimates are based on projected increases in property valuations. Please see their paper for details.
Vigdor (2004) provides a detailed review of the most prominent explanations for the existence of statewide limitations on local governments. As Vigdor notes, in the Tiebout (1956) model voters in an individual jurisdiction would never submit to limitations on their local government revenues and rates so long as residents have the power to dictate revenue and rate policy according to their preferences. The lack of motivation for limits when residents have control over policy suggests that perhaps residents have limited control over policy. This idea that residents have limited control over local policies on revenues and tax rates is often called the “Leviathan” model or agency loss theory of local government. In these models principal-agent problems, for example costly and imperfect monitoring of elected officials, allow local governments to collect property tax revenues that are excessive and inefficient. Expenditure and tax rate limitations imposed on wasteful local governments are then justified because they can increase the efficiency of local governments.

Vigdor (2004) proposes an alternative to these theories, noting that limitations on local government allow voters to influence local tax and expenditure decisions in jurisdictions where they do not reside. This motivation exists even when the local voters control presumably non-wasteful tax and expenditure policy within their own jurisdictions. Local voter control over their own jurisdiction’s policy requires that these limitations be implemented by the state legislature. Non-residents wish to limit local government behavior in order to make other communities more attractive to them in their roles as absentee landlords, wage earners, and possibly future residents. The non-resident explanation of limitations, unlike the Leviathan theory, can lead to inefficient local government when limits prevent local residents from selecting their optimal tax policy. The model in this paper, by considering volatility in tax payments, produces within-jurisdiction voter support for limitations in the absence of government waste.

The volatility of individual property tax payments is directly related to the tension between Smith’s maxims of equality and uncertainty. Smith’s equality maxim refers to the vertical and horizontal equity of the property tax system. That is, property owners with higher valued real estate should pay more in tax and taxpayers with equally valued properties should pay the same amount in taxes. Smith, however, recognizes that strict maintenance of equality in a property tax system requires frequent updates to property valuations and these updates have their own costs.

A land-tax assessed according to a general survey and valuation, how equal soever it may be at first, must, in the course of a very moderate period of time, become unequal. To prevent its becoming so would require the continual and painful attention of government to all the variations in the state and produce of

---

See also Bradbury, Mayer, and Case (2001), Cutler, Elmendorf, and Zeckhauser (1999), and McGuire (1999).

Another theory, the “state regime shift” theory does not require leviathan governments. Instead, the state regime shift theory focuses on voters’ desire to have the state finance a larger portion of necessary revenues with income and sales taxes, thus reducing reliance on local property taxes.

Smith describes equality in taxation as the condition where taxpayers pay taxes in proportion to their respective abilities; abilities that are measured by the revenue each taxpayer enjoys under the protection of the state.
every different farm in the country. The governments of Prussia, of Bohemia, of Sardinia, and the dutchy of Milan, actually exert an attention of this kind; an attention so unsuitable to the nature of government, that it is not likely to be of long continuance, and which, if it is continued, will probably in the long-run occasion much more trouble and vexation than it can possible bring relief to its contributors. (Book V, Chapter 2, Part I, article 1, pg 361)

Completely accurate and continually updated property values are required to maintain equality. The cost of this maintenance is more volatile, and possibly uncertain, individual property tax payments. In order to be uncertain, individual property tax liability must vary over time. Since the property tax is, for the most part, a single-rate ad valorem tax, individual property tax liability is equal to the product of the tax rate and the individual’s tax base (i.e., the taxable value of their real estate holdings). In the simplest property tax system only changes in tax rates and an individual’s property value cause changes in individual property tax payments. Thus even with completely accurate valuations of property, an owner’s uncertainty about their property’s value (or future values) can lead to uncertainty with regard to the amount of her tax payment. Of course this uncertainty can be exacerbated by inaccurate valuations.

Changes in the property tax rate is the other source of variation, and possible uncertainty, in an individual’s tax payment. Property tax rates with a single taxing jurisdiction exhibit far more variation over time than income tax or sales tax rates. While income tax and sales tax rates rarely change from year-to-year, property tax rates are constantly changing. The reason is that the local property is most often a “residual” tax, that is, as Bogart and Bradford (1990) explain, the property tax revenue each year is determined by the shortfall between desired expenditures and all other revenue sources (e.g., intergovernmental grants and parking fees). The fact that the tax base for a local property tax is known when expenditure levels are set allows for the property tax to be a ”residual tax.” This is perhaps why the policy variable for the local property tax is most often a selection of desired revenue level as opposed to the selection of a tax rate. The ex ante knowledge of total tax base allows for accurate predictions of actual ex post (i.e., collected) revenues. This ex ante tax base certainty provides local governments with the ability to set tax rates that, given the known tax base, imply a desired revenue level. With income and sales taxes, the tax base is not known with accuracy in advance and the setting of specific revenue levels is more

---

12 The difficulties of accurately updating property values to maintain equality in property taxation are by now well-understood and most would argue that substantial progress has been made in terms of the accuracy of these valuations. In the United States, appraisers of property are constantly evaluated as to the equality (i.e., uniformity) of their assessments.

13 Unless assessors follow the age-old maxim: To find a value good and true, Here are three things for you to do: Consider your replacement cost, Determine value that is lost, Analyze your sales to see what market value really should be. Now if these suggestions are not clear - Copy the figures you used last year!

14 For example, Anderson (forthcoming) shows that from 1995 to 1996, none of the 919 cities and townships in MN with population over 500 maintained the same tax rate.

15 The legality of whether rates or revenues are selected (although each one implies the other) is determined in each state’s constitution. We know of no studies that examine how differences in the legal policy variable might affect changes in revenues or rates.
problematic. Federal and state governments set tax rates and do not annually alter them in order to collect a desired amount of revenue.

Thus, unlike the federal income tax or state and local sales taxes, local property tax rates can and, most importantly, do vary from year-to-year. A property owner’s uncertainty about future tax rates thus depends on her expectation of future revenue requirements and the future values of all real estate in the jurisdiction (i.e., total tax base). This uncertainty about future tax rates creates uncertainty regarding property tax liability.

This concern over volatility and uncertainty leads Smith to offer the following choice with regard to property tax institutions

A tax upon the rent of land may either be imposed according to a certain canon, every district being valued at a certain rent, which valuation is not afterwards to be altered; or it may be imposed in such a manner as to vary with every variation in the real rent of the land, and to rise or fall with the improvement or declension of its cultivation. (page 352, Book V, Chapter 2, Part I, article I)

When valuations are “not afterwards to be altered” uncertainty in individual property tax payments is only caused by uncertainty about desired future revenues. In contrast, when valuations “vary with every variation in the real rent of the land,” an individual’s uncertainty about her tax payment may be caused by uncertainty about revenues, the value of her property, and the value of every property within her jurisdiction. While not necessarily tripling the amount of uncertainty, maintaining equality definitely triples the sources of uncertainty. Of course, valuations that are “not afterwards to be altered” have the potential to reduce equality.

The unbridled maintenance of equality, often called uniformity, of the property tax system produces changes in tax liability. If these change in property tax liability are large enough and somewhat unpredictable, some form of property tax limitation will be desirable even in the absence of any motivation to constrain excessive or inefficient government expenditures. In order to understand this possibility it is necessary to discuss how each of the three primary limits affect individual property tax payments. In particular, it is important to show how

\[\text{In this sense, in any given tax year the property tax resembles a lump-sum tax in that the amount to be collected from each taxpayer is set in advance. The individual can do nothing in this year to reduce their property tax burden. They could certainly move into a smaller house or to a different community, but this will only affect next year’s tax bill not this year’s tax bill. This is not the case with income and sales taxes where individuals can adjust their own current tax base in response to the current tax rate. It is possible that the unpopularity of the property tax is related to the unpopularity of lump-sum taxation. A recent example of the unpopularity of lump sum taxation is provided by Margaret Thatcher’s famous poll tax of 1990. The unpopularity of the poll tax led to Thatcher’s resignation and the eventual repeal of the poll tax.}\]

This idea is related to the personal finance theory of limit motivations cited by Cutler, Elmendorf, and Zeckhauser (1999). This theory states that voters interpret tax bill increases as a signal of government waste, especially when they observe no subsequent increases in service quality. As mentioned above and as will be made even more clear below, these tax increases can occur even without any increase in government revenues. Thus, increases in individual tax bills can be a very poor signal of increases in government expenditure. The research on personal finance theory, however, focuses on the level of property taxes, while this paper focuses on the variance of individual taxes.
individual tax payments change for reasons other than revenue changes. That is, individual tax payments can change even when a district’s revenue requirements are constant. These revenue-neutral changes in individual tax payments turn out to be key in understanding why some form of property tax limitation might exist without the existence or perception of wasteful or excessive government expenditures.

2 Tax Shares, Tax Prices, and Property Taxes

As mentioned above, individual tax payments will only vary if tax rates vary or if individual valuations vary. Although reasonably straightforward, the discussion of tax rates obscures the relationship between government expenditures and individual valuations. In particular, a focus on tax rates obscures the relationship between an individual’s tax payment and the distribution of individual valuations. The relationship between the distribution of valuations and the volatility of individual tax payments is an essential component of understanding why individuals would wish to place limits on property taxation.

The tax share is defined as individual $i$’s share of tax payments. We can denote each individual’s tax share at time $t$ as $s_{ti}$ and by definition the following equality must hold

$$\sum_{i=1}^{N} s_{ti} = 1.$$  

The tax price is defined as the expenditure required by the individual to provide another unit of public service to the community (e.g., to each individual in the community). This definition of tax price requires the inclusion of the costs of service provision which can vary across services and across communities. If individuals consider other, usually more indirect costs of taxation, such as the distortion of location decisions by individuals (i.e., migration) and the distortion of investment decisions by individuals, these indirect cost must also be incorporated into tax price.\footnote{See Wildasin (1989) and Crane (1990) for a discussion of these indirect costs and their inclusion in tax price.}

For any public service, an individual’s tax price is the product of the cost of that service and her tax share. For example, when the marginal cost of providing another unit of a public service to all residents is equal to $1$, the tax price for individual $i$ of providing another unit of the public service to all residents is equivalent to the tax share of individual $i$. This is clearly a special case, as an individual’s tax price does not always equal that individual’s tax share.

An individual’s budget constraint reflects the relationship between disposable income, $y$, and expenditures on a consumption good, $c$, whose price is normalized to one, and expenditures on local taxes,

$$y_i = c_i + t_i$$

where $t_i$ is taxes paid to the government to fund public expenditures, $E$.\footnote{See Wildasin (1989) and Crane (1990) for a discussion of these indirect costs and their inclusion in tax price.}
A local government’s budget constraint reflects the relationship between expenditures and revenues

\[ E = A + \frac{t_i}{s_i} = A + T \]  \hspace{1cm} (3)

where \( E \) is total expenditure, \( A \) is total lump-sum aid and \( s_i \) is the share of total taxes paid by voter \( i \), and \( T \) is total tax revenue received by the government. The government’s budget constraint reflects the following relationship between tax revenues and individual tax shares,

\[ T = \sum_{i}^{n} t_i \]
\[ \bar{t} = \frac{t_i}{s_i} \]  \hspace{1cm} (4)

Under a single-rate ad valorem property tax, \( s_i \) equals the ratio of the taxable value of an individual’s home, \( v_i \), to the total taxable value in their taxing district, \( \sum_{i} v_i \).

\[ s_i = \frac{t_i}{T} = \frac{\tau \cdot v_i}{\tau \cdot \sum_{i} v_i} = \frac{v_i}{\sum v_i} \]  \hspace{1cm} (5)

which follows from the following equalities describing tax revenue

\[ T = \tau \cdot \sum_{i} v_i \]  \hspace{1cm} (6)

and

\[ t_i = \tau \cdot v_i \forall i \]  \hspace{1cm} (7)

It is often easier to express the government budget constraint in terms of per-resident expenditure as per-resident expenditures are more readily comparable across jurisdictions. Per-resident (\( n \) = number of residents) expenditure can be be defined as

\[ e = \frac{E}{n} = a + \bar{t} \]  \hspace{1cm} (8)

where \( a = \frac{A}{n} \) and \( \bar{t} = \frac{T}{n} \).

After expressing \( \bar{t} \) in terms of the taxes paid by individual \( i \), \( t_i \),

\[ \frac{T}{n} = \tau \cdot \sum_{i} \frac{v_i}{n} = \left( \frac{t_i}{v_i} \right) \left( \sum_{i} \frac{v_i}{n} \right) \]  \hspace{1cm} (9)

\[ \bar{t} = t_i \cdot \frac{\sum_{r} v_r}{n} \cdot \frac{1}{v_i} \cdot \frac{\sum_{i} v_i}{\sum_{r} v_r} \]  \hspace{1cm} (10)

\[ \bar{t} = t_i \cdot \left[ \frac{\bar{v}_r}{v_i} \cdot \frac{\sum_{i} v_i}{\sum_{r} v_r} \right] \]  \hspace{1cm} (11)
it is now possible to rewrite the government budget constraint as

\[ e = a + t_i \cdot \left[ \frac{v_r}{v_i} \cdot \frac{\sum v_i}{\sum v_r} \right]. \]  

(12)

The government budget constraint (12) can now be combined with the individual budget constraint (2) by substituting for \( t_i \) in the individual budget constraint producing

\[
\begin{align*}
y = c + (e - a) \cdot \left[ \frac{v_i}{\bar{v}_r} \cdot \frac{\sum v_r}{\sum v_i} \right] \\
(y - a p_i) = c + p_i \cdot e
\end{align*}
\]

(13) (14)

where the price of per-resident expenditure, \( e \), is possibly different for each taxpayer

\[ p_i = \left[ \frac{v_i}{\bar{v}_r} \cdot \frac{\sum v_r}{\sum v_i} \right] \]  

(15)

which is also equal to her tax share divided by the resident population,

\[ p_i = s_i \cdot n. \]  

(16)

The tax price, \( p_i \), is equal to the cost to taxpayer \( i \) of increasing per-resident expenditure by $1.

It is more natural to define tax price in terms of service quality as opposed to defining it in terms of dollars of public expenditure (as above). Service quality is related to total expenditure, \( E \) through a cost function

\[
\begin{align*}
C_q(\cdot) \cdot q &= E \\
\frac{C_q(\cdot)}{n} \cdot q &= e \\
\Rightarrow \quad q &= e \cdot \frac{n}{C_q(\cdot)}
\end{align*}
\]

(17) (18) (19) (20)

where \( C_q(\cdot) \) is the marginal cost of providing a service quality of \( q \) to all residents.

By substituting for per capita expenditure, \( e \), in the individual’s budget constraint (equation 14), the tax price of a one unit increase in service quality is now equivalent to

\[
\begin{align*}
p_i &= \frac{C_q(\cdot)}{n} \cdot \left( \frac{v_i}{\bar{v}_r} \cdot \frac{\sum v_r}{\sum v_i} \right) \\
p_i &= C_q(\cdot) \cdot s_i
\end{align*}
\]

(21) (22)

19 The variable \( \bar{v}_r \) is the average value of residential properties, where \( r \) indicates that only residential homes are considered. This assumes that there are other types of property, but that it is most useful to consider expenditure on a per-resident, as opposed to per-property, basis. Furthermore, many assessment limits specifically address increasing residential tax shares and this equation demonstrates the effects of rising residential shares.

20 This assumes constant returns to scale technology so that marginal cost equals average cost at all levels of production. It is extremely important to remember that the composition of the tax base can affect \( C_q(\cdot) \). See, for example, Bradford, Malt, and Oates (1969).
Equation (22) demonstrates that each taxpayer’s tax price is the product of their tax share and the marginal cost of service quality. The relationship between residential tax share and tax price is illustrated in equation (21). By examining the individual’s budget constraint (14) it is clear that individual tax payments, $t_i = p_i \cdot q$, can change even if service quality or government expenditure does not change. Most importantly, individual tax payment volatility is a direct result of volatility in tax shares. As discussed above, empirical evidence suggests that this volatility in individual tax payments could be quite large.  

2.1 Limits

Tax limits constrain increases in individual property tax payments over time. Limits on property tax revenue and property tax rates are the most widespread property tax limitations with 34 states having some form of rate limits and 29 having some form of revenue limits. Limits on increases in assessed (i.e., taxable) value are also numerous with 20 states having some form of assessment limits in place as of 2006. Anderson (2006) provides a general overview of property tax limitations in the United States.

Tax revenue limits constrain annual increases in property tax revenues. Revenue limits vary across states but most limit growth in revenue to the rate of inflation or some other percentage. For example, Illinois’s Property Tax Extension Limitation Law limits annual increases in property tax revenues to the lower of 5% or inflation (as of 2006 this limit is in effect in 39 counties of Illinois 102 counties). Revenue limits will most effectively constrain increases in individual property tax payments in the presence of uniformly distributed increases in property values. Changes in the composition of the tax base, however, will cause individual tax payments to change even if property tax revenues were held constant. This may be why 10 of the 29 states with revenue limitations also have some form of assessment limitations.

Assessment limits constrain changes in individual property tax payments that would occur even if revenues remained constant. The exact nature and extent of the limitations on increases in taxable value vary across states. Twelve states have statewide limitation on increases in the taxable values of individual residential properties. For example, Florida limits increases in the taxable value of residential properties to the lower of 3% or inflation while Michigan limits increases to the lower of 5% or inflation. Georgia, Illinois, South Carolina provide counties with the option to explicitly limit taxable value increases and New York mandates limits in New York City and Nassau County. Iowa and Colorado limit increases in the total taxable value of residential property in the entire state. Only Connecticut

\footnote{The examples cited above are from Minnesota and Chicago. These are not places usually associated with so-called “hot” real estate markets or real estate bubbles. This suggests that tax shares and payments can be volatile even when property values are not excessively volatile.}

\footnote{These twelve states are AR, AZ, CA, FL, MD, MI, MN, MT, NM, OK, OR, and TX. CA, OR, and OK limit assessment increases regardless of property type. Except in Arizona and Oregon, assessment caps are removed at the time of a sale. O’Sullivan (2001) finds that nine states have limits on assessment increases. However, he never lists the nine states. Assessment limits approved by Washington voters have twice been ruled unconstitutional. In addition, Nevada recently instituted a cap on increases in individual tax payments.}
cut, Maryland, and South Carolina have assessment limits without explicit revenue or rate limitations. None of these three states require that property values be updated on an annual basis. In the remaining 17 states with assessment limits, nine have both levy limits and tax rate limits, one has only levy limits, and seven have only rate limits. All of these limitations on increases in taxable values constrain increases in individual property tax payments by restricting changes in the distribution of taxable values from year to year.

If assessment limits only apply to residential property, which is often the case, potential increases in tax payments by residential homeowners will be muted by the limitations. The desire to reduce tax payments is clearly strong but does not necessarily imply that individual voters want to limit the authority of government. In this paper the focus is turned to the variance of tax payments as opposed to changes in mean tax payments. The focus on variance in tax payments addresses a topic that has been neglected by the literature in local public finance. Discussion of the level of tax payments does not suffer from the same neglect.

Tax rate limits are in effect in 34 states with 23 of these 34 states also having revenue limits and 16 also having some form of assessment limits. Rate limits are either specific or general. Specific limits constrain the property tax rates of particular funds within taxing districts while general limits constrain the total property tax rate within taxing jurisdictions. Illinois has specific tax rate limitations that vary from fund to fund. California has a general limit that constrains the maximum tax rate to be less than or equal to 1% of taxable value.

Rate limits constrain individual property tax payments by setting a maximum value for the ratio of revenue to total tax base. If increases in total tax base and individual tax base are relatively small and relatively infrequent, tax rate limits will effectively constrain increases in individual property tax payments. For this reason, tax rate limits are often combined with revenue limits, assessment limits, or infrequent revaluations of property.

In fact, only five states have limitation on tax rates without any explicit statewide limits on either revenues or assessment increases. Alabama, North Carolina, and New York constitute three of these five states and they do not require annual updates to individual taxable values. Wyoming and Utah limit tax rates without other explicit limits and appear to legally require annual updates to taxable values. Thus, 3 of the 5 states with only rate limits appear to feature property tax systems that encourage infrequent changes in the taxable values of individual properties.

3 Voters Prefer Less Price Volatility

The analysis above emphasizes the fact that the tax price of public services can vary for any individual over time. This variation in tax price could possibly lead to uncertainty in tax prices. In order not to confuse voter responses to surely increasing tax prices with voter responses to an increased variance or uncertainty in tax prices, it is essential to discuss the variance of tax prices while holding the mean tax price constant. In other words, this analysis

Assessment limits that apply only to residential properties do not limit increases in the tax payments of individuals owning non-residential properties. Note also that states with non-annual assessment systems essentially cap annual assessment increases at zero in non-assessment years.
considers how voters respond to increases in the variance of tax price that do not affect the expected tax price for each voter.

Although possibly surprising at first, consumers, in general, benefit from mean-preserving increases in variance of prices. The literature in economics discussing the volatility or instability of prices features work by Waugh (1944), Oi (1961), Samuelson (1972), and Turnovsky, Shalit, and Shmitz (1980). Most succinctly, but perhaps not most intuitively, a consumer will gain from mean-preserving price instability if the indirect utility function is convex with respect to the price of the good in question.

Waugh (1944) was the first to demonstrate, using consumer surplus, that a consumer could gain from price instability. As Turnovsky, Shalit, and Shmitz (1980) demonstrate, consumers gain from price stability (i.e., consumers prefer less variance) if

$$s_j(\eta_j - \rho) - e_j < 0,$$

where \( \eta_j \) is the income elasticity of demand for the good in question, \( e_j \) is the price elasticity, \( s_j \) is the budget share of the good, and \( \rho \) is the coefficient of relative risk aversion. Empirical evidence suggests that this equality will be positive for almost any good. Conventional estimates of income and price elasticities for public services also suggest that this equality is positive. Thus, it appears highly unlikely that taxpayers would prefer less volatility in tax prices to more volatility in tax prices when they can freely choose the quantities of public services they consume. This does not suggest, however, that taxpayers would not prefer a lower average tax price or lower expected changes in tax price.

It is simplest to demonstrate this preference for price volatility in the context of a non-Giffen good whose price will vary about its mean. In this example consumers are risk-neutral.

**Theorem 1** An agent is made strictly better off by increased mean-preserving variance between any two prices for the same good if and only if the good is not a Giffen good.

**Proof.** Without loss of generality, suppose the first price of the same good is lower than the second: \( p_1 < p_2 \). Let \( \bar{p} = \frac{p_1 + p_2}{2} \). Then define \( \delta > 0 \) such that:

$$p_1 = \bar{p} - \delta \quad (24)$$
$$p_2 = \bar{p} + \delta \quad (25)$$

This analysis restricts discussion of variance to an analysis of \( \delta \). It should be clear to the reader that an increase in \( \delta \) is equivalent to an increase in variance.

---

24 Samuelson (1972) argued that the price instability discussed by Waugh (1944) and Oi (1961) was not realistic (i.e., feasible) and that feasible price instability did not benefit consumers. This discussion of feasibility is emphasized less in subsequent work and the work of Turnovsky, Shalit, and Shmitz (1980) considers mean-preserving price variance as feasible. The argument over feasibility is not relevant to the example of property taxation.

25 The price elasticity is certainly negative making the last term a positive number.

26 Consumers of a Giffen good will increase consumption of that good as its price increases.

27 See appendix, lemma section ?? for a demonstration of this equivalence.
As usual, define \( v(p, w) \) as the agent’s indirect utility at prices \( p \) and wealth \( w \). It is shown that the following two claims are equivalent:

1. The agent is made better off by an increase in price variance: \( \frac{\partial v}{\partial \delta} > 0 \), and

2. The good in question is not a Giffen good, that is the agent buys more of the good when it is cheaper: \( x_1^* > x_2^* \)

First note that:

\[
v(p, w) = v(p - \delta, p + \delta, p_3, \ldots, p_L, w)
\]

\[
\Rightarrow \frac{\partial v}{\partial \delta} = \frac{\partial v}{\partial p_2} - \frac{\partial v}{\partial p_1}
\]

then discover that \( \frac{\partial v}{\partial \delta} > 0 \) and \( x_1^* > x_2^* \) are equivalent:

\[
\frac{\partial v}{\partial \delta} > 0
\]

\[
\iff \frac{\partial v}{\partial p_2} - \frac{\partial v}{\partial p_1} > 0
\]

\[
\iff \frac{\partial v}{\partial p_2} > \frac{\partial v}{\partial p_1}
\]

\[
\iff -\frac{\partial v}{\partial p_2} < -\frac{\partial v}{\partial p_1}
\]

\[
\iff x_2^* < x_1^*
\]

by Roy’s Identity. This completes the proof.

The most crucial assumption in the above results is that consumers are able to freely select the quantity of the price-volatile good that they wish to consume. While this result certainly applies to the consumption of, for example, apples and oranges, it seems less applicable to consumption of public services such as education, law enforcement, fire protection, and street maintenance. Even when these are not pure public goods it is difficult to argue that each individual selects their specific level of consumption over time.

Individuals do select an exact quantity of public service consumption in models inspired by Tiebout (1956). These models (e.g., Hamilton (1975)) often feature communities whose taxpayers have identical tax prices. Although these models are incredibly insightful, they are also inherently static in nature. Thus they do not readily lend themselves to a consideration of individual tax prices changing over time. Indeed, the zoning restrictions assumed in these models would likely prevent changes in the distribution of tax shares over time.

Other models allow for individuals within a single community to have different tax shares (e.g., Epple and Platt (1998)). These models allow voters to have different tax shares than the decisive voter, implying that, given their location, all voters are not consuming their
most preferred level of public services. These are static models, however, and they do not address how voters might respond to volatility in their tax prices over time. Fernandez and Rogerson (1996) examine the same issues with an overlapping-generation model, thereby incorporating time explicitly into these types of models. They don’t, however, consider how tax prices within the same community might change over time. Furthermore, neither these models or the models inspired by Tiebout (1956) consider risks and uncertainty.

The evidence cited above from Minnesota and Chicago suggests that individual tax shares do change over time and that these changes can produce substantial changes in individual tax liabilities. Furthermore, it also the case that tax shares do vary within the same community at one point in time. Figure 1 displays the distribution of tax shares within the residential property class for four Chicago suburbs. The distributions exhibit substantial variation in tax shares, with both Evanston and Oak Park having much larger tails at the high-end of the tax share distribution. To provide some context, consider that using state-wide averages for property tax collections, a 0.01 difference in tax share implies over a $100 difference in property tax liability. The standard deviation of tax shares in Oak Park is almost 0.04, implying a one standard deviation move from the mean tax share implies an over $400 difference in property tax liability.

As is hopefully clear, changes over time in individual tax share result from differential rates of appreciation among properties within the same jurisdiction. If all properties appreciate by 10%, all tax prices will remain the same. If, however, some properties appreciate by 5% while others appreciate by 10%, the tax prices of high appreciating home will increase and the tax price of the low appreciating homes will decrease (assuming that the marginal costs of service provision remain the same). The empirical observations on tax payments in Chicago and Minnesota suggest that these differential rates of appreciation occur within the residential property class and between residential and commercial property within the same jurisdiction.

3.1 Uncertainty and Choice

As the Turnovsky, Shalit, and Shmitz (1980) result, equation (23), makes clear, when consumers with standard preferences have complete freedom to select consumption amounts, all else equal, these consumers prefer more variance in prices to less variance in prices. Rather than insist that preferences for public services are unusual or that taxpayers have an unusually high degree of risk-aversion, the simple model below explores how a lack of freedom to completely adjust the quantity of consumption in response to changing prices implies that

---

28 Across locations, an individual is in her most preferred community, with her most preferred tax policy. But this does not imply that she would not prefer more or less public service given her tax price within her current location.

29 Consider that in 2000, school districts in Illinois collected an average of $10,665,821 in property taxes in 2000. In this same year, residential property paid approximately 58% of the total property taxes. This suggests that a difference in tax share of 0.01 in the average school district would imply a difference in residential property tax of approximately $62. Considering that school district property tax revenues represented 58% of all property tax revenues, this implies an average total difference in property tax liability of $106.
taxpayers prefer lower variance in prices as opposed to higher variance in prices. This lack of freedom also creates a desire to limit government revenues.

As noted above, this lack of consumptive freedom is a feature of models that allow individuals in a single community to have different tax shares at one point in time. In the simplest model, the decisive voter (e.g., median voter) is able to select his or her most preferred level of public services. The level of public services most preferred by the decisive voter will be different from the most preferred bundle of the other voters in that same jurisdiction.

The first step in analyzing taxpayers preferences in the presence of variance in tax prices is to allow for tax prices to change by introducing time into a simple model of choice regarding private consumption and public services. Time is introduced by examining a two-period model. The Turnovsky, Shalit, and Shmitz (1980) result on price variance implies that if the decisive voter in one time period is certainly decisive in each subsequent period that this always-decisive taxpayer will prefer more price variance to less, all else equal. An always-decisive voter prefers higher price variance because she can always adjust her consumption in response to changes in prices. In the presence of an always-decisive voter, every other (non-decisive) voter will not be able to fully respond to changes in their tax prices by efficiently selecting a new perfectly optimal bundle. Taxpayers that are never-decisive or sometimes-decisive are classified as not-always-decisive. The model below demonstrates that any risk-averse taxpayer that is not-always-decisive will prefer less price variance to more variance. The analysis also demonstrates that given the potential for changing tax prices, that taxpayers may prefer to limit government revenues.

This analysis explores how risk-aversion combined with shocks to tax price create a desire for tax limits in the context of a specific model, with a particular shock to price, and a particular utility function. Thus, the issue is not discussed in full generality. The analysis highlights the effects of price uncertainty and rigid consumption choices on internal equilibria. It does not address the issue of external equilibria that presents itself in the context of multi-community sorting and voting models.

Suppose there is a decisive-in-revenue voter, voter \(i\), at time 1. (This decisive-in-revenue voter might be median in some parameter—presumably house value or tax price—but the parameter is not important for this result.) Decisiveness in revenue indicates that under the majority voting system, this time 1 decisive voter selects the level of government revenue in period 1. Selecting government revenue is equivalent to voters having preferences over service quality and assuming that the marginal cost of a unit of quality is $1.

All voters receive idiosyncratic shocks to real estate value in period 2. Hence, with virtual certainty, voter \(i\) will not be the decisive-in-revenue (e.g., median) voter at time 2 if the parameter determining decisiveness is either home value or tax price. But, regardless of

---

30 Again assuming convex indirect utility with respect to price.
31 This analysis ignores mobility across communities, but it is unlikely that any voter could always move and find the perfectly optimal tax-consumption bundle in every period. By optimal, we mean a particular voter’s optimal tax-consumption bundle within a community at their given tax price as opposed to their optimal selection of a community (described by a specific bundle) that is preferred over all other locations (and their specific bundles).
the mechanism, suppose this voter assumes that she will not be the decisive-in-revenue voter in period 2. This makes her a sometimes-decisive voter. Thus, voter $i$ views $R_2 \geq 0$ as out of her direct control. Voter $i$ is able to select private consumption in both periods and is able to transfer funds without cost across the two periods. All voters have the same two-period exogenous income and there is only one community. A CARA utility function introduces risk-aversion and the analysis assumes that shocks to tax price are normally distributed. This takes advantages of the well-known properties of the CARA utility function.

Below it is shown that this sometimes-decisive voter $i$ is interested in limiting the volatility of her tax price.

### 3.1.1 Assumptions

1. Suppose there are two goods: private consumption $c$ and government services (i.e., government revenue) $R$.

2. The level of $R$ is selected by majority vote in each period. With single-peaked preferences, the voter with median preferences will be decisive.

3. Suppose voter $i$ has CARA utility:
   
   $\text{utility}(c, R) = u(c) + u(R)$
   
   where $u(x) = -e^{-\alpha x}$

   where $\alpha$ is the coefficient of absolute risk aversion (which is constant.) (CARA=Constant Absolute Risk Aversion.)

4. Suppose voter $i$ is the decisive-in-revenue voter in period 1 and can choose:
   
   - Consumption $c_1$, and services $R_1$ at time 1. The voter can transfer assets, $a_1$, from period 1 to period 2 at no cost.
   - Consumption $c_2$ at time 2.

5. Suppose that tax price in period 1, $p_1$, is known for every voter and that the price of private consumption is normalized to one.

6. Tax prices in period 2, $p_2$, are subject to shocks and may be different than prices in period one.

7. Suppose revenue $R_2$ is known. Although she’s the decisive-in-revenue voter now, voter $i$ realizes that tax prices will shuffle around and that it is certain that she will not be the decisive-in-revenue voter next period. Rather then construct an expectation for $R_2$ it is assumed that the decisive voter (who is not the same individual in both periods) will have the same tax price in both periods. This implies that $R_1 = R_2$. Importantly, it is not the case that the same decisive voter chooses both $R_1$ and $R_2$.

8. Suppose the each voter has a fixed budget $B$ over the two-periods and there is no interest rate/discounting between periods.
3.1.2 Theorem and Proof

**Theorem 2** Suppose the assumptions 1-8 above.

Suppose the tax price is hit with a normal shock of the following form: \( p_2 = p_1(1 + \epsilon) \), where \( \epsilon \) is distributed \( N(\mu, \sigma^2) \).

Then voter \( i \)'s (a sometimes-decisive voter) utility is decreasing in \( \sigma^2 \).

**Proof.** Consider voter \( i \)'s problem as choosing, at time 1, three things: consumption \( c_1 \), services \( R_1 \), and savings to time 2, called assets \( a_1 \). Note that the government budget constraint (must balance its budget) has been substituted into the period 1 and period 2 budget constraints for the consumer. Then individual \( i \)'s problem is:

\[
\max_{c_1, a_1, R_1} u(c_1) + u(R_1) + Eu(c_2) + u(R_2) \tag{35}
\]

\[
c_1 + p_1 R_1 + a_1 \leq B, c_2 \leq a_1 - p_1(1 + \epsilon) R_2 \tag{36}
\]

Please note again that she does not consider that she can select \( R_2 \).

By local non-satiation (and substituting the budget constraints into the problem), this problem is equivalent to:

\[
\max_{c_1, a_1, R_1} u(c_1) + u(R_1) + u(a_1 - p_1(1 + \epsilon) R_2) + u(R_2) \tag{37}
\]

\[
c_1 + p_1 R_1 + a_1 = B \tag{38}
\]

Using a property of CARA utility and the fact that \( \epsilon \) is distributed \( N(\mu, \sigma^2) \), makes this is equivalent to:

\[
\max_{c_1, a_1, R_1} u(c_1) + u(R_1) + u \left( a_1 - p_1 R_2 - p_1 R_2 \mu - \frac{1}{2} \alpha(p_1 R_2) 2 \sigma^2 \right) + u(R_2) \tag{39}
\]

\[
c_1 + p_1 R_1 + a_1 = B \tag{40}
\]

Consider, now, the voter’s indirect utility as a function of \( \sigma^2 \):

\[
v(\sigma^2) = u(c_1^*) + u(R_1^*) + u(R_2) + u \left( a_1^* - p_1 R_2 - p_1 R_2 \mu - \frac{1}{2} \alpha(p_1 R_2) 2 \sigma^2 \right) \tag{41}
\]

\[
\Rightarrow \frac{\partial v}{\partial \sigma^2} = u' \left( a_1^* - p_1 R_2 - p_1 R_2 \mu - \frac{1}{2} \alpha(p_1 R_2) 2 \sigma^2 \right) \left( -\frac{1}{2} \alpha(p_1 R_2) 2 \right) < 0 \tag{42}
\]

(By the envelope theorem, the indirect effects through \( c_1^* \), etc. can be safely ignored. Note that \( \sigma^2 \) does not appear in the budget constraint, and hence can also be ignored in the derivative of the Lagrangean.)

Hence, voter \( i \) (a sometimes-decisive voter) is made worse off by tax price variance. ■

---

\[\text{\textsuperscript{32}}\]The government budget constraint in each period is \( R_t = \frac{t}{s_t} \), where \( C_q = 1 \) and property taxes are the only revenue source, implying that \( p_{it} = s_{it} \).
The choice of voter \( i \) is arbitrary and that this is a two-period model is also arbitrary. This result demonstrates that any not-always-decisive (in revenue) voter prefers now to reduce price variance in the future. This applies to any not-always-decisive (in revenue) voter at any time she has the opportunity to vote.

Note also that voter \( i \) will be made more worse off by tax price variance as the level of risk aversion \( \alpha \) increases.

\[
\frac{\partial v}{\partial \sigma_2 \partial R_2} = u'' \left( a_1^* - p_1 R_2 - \frac{1}{2} \alpha(p_1 R_2) 2 \sigma^2 \right) \left( \frac{1}{4} \alpha(p_1 R_2) 4 \sigma^2 \right) + u' \left( a_1^* - p_1 R_2 - \frac{1}{2} \alpha(p_1 R_2) 2 \sigma^2 \right) \left( -\frac{1}{2} (p_1 R_2)^2 \right) < 0 \quad (43)
\]

Furthermore, it is the case that higher second period revenue makes voter \( i \) worse off for a given level of price variance

\[
\frac{\partial v}{\partial \sigma_2 \partial R_2} = u'' \left( a_1^* - p_1 R_2 - \frac{1}{2} \alpha(p_1 R_2) 2 \sigma^2 \right) \left( -\frac{1}{2} \alpha(p_1 R_2)^2 \right) \left( -(p_1 + p_1 \mu + \alpha p_2^2 R_2 \sigma^2) \right) + u' \left( a_1^* - p_1 R_2 - \frac{1}{2} \alpha(p_1 R_2) 2 \sigma^2 \right) \left( -\alpha p_2^2 R_2 \right) < 0. \quad (44)
\]

Equation 44 demonstrates that a higher level of revenue in period 2 exacerbates the costs of variance in prices for the non-decisive voter. Variances in prices are also seen to exacerbate the potential costs to a non-decisive voter of increases in revenue. Increases in revenues in a period when voter \( i \) is not decisive, \( R_2 \), can have negative or positive effects on individual utility even without variance in tax prices. Positive variance in prices, however, makes it more likely that increases in \( R_2 \) will decrease an individual’s indirect utility.\(^{33}\) Letting \( X(\mu, \sigma_2, p_1, R_2) = p_1 R_2 \mu + \frac{1}{2} \alpha(p_1 R_2) 2 \sigma^2 \),

\[
\frac{\partial v}{\partial R_2} = \frac{u'(R_2)}{1} + \frac{u' \left( a_1^* - p_1 R_2 - X(\mu, \sigma_2, p_1, R_2) \right) \times \left( -p_1 - p_1 \mu - \alpha p_2^2 R_2 \sigma^2 \right)}{u' \left( a_1^* - p_1 R_2 - X(\mu, \sigma_2, p_1, R_2) \right)} \quad \Rightarrow
\]

\[
\frac{\partial v}{\partial R_2} < 0 \quad \text{if} \quad u'(R_2) < \frac{u' \left( a_1^* - p_1 R_2 - X(\mu, \sigma_2, p_1, R_2) \right) \times \left( p_1 + p_1 \mu + \alpha p_2^2 R_2 \sigma^2 \right)}{u' \left( a_1^* - p_1 R_2 - X(\mu, \sigma_2, p_1, R_2) \right)} \quad (45)
\]

or
\[
\frac{u'(R_2)}{u' \left( a_1^* - p_1 R_2 - X(\mu, \sigma_2, p_1, R_2) \right)} < \left( p_1 + p_1 \mu + \alpha p_2^2 R_2 \sigma^2 \right. \quad (46)
\]

\(^{33}\)In a model with constant tax prices over time and an always-decisive voter there is no reason to expect changes in \( R \) over time unless something else changes to alter demand (e.g., income).

18
To see that the effect of an increase in $R_2$ is more likely to be negative when there are shocks (even when shocks are mean zero) more formally, examine

\[
\frac{\partial^2 v}{\partial R_2 \partial \sigma^2} = u''(\cdot) \left( -p_1 - p_1 \mu - \alpha p_1^2 R_2 \sigma^2 \right) \left( -\frac{1}{2} \alpha p_2^2 R_2^2 \right) + u'(\cdot) \left( -\alpha p_1^2 R_2 \right) < 0. \tag{48}
\]

Increases in revenue in period 2 will be less desirable when there is variance in tax prices. Of course, it is trivial to demonstrate that consumers would prefer to reduce $\mu$ as all individuals would prefer a lower expected value for price shocks, all else equal.

These results demonstrate that not-always-decisive (in revenue) voters will support measures to limit tax revenues, $R_2$, given variance in their tax prices. Since at any moment non-decisive-in-revenue voters out-number even an always-decisive-in-revenue voter, limits on tax revenues can win a majority vote. Of course, this is in the context of a simple median-voter model, where one voter is decisive in the choice of revenue $R$ in each period. Importantly, just because this voter is decisive in revenue does not imply that they are decisive on all issues, as the analysis above indicates. In fact, an always-decisive (in revenue) voter is the only voter that would always reject any limits on revenue growth in the presence of variance in prices.

This result proves a limit on tax revenues can win a majority vote in the context of a median voter model. There is no waste or inefficiency in this model as the government selects $R$ by majority vote in each period. The not-always-decisive voters must pass legislation limiting revenue since an always-decisive in revenue voter (or any decisive voter in revenue in any period), can always win a majority vote in the current period to receive their preferred level of public revenues and expenditures. That is, the non-decisive voters will not be able to effectively limit changes in revenues in each period.

It is also clear that measures limiting variance in prices, all else equal, will also win in a majority voting framework. Assessment limits might resemble such measures, although assessment limits are likely to affect expected values of changes in tax prices as well as the variance of tax price changes.

The desire for limits on revenues and price variance arises out of the preference for smoothing marginal utility across periods that is common in multiple period models. While a revealed lower price in period 2 offers a benefit to a consumer in the form of increased private consumption (assuming the same $R$), a revealed higher price in period 2 costs the consumer private consumption. Risk-aversion then implies that the consumer will prefer to avoid the possible loss, and importantly, forgo the possible benefit in order to better equate marginal utility across periods. This argument, of course, does not apply to a consumer that can optimally adjust $R_2$ to any change in prices.

One important caveat to these results is the availability of overrides. An override gives voters an opportunity to collect revenues or increase tax rates in excess of the legislated limit, often only temporarily. Overrides are an important component in most revenue limitation legislation. As the analysis above indicates, within any single period a decisive-in-revenue voter should always win a majority vote to exceed the limit if this is what she desires.
This would seem to make revenue limits completely useless in the context of this model. The actual override process, however, often also limits the amount by which revenue can exceed a limit. For example, in Illinois school districts many tax rate limitations can be exceeded for one period but only up to a new maximum rate level that cannot be exceeded. In Massachusetts voters can override revenue limitations with a simple majority but they cannot exceed tax rate limitations. These types of super-limits insure that the non-decisive voters have protections against large and sudden increases in property taxes. It is important, however, to have overrides if, for example, a common shock to income were to affect taxpayers in a jurisdiction and they all wish to spend more than previously. The limits, even if they are not designed with only this specific purpose in mind, protect voters from particularly bad outcomes caused by shocks to price (or something else) that are not equally distributed across the jurisdiction’s population.

The lack of an ability to adjust consumption of public services, \( R \), is the driving force behind the conclusions thus far. Of course, the lack of ability to adjust consumption of \( R \) could arise from sources different from what is described above. For example, if a decisive voter was forced to choose a constant level of revenue or service quality for several periods, she may vote to restrict variance in prices.

Yet another possible reason for revenue limits might be causal confusion on the part of voters (see Pape (2006)). It is expensive to monitor local government and determine if expenditures are wasteful or inefficient. A good signal of increased waste might be an increase in an individual tax bill that is not followed by a corresponding increase in service quality. If a voter is unsure of the cause of the increase in her property tax bill, or incorrectly assumes that the increase must be caused by increased government revenue and waste, she may support limitations on revenue even if the government is not wasteful.

In conclusion, the traditional Leviathan model provides a motivation for voters to limit variance in prices. If voters cannot force the government to collect less revenue, variance in prices would only make revenue limits more of an urgent priority. But these results demonstrate that a voter belief in wasteful government is not necessary for the existence of limitations on revenue, tax rates, and assessed values.

4 Conclusion

Revenue limits serve to dampen the effects of the changes in real estate value that can cause volatility in individual tax payments. When direct policy measures limiting the mean and variance of price changes are not available the majority of voters within a jurisdiction will support at least some form of revenue limitation. At this point these results neglect to formally demonstrate the existence of a political equilibrium where revenue limits survive a majority vote. It is, however, demonstrated formally that individuals prefer changes in tax prices to have less variance and lower expected values and that revenue increases exacerbate the potential negative effects of these price changes.

The next step in this research is to demonstrate empirically that changes in tax prices are large enough to engender these types of limitations. In the appendix it is demonstrated that
small changes in value can produce large changes in tax shares. Thus tax shares can exhibit substantial variation without extreme variations in property valuation. While evidence from Minnesota and Illinois suggests that tax shares are volatile, there is still a need for more empirical studies that measure this extent of the problem. It is still possible that while theoretically important the variance of tax prices is just not large enough to be empirically relevant. We find this unlikely and hope to demonstrate that these shocks to tax price can be surprisingly large. Of course, if voters are concerned (even mistakenly) about shocks to tax price they may support certain limitations even when the likelihood of the shocks is relatively small.

An important assumption as yet undiscussed is the affects of increases in property value on the wealth of consumers. Even if wealth increases, an unequal change in tax prices still appears to cause the same problems for the non-decisive voters. They cannot optimally respond to their changes in wealth by selecting their most desired reallocation of income between private consumption and government services. It is the non-decisiveness that produces the limitation results, not the inability of taxpayers to finance consumption with gains in the asset value of their property.

An important extension to this research is a discussion of how the different sources of variation in local property tax payments make it more difficult for individuals to monitor local government. As the above analysis makes clear an increase in an individual’s tax bill is not necessarily a signal of increased government expenditure. If voters are unsure how to interpret changes in their tax bills they may support constraints on government spending because they falsely interpret the causal mechanism of changes in their tax bills.

References


A Appendix

The appendix features a proof of delta and variance equivalence as well as a detailed discussion of the role of assessment limits. We do not include these discussions in the main text because they are much more speculative and, at the moment, seem to detract from our main point above concerning revenue limitations.

A.1 Delta, Variance Equivalence

Lemma 1 Evaluating a change in $\delta$ is equivalent in sign to evaluating a change in variance.

Proof. The variance between the prices is:

$$\text{var}(p_1, p_2) = (p_1 - \bar{p})^2 + (p_2 - \bar{p})^2$$

$$= \delta^2 + \delta^2 = 2\delta^2$$

$$\implies \delta = \frac{1}{2} \sqrt{\text{var}}$$

$$\implies \frac{\partial \delta}{\partial \text{var}} = \frac{1}{4\sqrt{\text{var}}}$$

$$\implies \frac{\partial \delta}{\partial \text{var}} > 0 \iff \text{var} > 0$$

This establishes that an increase in var is equivalent to an increase in $\delta$; hence, to evaluate the effect of an increase in variance, it is equivalent to evaluate the effect of an increase in $\delta$.

A.2 Assessment Limits and Price Variance

As of 2006, 20 states have some form of assessment limit for property taxation. Although idiosyncratic to each state, most assessment limitations designed to limit increases in the taxable value of individual pieces of real estate. Since the policies are not explicitly designed to limit variance of tax prices it is important to understand how assessment limitations might affect the variance of individual tax prices.

Assessment limitations affect the expected value of an individual’s future tax price as well as its variance. If a policy lowers variance in tax price but increases its expected value voters will be more likely to oppose the assessment limitation. Of course, if a policy lowers variance in tax price and also decreases its expected value, voters will be more likely to support the assessment limitation.

In order to understand how assessment limitations affect the variance and expected value of individual tax prices and tax bills, it is necessary to return to detailed discussion of tax
share, the major component of tax price and the primary cause of changes to tax prices within a community over time.

Note again that the tax shares of all individuals must sum to one.

\[ \sum_{i=1}^{N} s_{ti} = 1 \]

Does this place any constraints on the distribution of changes to individual tax shares within a community? If each taxpayer’s change in value at time \( t \) is equal to \( \phi_{ti} \) it must be the case that

\[ \sum_{i=1}^{N} s_{ti} = 1 \quad \forall \ t \]

\[ \sum_{i=1}^{N} s_{ti} + \phi_{t'i} = 1 = \sum_{i=1}^{N} s_{t'i} \quad \forall \ t \neq t' \]

\[ 1 + \sum_{i=1}^{N} \phi_{t'i} = 1 \]

\[ \sum_{i=1}^{N} \phi_{t'i} = 0 \quad (54) \]

Redistributions in tax share are zero-sum. Note that this also implies that the mean share in all periods will equal the mean share in period \( t \), given a constant number of taxpayers, \( N \).

\[ \sum_{i=1}^{N} s_{ti} = 1 \quad \forall \ t \]

\[ \implies \sum_{i=1}^{N} s_{ti} = \sum_{i=1}^{N} s_{t'i} \]

\[ \frac{\sum_{i=1}^{N} s_{ti}}{N} = \frac{\sum_{i=1}^{N} s_{t'i}}{N} \]

\[ \bar{s}_t = \bar{s}_{t'} \quad \forall \ t, t' \quad (55) \]

\[ \bar{s}_t = \frac{1}{N_t} \quad \forall \ t \quad (56) \]

Assessment limitations involve lump-sum tradeoffs across individual taxpayers. Every decrease in one individual’s tax share must result in the increase of at least one other individual’s tax share.
A.2.1 Multiplicative Shocks to Value

Since most assessment limits constrain percentage increases in taxable value it is helpful to discuss how percentage shocks to value affect tax shares and thus tax price. In this section an effort is made to consistently adhere to the following notation: *Capital* letters are random variables, and *small* letters are known outcomes (i.e., small letters are not random variables).

**Assumption 1** Suppose the home value in the previous (known) period, \(v_{i(t-1)}\), is hit with a normal shock of the following form: 

\[ V_{it} = v_{i(t-1)}(1 + \epsilon) \]

where \(\epsilon\) is distributed \(N(\mu_\epsilon, \sigma^2)\).

This is an multiplicative shock to value. The assumption of normality does not appear to be important with regard to the derivation of the expected value and variance of tax share, but is important for their distributions.

**Definition 1** Any individual’s tax share at time \(t\) can be described by the random variable

\[ S_{it} = \frac{V_{it}}{\sum_{i=1}^{N} V_{it}} \]  

which is a function of many random variables, \(V_{it}\). If the shocks to individual value are normally distributed it is straightforward to describe the distribution of \(V_{it}\) and even \(\sum_i V_{it}\). Things become complicated, however, when discussion turns to the distribution of \(S_{it}\). Remember, the changes in \(S_{it}\) that are the result of the shocks to value, must preserve the mean of \(S_{it}\) (i.e., tax share) upon its realization.

Given the definition of \(V_{it}\), its expected value and variance are

\[
E[V_{it}] = E[v_{i(t-1)}(1 + \epsilon)] \\
E[V_{it}] = v_{i(t-1)} + v_{i(t-1)}E[\epsilon] \\
E[V_{it}] = v_{i(t-1)} + v_{i(t-1)} \cdot \mu_\epsilon. 
\]

\[
VAR(V_{it}) = VAR(v_{i(t-1)} + v_{i(t-1)} \cdot \epsilon) \\
VAR(V_{it}) = v_{i(t-1)}^2 \cdot VAR(\epsilon) \\
VAR(V_{it}) = v_{i(t-1)}^2 \cdot \sigma^2_\epsilon. 
\]

Using the assumption of the normally distributed errors, and the fact that 

\[
\epsilon \sim N(\mu_\epsilon, \sigma^2) \implies a\epsilon + b \sim N(a\mu_\epsilon + b, a^2\sigma^2_\epsilon)
\]

it is the case that

\[
V_{it} = v_{i(t-1)}\epsilon + v_{i(t-1)} \forall t, \\
\implies V_{it} \sim N(v_{i(t-1)} + v_{i(t-1)}\mu_\epsilon, (v_{i(t-1)})^2\sigma^2_\epsilon). 
\]
Definition 2 A random variable also describes tax base: \( B_t = \sum_{i=1}^{N} V_{it} \)

The mean and variance of the tax base, \( B_t \) are

\[
E(B_t) = \sum_{i=1}^{N} E(V_{it})
\]

\[
E(B_t) = \sum_{i=1}^{N} \left( v_i(t-1) + v_i(t-1) \cdot \mu \right)
\]

\[
E(B_t) = \sum_{i=1}^{N} v_i(t-1) + \mu \cdot \sum_{i=1}^{N} v_i(t-1)
\]

(63)

\[
E(B_t) = b_{t-1} + \mu b_{t-1}
\]

(64)

What is the variance of \( B_t \)? Since the errors are independent, each value \( V_{it} \) is independent.

\[
VAR(B_t) = Var \left( \sum_{i=1}^{N} V_{it} \right)
\]

\[
VAR(B_t) = \sum_{i=1}^{N} Var(V_{it})
\]

\[
VAR(B_t) = \sum_{i=1}^{N} (v_i(t-1))^2 \sigma^2
\]

(65)

Since it also the case that the sum of independent, identically distributed normal random variables is normal, the tax base in the future (time \( t \)) is distributed, normally as

\[
B_t \sim N \left( b_{t-1} + \mu b_{t-1}, \sum_{i=1}^{N} (v_i(t-1))^2 \sigma^2 \right).
\]

(66)

A.2.2 Dependence of \( V \) and \( B \)

At any point in time, \( t-1 \), when tax shares are known, the tax share of individual \( i \) is equal to

\[
s_{i(t-1)} = \frac{v_i(t-1)}{b_{t-1}}
\]

(67)

There is also the random variable of tax share yet to be revealed at some time \( t \)

\[
S_{it} = \frac{V_{it}}{B_t}
\]

(68)
Both $V_{it}$ and $B_{t}$ are normally distributed. Unfortunately, just because the expectation of both components of the tax share is defined does not imply that the expectation of their ratio is defined.\footnote{If these two normally distributed variables were independent their ratio would follow the Cauchy distribution which has no mean or variance.}

Are $V_{it}$ and $B_{t}$ independent given the assumptions? The answer appears to be no. First, remember that to calculate covariance it is often easier to express it in the following way

\[
COV(V_{it}, B_{t}) = E[(V_{it} - \mu_v)(B_{t} - \mu_b)] \\
= E[V_{it}B_{t} - \mu_vB_{t} - \mu_vB_{t} + \mu_v\mu_b] \\
= E[V_{it}B_{t}] - \mu_bE[V_{it}] - \mu_vE[B_{t}] + \mu_v\mu_b \\
= E(V_{it}B_{t}) - \mu_v\mu_b.
\]

Solving for the first term on the right-hand side

\[
E(V_{it}B_{t}) = E \left[ (v_{i(t-1)} + v_{i(t-1)}) \left( \sum_{j=1}^{N} v_{jt} \right) \right] \\
E(V_{it}B_{t}) = E \left[ (v_{i(t-1)} + v_{i(t-1)}) \left( \sum_{j=1}^{N} (v_{j(t-1)} + v_{j(t-1)}) \right) \right] \\
E(V_{it}B_{t}) = \sum_{j=1}^{N} (v_{i(t-1)} + v_{i(t-1)}) \left( v_{j(t-1)} + v_{j(t-1)} \right) \\
E(V_{it}B_{t}) = \sum_{j=1}^{N} (v_{i(t-1)}v_{j(t-1)}\epsilon^2 + 2v_{i(t-1)}v_{j(t-1)}\epsilon + v_{i(t-1)}v_{j(t-1)}) \\
E(V_{it}B_{t}) = \sum_{j=1}^{N} (v_{i(t-1)}v_{j(t-1)}\epsilon^2) + \sum_{j=1}^{N} (2v_{i(t-1)}v_{j(t-1)}\epsilon) + \sum_{j=1}^{N} (v_{i(t-1)}v_{j(t-1)}) \\
E(V_{it}B_{t}) = \sum_{j=1}^{N} (v_{i(t-1)}v_{j(t-1)}\epsilon^2) + E \left[ \sum_{j=1}^{N} (2v_{i(t-1)}v_{j(t-1)}\epsilon) \right] + E \left[ \sum_{j=1}^{N} (v_{i(t-1)}v_{j(t-1)}) \right] \\
E(V_{it}B_{t}) = v_{i(t-1)} \sum_{j=1}^{N} (v_{j(t-1)}E[\epsilon^2]) + 2v_{i(t-1)} \sum_{j=1}^{N} (v_{j(t-1)}E[\epsilon]) + v_{i(t-1)} \sum_{j=1}^{N} v_{j(t-1)} \\
E(V_{it}B_{t}) = E[\epsilon^2]v_{i(t-1)}b_{t-1} + 2E[\epsilon]v_{i(t-1)}b_{t-1} + v_{i(t-1)}b_{t-1} \quad (69)
\]
Reinserting the result for $E(V_i t B_t)$ into the equation for covariance produces

$$COV(V_{it}, B_t) = E[\epsilon^2]v_{i(t-1)}b_{t-1} + 2E[\epsilon]v_{i(t-1)}b_{t-1} + v_{i(t-1)}b_{t-1} - \mu_v \cdot \mu_b$$

$$COV(V_{it}, B_t) = (E[\epsilon^2] + 2E[\epsilon] + 1) (v_{i(t-1)}b_{t-1}) - (v_{i(t-1)} + v_{i(t-1)}\mu_e) (b_{t-1} + b_{t-1} \mu_e)$$

$$COV(V_{it}, B_t) = (v_{i(t-1)}b_{t-1}) (E[\epsilon^2] - [E(\epsilon)]^2)$$

$$COV(V_{it}, B_t) = (v_{i(t-1)}b_{t-1}) \sigma_\epsilon^2 > 0, \quad (70)$$

It is clear that these two normally distributed variables are not independent.

### A.2.3 Changes in Tax Share

When asking questions about the future, an individual might inquire as to the probability that tax share this period will be higher than tax share last period. Since last period is known, the only random variable is tax share this period. The simple formulation of the question asks

$$P \left( S_{it} > s_{i(t-1)} \right) \quad (71)$$

Unfortunately, the distribution of tax share is unknown, as the ratio of two normally distributed variables is not necessarily normal. From the definition of tax share, however, it is possible to derive conditions consistent with tax share increasing or decreasing from period to period. For example, if the individual’s value increases by a larger percentage than the entire base the individual’s tax share will increase. The probability that the tax share increases is

$$P \left( \% \Delta V_{it} - \% \Delta B_t > 0 \right) \quad (72)$$

To answer this question formally the distribution of $(\% \Delta V_{it} - \% \Delta B_t)$ must be known. Unfortunately, although these two random variables are normally distributed, they are not independent so we cannot know their distribution using the properties previously discussed.

The random variable $(\% \Delta V_{it} - \% \Delta B_t)$ is a good approximation of the percentage change in tax share. For instance, using the natural log of tax share

$$ln(S_{it}) = ln(V_{it}) - ln(B_t) \quad (73)$$

$$d \left[ ln(v_{i(t-1)}) - ln(b_{t-1}) \right] = \frac{1}{v_{i(t-1)}} dv_{i(t-1)} + \frac{1}{b_{t-1}} db_{t-1}$$

$$d \left[ ln(v_{i(t-1)}) - ln(b_{t-1}) \right] = \frac{dv_{i(t-1)}}{v_{i(t-1)}} + \frac{db_{t-1}}{b_{t-1}} \quad (74)$$

The percentage change in individual value is a random variable

$$\% \Delta V_{it} = \frac{V_{it} - v_{i(t-1)}}{v_{i(t-1)}}$$

$$\% \Delta V_{it} = \frac{V_{it}}{v_{i(t-1)}} - 1. \quad (75)$$
The expected value of $\%\Delta V_{it}$

\[
E[\%\Delta V_{it}] = \frac{1}{v_{i(t-1)}} E[V_{it}] - 1
\]

\[
E[\%\Delta V_{it}] = \frac{1}{v_{i(t-1)}} \cdot (v_{i(t-1)} + v_{i(t-1)}\mu_{\epsilon}) - 1
\]

\[
E[\%\Delta V_{it}] = 1 + \mu_{\epsilon} - 1 = \mu_{\epsilon}
\] (76)

The variance of $\%\Delta V_{it}$

\[
VAR(\%\Delta V_{it}) = VAR\left[\frac{V_{it}}{v_{i(t-1)}}\right]
\]

\[
VAR(\%\Delta V_{it}) = \frac{1}{v_{i(t-1)}^2} VAR(V_{it})
\]

\[
VAR(\%\Delta V_{it}) = \frac{1}{v_{i(t-1)}^2} \cdot (v_{i(t-1)}^2\sigma_{\epsilon}^2)
\]

\[
VAR(\%\Delta V_{it}) = \sigma_{\epsilon}^2.
\] (77)

Since $V_{it}$ is normally distributed

\[
\%\Delta V_{it} \sim N(\mu_{\epsilon}, \sigma_{\epsilon}^2)
\] (78)

The distribution of the percentage change in $V_{it}$ is the same as the distribution for $\epsilon$. This is because $\epsilon$ was defined as the percentage change in value.

The expected value and variance of $\%\Delta B_t$.

\[
\%\Delta B_t = \frac{B_t - b_{t-1}}{b_{t-1}}
\]

\[
\%\Delta B_t = \frac{B_t}{b_{t-1}} - 1
\] (79)

\[
E[\%\Delta B_t] = \frac{1}{b_{t-1}} E[B_t] - 1
\]

\[
E[\%\Delta B_t] = \frac{1}{b_{t-1}} \cdot (b_{t-1} + b_{t-1} \cdot \mu_{\epsilon}) - 1
\]

\[
E[\%\Delta B_t] = 1 + \mu_{\epsilon} - 1 = \mu_{\epsilon}
\] (80)
The variance

\[ \text{VAR} [\% \Delta B_t] = \frac{1}{b_{t-1}^2} \text{Var}(B_t) \]

\[ \text{VAR} [\% \Delta B_t] = \frac{1}{b_{t-1}^2} \cdot \sum_{i=1}^{N} (v_{i(t-1)})^2 \sigma_{\epsilon}^2 \]

\[ \text{VAR} [\% \Delta B_t] = \frac{\sum_{i=1}^{N} (v_{i(t-1)})^2}{\left( \sum_{i=1}^{N} (v_{i(t-1)}) \right)^2} \cdot \sigma_{\epsilon}^2 \]

\[ \text{VAR} [\% \Delta B_t] = \lambda_b \cdot \sigma_{\epsilon}^2 \]  

where

\[ \lambda_b = \frac{\sum_{i=1}^{N} (v_{i(t-1)})^2}{\left( \sum_{i=1}^{N} (v_{i(t-1)}) \right)^2} < 1. \]  

Given the assumption that \( B_t \) is normally distributed,

\[ \% \Delta B_t \sim N(\mu_{\epsilon}, \lambda_b \cdot \sigma_{\epsilon}^2). \]  

The distribution of \( \% \Delta V_t \) has the same mean, but a larger variance than the distribution of \( \% \Delta B_t \).\(^{35}\) This makes some intuitive sense as the errors in the base have a chance to mute each other and dampen the total percentage change in tax base, \( B_t \).

Fortunately, it’s possible to solve for the expected value and variance of the random variable \( \% \Delta V_t - \% \Delta B_t \).

\[ E(\% \Delta V_t - \% \Delta B_t) = E \left[ \frac{V_t}{v_{i(t-1)}} - \frac{B_t}{b_{t-1}} \right] \]

\[ E(\% \Delta V_t - \% \Delta B_t) = E \left[ \frac{V_t}{v_{i(t-1)}} \right] - E \left[ \frac{B_t}{b_{t-1}} \right] \]

\[ E(\% \Delta V_t - \% \Delta B_t) = \frac{1}{v_{i(t-1)}} E[V_t] - \frac{1}{b_{t-1}} E[B_t] \]

\[ E(\% \Delta V_t - \% \Delta B_t) = \frac{1}{v_{i(t-1)}} (v_{i(t-1)} + v_{i(t-1)}\mu_{\epsilon}) - \frac{1}{b_{t-1}} (b_{t-1} + b_{t-1}\mu_{\epsilon}) \]

\[ E(\% \Delta V_t - \% \Delta B_t) = 1 + \mu_{\epsilon} - 1 - \mu_{\epsilon} \]

\[ E(\% \Delta V_t - \% \Delta B_t) = 0 \]  

Now, the variance can be calculated by remembering

\[ \text{VAR}(aX + bY) = a^2 \text{VAR}(X) + b^2 \text{VAR}(Y) + 2 \cdot a \cdot b \cdot \text{COV}(X, Y) \]  

\(^{35}\)To see this, note that \( \sum x_i^2 < (\sum x_i)^2 \) when \( x_i > 0 \ \forall \ i \). For an example, try it with \( i = 2, x_1^2 + x_2^2 < x_1^2 + x_2^2 + 2x_1x_2. \)
Some algebra demonstrates

\[
\text{VAR} (\% \Delta V_t - \% \Delta B_t) = \text{Var} \left[ \frac{V_t}{v_i(t-1)} - \frac{B_t}{b_{t-1}} \right]
\]

\[
\text{VAR} (\% \Delta V_t - \% \Delta B_t) = \frac{1}{v_i(t-1)} \text{Var}(V_t) + \frac{1}{b_{t-1}^2} \text{Var}(B_t) + 2 \left( \frac{1}{v_i(t-1)} \right) \left( \frac{1}{b_{t-1}} \right) \text{Cov}(V_t, B_t)
\]

\[
\text{VAR} (\% \Delta V_t - \% \Delta B_t) = \frac{1}{v_i(t-1)} \left( v_i(t-1)^2 \sigma^2_\epsilon \right) + \frac{1}{b_{t-1}^2} \sum_{i=1}^{N} v_i(t-1)^2 \sigma^2_\epsilon + \frac{2}{v_i(t-1)b_{t-1}} \text{Cov}(V_t, B_t)
\]

\[
\text{VAR} (\% \Delta V_t - \% \Delta B_t) = \sigma^2_\epsilon + \sigma^2_\epsilon \lambda_b + \frac{2}{v_i(t-1)b_{t-1}} (v_i(t-1)b_{t-1}\sigma^2_\epsilon)
\]

\[
\text{VAR} (\% \Delta V_t - \% \Delta B_t) = \sigma^2_\epsilon + 2 \sigma^2_\epsilon \lambda_b + 2 \sigma^2_\epsilon
\]

\[
\text{VAR} (\% \Delta V_t - \% \Delta B_t) = \sigma^2_\epsilon (3 + \lambda_b) < 4 \sigma^2_\epsilon.
\]  

(86)

The variance of changes in individual tax share is much larger than the variance of changes in value or changes in base. Thus, relatively small changes in values can create a large variance in tax shares and thus large variance in tax prices. It also stands to reason that limiting the variance of shocks to an individual’s value, \( \sigma^2_\epsilon \) will serve to limit the variance of changes to an individual’s tax share. When individuals desire more stable prices they will prefer a policy that limits variance to a policy that does not limit variance.

### A.2.4 Simulating the Effects of Assessment Limits

In order to understand how assessment limits limit variance and affect the mean of individual tax shares it is helpful to simulate the effects of assessment limits under a different assumptions about the nature of shocks to value. We ran simulations based on the assessed values of residential properties in Oak Park, IL from the year 2000. The values in Oak Park were initial values for housing. These initial values were then subjected to shocks to value over 10 periods (e.g., years). In each period new tax shares were calculated for each home. Each simulation was run 10 times and the results discussed here are the average results from the 10 simulations.

Figures A1 – A3 display results assuming that shock in each period is randomly drawn from a normal distribution with mean zero and variance 0.04. Each year’s shock is independent of the previously year’s shock and shocks do not depend on previous value of the home. There is no reason to believe that this is an accurate portrayal of housing price changes within a community. In fact, we suspect there are reasons to believe that this is not an accurate portrayal of price changes. We hope, however, that these simulations provide an example of how assessment limits might affect tax shares over time. These simulations were also run assuming independent uniformly distributed shocks across homes with similar results.

\footnote{We’ve also examined simulations using an AR(1) process to generate the percentages changes to each home value. This has produces slightly different results than those discussed above. The assessment limits}
Under these very strong assumptions, Figure A1 demonstrates that over 90% of the homes in the simulation experience a lower average percentage change in tax price across the 10 periods with limits than without limits. Thus, for most taxpayers the limits would reduce the average magnitude of tax price changes. This figure also demonstrates that the variance of percentage changes in prices across all the periods was reduced for over half of the homes. Figure A2 demonstrates that within each period, the standard deviation of price changes across individuals is lower under limits than without limits. Figure A3 demonstrates that within each period the average percentage change in individual tax price is lower under limits than without limits.

There is limited empirical evidence on the affects of assessment limitations. Furthermore, most of the existing research focuses on so-called ”tax shifts” rather than any changes in the variance of either tax shares or tax payments over time. The clearest implication of the evidence is that substantial tax-shifting does occur between classes of property. There is not much research and therefore less evidence regarding the extent of within-class shifting of statutory tax burdens. The next step is clearly to examine changes in individual tax shares over time in jurisdictions where limits exist. These changes under limits could in principal be compared to some estimation of the tax shares that would have existed if not for the limits.

A.3 Theoretical Aside: Radner Equilibrium and Limiting Price Variance

The concept of Radner Equilibrium describes limits on assessment extremely well. As discussed above, assessment limits redistribute the statutory property tax burden. This process is perfectly zero-sum as one individual’s tax share decrease must be followed by an increase in the tax share of at least one individual. The extent of redistribution depends on the outcome of property values in each period. This resembles Radner equilibrium where individuals make promises in period 1 to redistribute income in period 2. The exact nature of distribution depends on the state of the world realized in period 2 but the redistributions are by definition zero sum in nature.

To see this, consider a simple example of a Radner equilibrium. Assume that \( R \) (government revenue) is fixed across periods or, in the terminology used above, that neither of the consumers considered below are decisive in revenue.

Consider a world with two consumers at time 0. These consumers have preferences over government revenues and consumption. In the simplest case two different states can occur at time 1 (\( t = 1 \)). These two different states reflect the effects of changing tax shares on voters. The assumption of fixed revenues highlights the effects of changing tax shares, but also neglects the potential ability of these consumers to vote for less revenue. In state A, consumer 1 has a tax share of \( h \) and consumer 2 has a tax share of \( l \), where \( h > l \). In state B, consumer 1 has a tax share of \( l \) and consumer two a tax share of \( h \). Suppose that there only effective in reducing the mean and variance for roughly half the population and the average variance in each period is not significantly affected.
are two assets, each of which pay out one unit of consumption in state A or state B. Asset a pays out one unit of consumption in state A and asset b one unit of consumption in state B. There is equal probability of either state A or state B occurring.

Two people 1,2

Two states A,B

State A: \( p_1 = h, p_2 = l \), where \( h > l \)

State B: the reverse

Two assets: Asset a: pays \((1, 0)\), that is, one unit in state A, zero in B. It can be bought or sold at period 0.

Asset b: pays \((0,1)\).

Then the problem is, for each \( i = 1, 2 \), where \( M_i \) is the money belonging to person \( i \):

\[
\max .5u_i(c_i^A, R) + .5u_i(c_i^B, R) \quad (87)
\]

\( M_i - p_i^A R + a_i = c_i^A \) (state A budget constraint, associated with lagrange multiplier \( \lambda_a \))(88)

\( M_i - p_i^B R + b_i = c_i^B \) (state B budget constraint, associated with \( \lambda_b \))(89)

\( q_a a_i + b_i = 0 \) (period 0 budget constraint, associated with \( \gamma \))(90)

where \( q_a \) is the price of asset a at period 1, and the price of asset b at period 0 is normalized to one, as is the price of consumption.

A.3.1 Log Utility

If utility is log (implying risk-averse consumers), i.e. \( u_i(c, R) = \ln(c) + \ln(R) \), then, using the budget constraints for states A and B, we can show that the indirect utility function for consumer \( i \) is

\[
V(p, M_i) = .5\ln(M_i + a_i - p_i^A R) + .5\ln(M_i + b_i - p_i^B R) \quad (91)
\]

This indirect utility function is maximized with respect to the individual consumer’s asset budget constraint at period 0 (when they have no money)

\( q_a a_i + b_i = 0. \quad (92) \)

Plugging in the constraint (substituting for \( b_i \)) into the indirect utility function we maximize the following function, choosing \( a_i \), without constraint

\[
.5\ln(M_i + a_i - p_i^A R) + .5\ln(M_i - q_a a_i - p_i^B R) \quad (93)
\]
The first order conditions imply that
\[
\frac{.5}{M_i + a_i - p_i^A R} = \frac{.5q_a}{M_i - q_a a_i - p_i^B R} \quad (94)
\]
\[
a_i = \frac{M_i}{2} \left( \frac{1}{q_a} - 1 \right) + \frac{R}{2} \left( p_i^A - p_i^B \frac{q_a}{q_a} \right) \quad (95)
\]
The asset budget constraint \((q_a a_i + b_i = 0)\) implies that
\[
b_i = \frac{M_i}{2} (q_a - 1) + \frac{R}{2} (p_i^B - p_i^A q_a) \quad (96)
\]
Now all the asset demands equation are a functions of money and prices. The asset market clearing assumption \((a_1 + a_2 = 0; b_1 + b_2 = 0)\) solves for equilibrium asset prices \((q_a)\). Only the relative prices, \(q_a\) will be determined. Using \(a_1 + a_2 = 0\) and substituting in our solutions for \(a_i\), produces
\[
\frac{M_1}{2} \left( \frac{1}{q_a} - 1 \right) + \frac{R}{2} \left( p_i^A - p_i^B \frac{q_a}{q_a} \right) = \frac{M_2}{2} \left( \frac{1}{q_a} - 1 \right) + \frac{R}{2} \left( p_i^A - p_i^B \frac{q_a}{q_a} \right) \quad (97)
\]
\[
q_a = \frac{R(p_i^B + p_i^B) - M_1 - M_2}{R(p_i^A + p_i^B) - M_1 - M_2} \quad (98)
\]
Given our assumptions about \(p_i^S S = A, B\), we can say that \(q_a = 1\). Thus relative prices are equal to one, both assets have the same price. We can, without loss of generality, set these prices both equal to one.

A.3.2 Asset Demand

What is the demand for consumption and the demand for the assets, given our assumptions about price? Can we say anything more general or more specific to our issue of tax price volatility?

The demand for assets equals
\[
a_1 = R \cdot \frac{(h - l)}{2} \quad (99)
\]
\[
b_1 = R \cdot \frac{(l - h)}{2} \quad (100)
\]
\[
a_2 = R \cdot \frac{(l - h)}{2} \quad (101)
\]
\[
b_2 = R \cdot \frac{(h - l)}{2} \quad (102)
\]
These equations satisfy the asset budget constraint in period 0 since \(q_a a_i + b_i = 0\). Next we can solve for consumption by both consumers in both states
\[
c_i^S = M_i - \frac{(h + l)}{2} \cdot R \quad \text{for} \quad i = 1, 2; \quad S = A, B \quad (103)
\]

34
Consumers smooth out consumption and also tax payments over time. In a world where tax shares change individuals will demand assets to smooth out their tax payments over time. Although assessment limits do not directly address changes in prices in the same way as these assets, these results suggest that it would be optimal to insure against price risk when $R$ is not under a taxpayer’s control.
Figure 1: Distribution of Tax Shares in Chicago Suburbs (2000)

Source: Author’s tabulations using data from Cook County Assessor’s Office.
Kernel: Epanechnikov, kernel half−width=0.005.
All properties within each suburb are assessed at the same time but assessment occurs at different times in different suburbs.
Tax Shares reflect an individual residential property’s share of each $1,000 in property taxes remitted by owners of residential property.
Using Illinois state−wide averages for property tax revenue, a 0.01 difference in tax share per $1,000 results in an approximate property tax liability difference of over $100.
Annual shocks to individual real estate value are assumed to be normally distributed with mean 0 and variance 0.04. In period 0, all real estate values are from Oak Park, IL in 2000. The shocks begin in period 1 and are independent across time and over pieces of real estate.
Annual percentage shocks to individual real estate value are assumed to be normally distributed with mean 0 and variance 0.04. In period 0, property values are the values in Oak Park, IL in 2000. The shocks begin in period 1 and are independent across time and over individual pieces of real estate.
Annual percentage shocks to individual real estate value are assumed to be normally distributed with mean 0 and variance 0.04. In period 0, the property values are the property values in Oak Park, IL in 2000. The shocks begin in period 1 and are independent across time and over individual pieces of real estate.