# An Insurance Model of Property Tax Limitations

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**Abstract**

Forty-three states in the United States have legal limits on local property taxation. These limits constrain increases in local government revenues, increases in property values, and the level of tax rates. A large literature cites the desire of taxpaying voters to constrain wasteful local government expenditures as the primary motivation for these limitations. The ability of tax limitations to insure voters against unexpected increases in property tax payments, however, offers an alternative motivation. Introducing risk into standard models of local service demand demonstrates that voters may support tax limitations in the absence of any perception of wasteful government expenditures.

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# 1 Introduction

> The tax which each individual is bound to pay ought to be certain, and not arbitrary. The time of payment, the manner of payment, the quantity to be paid, ought all to be clear and plain to the contributor, and to every other person. [...] The certainty of what each individual out to pay is, in taxation, a matter of so great importance, that a very considerable degree of inequality, it appears, I believe, from the experience of all nations, is not near so great an evil as a very small degree of uncertainty.
>
> Adam Smith
> *The Wealth of Nations (Book V)*

No other tax in the United States exemplifies the tension between Smith's maxims of equality and uncertainty more than the property tax. This paper explores the relationship between the equality and uncertainty of the property tax and the widespread limitations placed on aspects of property taxation in the United States. In terms of revenues collected, the local property tax is the second largest tax in the United States with over $286 billion collected by local governments in 2002; representing approximately $80 billion more in revenues than federal corporate income tax; $10 billion more than state sales taxes; and $70 billion more than states collected in individual and corporate income taxes combined.[1] Despite of or because of its size, the ability of local governments to collect property tax revenue is often constrained and the tax itself is extremely unpopular, especially when compared to vacations in the south of France, but even when compared to other taxes.[2]

As of 2006, only 5 of the 48 states in the continental United States have absolutely no explicit limits on some aspect of property taxation.[3] The remaining 43 states each have at least one of the three most common limitations in place. The most common types of limitations, revenue and tax rate limits, explicitly constrain local government behavior. The third type, limitations on increases in the taxable value of property, is less common and does not explicitly constrain local government behavior.[4] What these limits have in common is their potential to reduce variation in individual property tax payments. Previous research

[1]Although the distribution of revenues varies greatly across states, more than 40% of local property tax revenues collected in 2002 financed school districts; 20% to municipalities and townships, 24% to counties, with the remaining funds allocated to special districts. Sources: Internal Revenue Service and US Census Bureau.

[2]The property tax is the most unpopular state or local tax and rivals only the federal income tax in terms of its unpopularity. In the most recent annual survey by the Tax Foundation, 38% of respondents listed the property tax as the worst tax state and local tax. In a virtual tie for a distant second place, income and sales taxes received 20% and 19% of the vote for worst state and local tax. This is not a recent phenomenon. A 1974 survey by the Advisory Committee on Intergovernmental Relations had the property tax virtually tied with the federal income tax as the worst tax.

[3]The five states are ME, NH, TN, VT, VA. Utah might also be considered as a state without any stringent limitations on property taxation. Of the five states with none of three major limits on property taxes not one has a statewide mandatory annual assessment system. Maine recently passed tax reforms that included "circuit-breakers" designed to keep property tax burdens below a certain percentage of income.

[4]For a more detailed summary of these limits see Anderson (2006b).

on the motivations for property tax limitations has focused on constraint of possibly wasteful government expenditures.[5]

This paper investigates the effect that property tax limitations have on the volatility of individual property tax payments. When individual tax payments are volatile, voters will support property tax limitation measures within their own jurisdictions and in state-wide referenda even when government expenditures are not wasteful.

The idea of property tax limitation as insurance is motivated by a simple example. Suppose there is a jurisdiction with only two homes, each valued at $200. If the local government requires $50 of revenue, each taxpayer will pay $25 under a pure current value assessment system. Next year, the government again needs $50 but one of the homes has gone up in value to $400 (the other is still valued at $200). Under this scenario, with government revenue still at $50, the first taxpayer will pay about $33 while the other will pay about $17. With no increase in government revenue each taxpayer has experienced a 32% change in their tax payment. The increase in tax payments faced by the appreciating home would be higher still if government revenue had not remained constant. Perhaps the owner of the non-appreciating home believes that there is a non-trivial risk that she might experience such a tax increase in the future. She certainly would be willing to pay some amount to ensure that such a thing would not happen to her in the future. She might support limiting revenue increases to guarantee that increases in her tax payments would not be too large or she might support assessment limitations that restrict increases in the value of her home. These limitations may be supported even if she believes that her government is benevolent and that the entire $50 is extremely well-spent.

There is little to no prior economics research on the volatility and uncertainty of individual property tax payments.[6] Fortunately there does exist some suggestive evidence that provides an idea about the magnitude of volatility, if not the degree of uncertainty. For example, from 2002 to 2006, total property taxes paid by owners of residential property in Minnesota increased by an average of 58% across all the counties in state. This increase was not caused by increased property tax revenues; taxing jurisdictions in Minnesota only collected 14% more property taxes in 2006 than they did in 2002. The majority of the increase was caused by large increases in the value of residential property that far outpaced increases in the value of other types of property, such as commercial and industrial real estate.[7] The large residential property tax increases in Minnesota represent increases in the mean tax payment for residential properties. Although insightful, these data do not inform as to changes in the distribution of tax payments within the residential property class that might produce even more volatility an uncertainty.

A recent study by Dye, McMillen, and Merriman (2006) provides estimates of within-residential class volatility in individual property tax bills. They estimate that for 2003 individual property tax bills for residential owners in Chicago increased by an average of 50% in some areas of Chicago and decreased by as much as 10% in other areas of Chicago. The average estimated annual increase in tax bills across some neighborhoods was larger than $400 for each year from 2003 to 2005. Many neighborhoods in Chicago had average

[5]Wasteful usually implies bureaucrats motivated by a goal to maximize their budgets with little to no concern regarding the provision of services to voters.

[6]See Allen (2006) for an excellent study of individual property tax burdens in Maine at one point in time.

[7]Source: Minneapolis Star Tribune, October 15, 2006.

annual decreases in tax bills of over $100 from 2003 to 2005. Even within the same neighborhood, some individuals experience large increases in property taxes while others experience decreases. Perhaps most surprisingly, these estimates hold tax revenue constant, implying that these changes in tax bills are derived solely from fluctuations in property valuations.[8]

The idea that tax payment volatility and uncertainty motivate tax limitations is in stark contrast with the conventional analysis that in the absence of wasteful expenditures, no rational voter would support tax limitations within their own jurisdiction. Vigdor (2004) provides a detailed review of the most prominent explanations for the existence of statewide limitations on local governments.[9] In the Tiebout (1956) model voters in an individual jurisdiction would never submit to limitations on their local government revenues and rates so long as residents have the power to dictate revenue and rate policy according to their preferences. The lack of motivation for limits when residents have control over policy suggests that perhaps residents have limited control over policy. This idea that residents have limited control over local policies on revenues and tax rates is often called the "Leviathan" model or agency loss theory of local government. In these models, principal-agent problems, for example costly and imperfect monitoring of elected officials, foce local government property tax revenue collection to be excessive and inefficient. Expenditure and tax rate limitations imposed on wasteful local governments are then justified because they can increase the efficiency of local governments.[10]

Another important theory is that school finance reforms may have contributed to property tax limitations. As school finance was made largely a state government as opposed to local government responsibility, many of the attractive attributes of the property tax were no longer applicable. This is a compelling argument, and is outlined in Fischel (1989).

Vigdor (2004) proposes an alternative to these theories, noting that limitations on local government allow voters to influence local tax and expenditure decisions in jurisdictions where they do not reside. The motivation to influence the policies of other local governments exists even when the local voters control presumably non-wasteful tax and expenditure policy within their own jurisdictions. Local voter control over their own jurisdiction's policy requires that these limitations be implemented by the state legislature. Non-residents wish to limit local government behavior in order to make other communities more attractive to them in their roles as absentee landlords, wage earners, and possibly future residents. The non-resident explanation of limitations, unlike the Leviathan theory, can lead to inefficient local government when limits prevent local residents from selecting their optimal tax policy.

The model in this paper, however, produces within-jurisdiction voter support for limitations in the absence of government waste. The volatility of individual property tax payments is directly related to the tension between Smith's maxims of equality and uncertainty. Smith's

[8]These numbers reflect the annual changes in property tax liability that would have occurred if Chicago was not covered by Cook County's cap on increased in property valuations. Thus, they are lower bounds on tax increases and decreases since the cap on valuation increases did not eliminate all increases, implying a certain level of redistribution even under the assessment cap. The estimates are based on projected increases in property valuations. Please see their paper for details.

[9]See also Bradbury, Mayer, and Case (2001), Cutler, Elmendorf, and Zeckhauser (1999), and McGuire (1999).

[10]Another theory, the "state regime shift" theory does not require leviathan governments. Instead, the state regime shift theory focuses on voters' desire to have the state finance a larger portion of necessary revenues with income and sales taxes, thus reducing reliance on local property taxes.

equality maxim refers to the vertical and horizontal equity of the property tax system. That is, property owners with higher valued real estate should pay more in tax and taxpayers with equally valued properties should pay the same amount in taxes.[11] Smith, however, recognizes that strict maintenance of equality in a property tax system requires frequent updates to property valuations and these updates have their own costs.

> A land-tax assessed according to a general survey and valuation, how equal soever it may be at first, must, in the course of a very moderate period of time, become unequal. To prevent its becoming so would require the continual and painful attention of government to all the variations in the state and produce of every different farm in the country. The governments of Prussia, of Bohemia, of Sardinia, and the dutchy of Milan, actually exert an attention of this kind; an attention so unsuitable to the nature of government, that it is not likely to be of long continuance, and which, if it is continued, will probably in the long-run occasion much more trouble and vexation than it can possible bring relief to its contributors. (Book V, Chapter 2, Part I, article 1, pg 361)

Completely accurate and continually updated property values are required to maintain equality. The cost of this maintenance is more volatile, and possibly uncertain, individual property tax payments.[12] In order to be uncertain, individual property tax liability must vary over time. Since the property tax is, for the most part, a single-rate ad valorem tax, individual property tax liability is equal to the product of the tax rate and the individual's tax base (i.e., the taxable value of their real estate holdings). In the simplest property tax system only changes in tax rates and an individual's property value cause changes in individual property tax payments. Thus even with completely accurate valuations of property, an owner's uncertainty about their property's value (or future values) can lead to uncertainty with regard to the amount of her tax payment. Of course this uncertainty can be exacerbated by inaccurate valuations.[13]

Changes in the property tax rate is the other source of variation, and possible uncertainty, in an individual's tax payment. Property tax rates with a single taxing jurisdiction exhibit far more variation over time than income tax or sales tax rates. While income tax and sales tax rates rarely change from year-to-year, property tax rates are constantly changing.[14] The reason is that the local property is most often a "residual" tax, that is, as Bogart and Bradford (1990) explain, the property tax revenue each year is determined by the shortfall between desired expenditures and all other revenue sources (e.g., intergovernmental grants

[11] Smith describes equality in taxation as the condition where taxpayers pay taxes in proportion to their respective abilities; abilities that are measured by the revenue each taxpayer enjoys under the protection of the state.

[12] The difficulties of accurately updating property values to maintain equality in property taxation are by now well-understood and most would argue that substantial progress has been made in terms of the accuracy of these valuations. In the United States, appraisers of property are constantly evaluated as to the equality (i.e., uniformity) of their assessments.

[13] Unless assessors follow the age-old maxim: To find a value good and true, Here are three things for you to do: Consider your replacement cost, Determine value that is lost, Analyze your sales to see what market value really should be. Now if these suggestions are not clear - Copy the figures you used last year!

[14] For example, Anderson (2006a) shows that from 1995 to 1996, none of the 919 cities and townships in MN with population over 500 maintained the same tax rate.

and parking fees). The fact that the tax base for a local property tax is known when expenditure levels are set allows for the property tax to be a "residual tax." This is perhaps why the policy variable for the local property tax is most often a selection of desired revenue level as opposed to the selection of a tax rate.[15]

The ex ante knowledge of total tax base allows for accurate predictions of actual ex post (i.e., collected) revenues. This ex ante tax base certainty provides local governments with the ability to set tax rates that, given the known tax base, imply a desired revenue level. With income and sales taxes, the tax base is not known with accuracy in advance and the setting of specific revenue levels is more problematic.[16] Federal and state governments set tax rates and do not annually alter them in order to collect a desired amount of revenue.

Thus, unlike the federal income tax or state and local sales taxes, local property tax rates can and, most importantly, do vary from year-to-year. A property owner's uncertainty about future tax rates thus depends on her expectation of future revenue requirements and the future values of all real estate in the jurisdiction (i.e., total tax base). This uncertainty about future tax rates ultimately creates uncertainty regarding property tax liability.

This concern over volatility and uncertainty leads Smith to offer the following stark choice with regard to property tax institutions

> A tax upon the rent of land may either be imposed according to a certain canon, every district being valued at a certain rent, which valuation is not afterwards to be altered; or it may be imposed in such a manner as to vary with every variation in the real rent of the land, and to rise or fall with the improvement or declension of its cultivation. (page 352, Book V, Chapter 2, Part I, article I)

When valuations are "not afterwards to be altered" uncertainty in individual property tax payments is only caused by uncertainty about future tax rates. In contrast, when valuations "vary with every variation in the real rent of the land," an individual's uncertainty about her tax payment may be caused by uncertainty about revenues, the value of her property, and the value of every property within her jurisdiction. While not necessarily tripling the amount of uncertainty, maintaining equality definitely triples the sources of uncertainty. Of course, valuations that are "not afterwards to be altered" have the potential to reduce equality.

An unbridled maintenance of equality, often called uniformity, of the property tax system produces changes in tax liability. If these changes in property tax liability are large enough and somewhat unpredictable, some form of property tax limitation will be desirable even in

[15]The legality of whether rates or revenues are selected (although each one implies the other) is determined in each state's constitution. We know of no studies that examine how differences in the legal policy variable might affect changes in revenues or rates.

[16]In this sense, *in any given tax year* the property tax resembles a lump-sum tax in that the amount to be collected from each taxpayer is set in advance. The individual can do nothing in this year to reduce their property tax burden. They could certainly move into a smaller house or to a different community, but this will only affect next year's tax bill not this year's tax bill. This is not the case with income and sales taxes where individuals can adjust their own current tax base in response to the current tax rate. It is possible that the unpopularity of the property tax is related to the unpopularity of lump-sum taxation. A recent example of the unpopularity of lump sum taxation is provided by Margaret Thatcher's famous poll tax of 1990. The unpopularity of the poll tax led to Thatcher's resignation and the eventual repeal of the poll tax.

the absence of any motivation to constrain excessive or inefficient government expenditures.[17] All three of the most popular types of property tax limitations can provide insurance against undesirable changes in property tax liability. Each of the three primary limits can constrain changes in individual property tax payments. A demand for more predictable property tax payments explains why some form of property tax limitation might exist without the existence or perception of wasteful or excessive government expenditures.

# 2 Background: The Mechanics of Property Taxes

## 2.1 Tax Shares and Tax Prices

As mentioned above, individual tax payments will only vary over time only if tax rates vary or individual valuations vary. Although reasonably straightforward, the discussion of tax rates obscures the relationship between government expenditures and individual valuations. In particular, a focus on tax rates obscures the relationship between an individual's tax payment and the distribution of individual valuations. The relationship between the distribution of valuations and the volatility of individual tax payments is an essential component of understanding why individuals would wish to place limits on property taxation.

The tax share is defined as individual $i$'s share of tax payments. We can denote each individual's tax share at time $t$ as $s_{ti}$ and by definition the following equality must hold

$$\sum_{i=1}^{N} s_{ti} = 1. \tag{1}$$

The tax price is defined as the expenditure required by the individual to provide another unit of public service to the community (e.g., to each individual in the community). This definition of tax price requires the inclusion of the costs of service provision which can vary across services and across communities. If individuals consider other, usually more indirect costs of taxation, such as the distortion of location decisions by individuals(i.e., migration) and the distortion of investment decisions by individuals, these indirect cost must also be incorporated into tax price.[18]

For any public service, an individual's tax price is the product of the cost of that service and her tax share. For example, when the marginal cost of providing another unit of a public service to all residents is equal to $1, the tax price for individual $i$ of providing another unit of the public service to all residents is equivalent to the tax share of individual $i$. This is clearly a special case, as an individual's tax price does not always equal that individual's tax share.

[17] This idea is related to the personal finance theory of limit motivations cited by Cutler, Elmendorf, and Zeckhauser (1999). This theory states that voters interpret tax bill increases as a signal of government waste, especially when they observe no subsequent increases in service quality. As mentioned above and as will be made even more clear below, these tax increases can occur even without any increase in government revenues. Thus, increases in individual tax bills can be a very poor signal of increases in government expenditure. The research on personal finance theory, however, focuses on the level of property taxes, while this paper focuses on the variance of individual taxes.

[18] See Wildasin (1989) and Crane (1990) for a discussion of these indirect costs and their inclusion in tax price.

An individual's budget constraint reflects the relationship between disposable income, $y$, and expenditures on a consumption good, $c$, whose price is normalized to one, and expenditures on local taxes,

$$y_i = c_i + t_i \tag{2}$$

where $t_i$ is taxes paid to the government to fund public expenditures, $E$.

A local government's budget constraint reflects the relationship between expenditures and revenues

$$E = A + \frac{t_i}{s_i} = A + T \tag{3}$$

where $E$ is total expenditure, $A$ is total lump-sum aid and $s_i$ is the share of total taxes paid by voter $i$, and $T$ is total tax revenue received by the government. The government's budget constraint reflects the following relationship between tax revenues and individual tax shares,

$$\begin{aligned} T &= \sum_i^n t_i \\ T &= \frac{t_i}{s_i}. \end{aligned} \tag{4}$$

Under a single-rate ad valorem property tax, $s_i$ equals the ratio of the taxable value of an individual's home, $v_i$, to the total taxable value in their taxing district, $\sum_i v_i$.

$$s_i = \frac{t_i}{T} = \frac{\tau \cdot v_i}{\tau \cdot \sum_i v_i} = \frac{v_i}{\sum v_i} \tag{5}$$

which follows from the following equalities describing tax revenue

$$T = \tau \cdot \sum_i v_i \tag{6}$$

and

$$t_i = \tau \cdot v_i \;\; \forall\, i. \tag{7}$$

It is often easier to express the government budget constraint in terms of per-resident expenditure as per-resident expenditures are more readily comparable across jurisdictions. Per-resident ($n$ = number of residents) expenditure can be be defined as

$$e = \frac{E}{n} = a + \bar{t} \tag{8}$$

where $a = \frac{A}{n}$ and $\bar{t} = \frac{T}{n}$.

After expressing $\bar{t}$ in terms of the taxes paid by individual $i$, $t_i$,

$$\frac{T}{n} = \frac{\tau \cdot \sum_i v_i}{n} = \left(\frac{t_i}{v_i}\right)\left(\frac{\sum v_i}{n}\right) \tag{9}$$

$$\bar{t} = t_i \cdot \frac{\sum v_r}{n} \cdot \frac{1}{v_i} \cdot \frac{\sum v_i}{\sum v_r} \tag{10}$$

$$\bar{t} = t_i \cdot \left[\frac{\bar{v}_r}{v_i} \cdot \frac{\sum v_i}{\sum v_r}\right] \tag{11}$$

it is now possible to rewrite the government budget constraint as[19]

$$e = a + t_i \cdot \left[ \frac{\bar{v}_r}{v_i} \cdot \frac{\sum v_i}{\sum v_r} \right]. \tag{12}$$

The government budget constraint (12) can now be combined with the individual budget constraint (2) by substituting for $t_i$ in the individual budget constraint producing

$$y = c + (e - a) \cdot \left[ \frac{v_i}{\bar{v}_r} \cdot \frac{\sum v_r}{\sum v_i} \right] \tag{13}$$

$$(y - ap_i) = c + p_i \cdot e \tag{14}$$

where the price of per-resident expenditure, $e$, is possibly different for each taxpayer

$$p_i = \left[ \frac{v_i}{\bar{v}_r} \cdot \frac{\sum v_r}{\sum v_i} \right] \tag{15}$$

which is also equal to her tax share divided by the resident population,

$$p_i = s_i \cdot n. \tag{16}$$

The tax price, $p_i$, is equal to the cost to taxpayer $i$ of increasing per-resident expenditure by \$1.

It is more natural to define tax price in terms of service quality as opposed to defining it in terms of dollars of public expenditure (as above). Service quality is related to total expenditure, $E$ through a cost function

$$C_q(\cdot) \cdot q = E \tag{17}$$

$$\frac{C_q(\cdot)}{n} \cdot q = e \tag{18}$$

$$\Longrightarrow q = e \cdot \frac{n}{C_q(\cdot)} \tag{19}$$

where $C_q(\cdot)$ is the marginal cost of providing a service quality of $q$ to all residents.[20]

By substituting for per capita expenditure, $e$, in the individual's budget constraint (equation 14), the tax price of a one unit increase in service quality is now equivalent to

$$p_i = \frac{C_q(\cdot)}{n} \cdot \left( \frac{v_i}{\bar{v}_r} \cdot \frac{\sum v_r}{\sum v_i} \right) \tag{20}$$

$$p_i = C_q(\cdot) \cdot s_i \tag{21}$$

[19] The variable $\bar{v}_r$ is the average value of residential properties, where $r$ indicates that only residential homes are considered. This assumes that there are other types of property, but that it is most useful to consider expenditure on a per-resident, as opposed to per-property, basis. Furthermore, many assessment limits specifically address increasing residential tax shares and this equation demonstrates the effects of rising residential shares.

[20] This assumes constant returns to scale technology so that marginal cost equals average cost at all levels of production. It is extremely important to remember that the composition of the tax base can affect $C_q(\cdot)$. See, for example, Bradford, Malt, and Oates (1969).

Equation (21) demonstrates that each taxpayer's tax price is the product of their tax share and the marginal cost of service quality. The relationship between residential tax share and tax price is illustrated in equation (20). By examining the individual's budget constraint, (14), it is clear that individual tax payments, $t_i = p_i \cdot q$, can change even if service quality or government expenditure does not change. Most importantly, individual tax payment volatility is a direct result of volatility in tax shares. As discussed above, empirical evidence suggests that this volatility in individual tax payments could be quite large.[21]

## 2.2 Tax Limits: Assessment and Revenue Limits

Tax limits constrain increases in individual property tax payments over time. Limits on property tax revenue and property tax rates are the most widespread property tax limitations with 34 states having some form of rate limits and 29 having some form of revenue limits. Limits on increases in assessed (i.e., taxable) value are also numerous with 20 states having some form of assessment limits in place as of 2006. Anderson (2006b) provides a general overview of property tax limitations in the United States.

Tax revenue limits constrain annual increases in property tax revenues. Revenue limits vary across states but most limit growth in revenue to the rate of inflation or some other percentage. For example, Illinois's Property Tax Extension Limitation Law limits annual increases in property tax revenues to the lower of 5% or inflation (as of 2006 this limit is in effect in 39 counties of Illinois 102 counties). Revenue limits will most effectively constrain increases in individual property tax payments in the presence of uniformly distributed increases in property values. Changes in the composition of the tax base, however, will cause individual tax payments to change even if property tax revenues were held constant. This may be why 10 of the 29 states with revenue limitations also have some form of assessment limitations.

Assessment limits constrain changes in individual property tax payments that would occur even if revenues remained constant. The exact nature and extent of the limitations on increases in taxable value vary across states. Twelve states have statewide limitation on increases in the taxable values of individual residential properties.[22] For example, Florida limits increases in the taxable value of residential properties to the lower of 3% or inflation while Michigan limits increases to the lower of 5% or inflation. Georgia, Illinois, South Carolina provide counties with the option to explicitly limit taxable value increases and New York mandates limits in New York City and Nassau County. Iowa and Colorado limit increases in the total taxable value of residential property in the entire state. Only Connecticut, Maryland, and South Carolina have assessment limits without explicit revenue or rate

[21] The examples cited above are from Minnesota and Chicago. These are not places usually associated with so-called "hot" real estate markets or real estate bubbles. This suggests that tax shares and payments can be volatile even when property values are not excessively volatile.

[22] These twelve states are AR, AZ, CA, FL, MD, MI, MN, MT, NM, OK, OR, and TX. CA, OR, and OK limit assessment increases regardless of property type. Except in Arizona and Oregon, assessment caps are removed at the time of a sale. O'Sullivan (2001) finds that nine states have limits on assessment increases. However, he never lists the nine states. Assessment limits approved by Washington voters have twice been ruled unconstitutional. In addition, Nevada recently instituted a cap on increases in individual tax payments.

limitations. None of these three states require that property values be updated on an annual basis. In the remaining 17 states with assessment limits, nine have both levy limits and tax rate limits, one has only levy limits, and seven have only rate limits. All of these limitations on increases in taxable values constrain increases in individual property tax payments by restricting changes in the distribution of taxable values from year to year.

If assessment limits only apply to residential property, which is often the case, potential increases in tax payments by residential homeowners will be muted by the limitations.[23] The desire to reduce tax payments is clearly strong but does not necessarily imply that individual voters want to limit the authority of government. In this paper the focus is turned to the variance of tax payments as opposed to changes in mean tax payments. The focus on variance in tax payments addresses a topic that has been neglected by the literature in local public finance. Discussion of the level of tax payments does not suffer from the same neglect.

Tax rate limits are in effect in 34 states with 23 of these 34 states also having revenue limits and 16 also having some form of assessment limits. Rate limits are either specific or general. Specific limits constrain the property tax rates of particular funds within taxing districts while general limits constrain the total property tax rate within taxing jurisdictions. Illinois has specific tax rate limitations that vary from fund to fund. California has a general limit that constrains the maximum tax rate to be less than or equal to 1% of taxable value.

Rate limits constrain individual property tax payments by setting a maximum value for the ratio of revenue to total tax base. If increases in total tax base and individual tax base are relatively small and relatively infrequent, tax rate limits will effectively constrain increases in individual property tax payments. For this reason, tax rate limits are often combined with revenue limits, assessment limits, or infrequent revaluations of property.

In fact, only five states have limitation on tax rates without any explicit statewide limits on either revenues or assessment increases. Alabama, North Carolina, and New York constitute three of these five states and they do not require annual updates to individual taxable values. Wyoming and Utah limit tax rates without other explicit limits and appear to legally require annual updates to taxable values.Thus, 3 of the 5 states with only rate limits appear to feature property tax systems that encourage infrequent changes in the taxable values of individual properties.

# 3 A Dynamic Model of Property Tax Limits

This section presents a simple two-period dynamic voting model. The model produces two key results: that voters prefer less tax-price volatility and are willing to vote to restrict volatility (section 3.3), and, under more restrictive assumptions, that voters also prefer to limit future tax revenue as well (section 3.4). It may seem intuitive that voters would prefer less tax-price volatility, but that is not necessarily true: below, in section 3.1, it is shown that in general, consumers prefer *more* price variance.

[23] Assessment limits that apply only to residential properties do not limit increases in the tax payments of individuals owning non-residential properties. Note also that states with non-annual assessment systems essentially cap annual assessment increases at zero in non-assessment years.

### 3.1 A paradox: Consumers usually prefer more price volatility

The analysis in section 2.1 emphasizes that the tax price of public services can vary for an individual over time. This variation in tax price can lead to uncertainty in tax prices. This section investigates the circumstances required for voters to prefer tax price volatility. It is demonstrated that voters prefer more price variance in a seemingly general set of circumstances. It is shown in section 3.3, however, that in a more realistic, dynamic tax and voting model, voters may prefer less volatility.

In order to separate voter responses to surely increasing tax prices from voter responses to an increased variance or uncertainty in tax prices, this section considers altering the variance of tax prices while holding the mean tax price constant. The following analysis considers how voters respond to increases in the variance of tax price that do not affect the expected tax price for each voter.

Consumers, in general, *benefit* from mean-preserving increases in variance of prices. The literature in economics discussing the volatility or instability of prices features work by Waugh (1944), Oi (1961), Samuelson (1972), and Turnovsky, Shalit, and Shmitz (1980). Most succinctly, a consumer will gain from mean-preserving price *instability* if the indirect utility function is convex with respect to the price of the good in question. Why should a consumer prefer price variance? Consider a consumer for which apples and oranges are perfect substitutes. Suppose the mean price of fruit is one dollar. If there is no price variance, then the consumer pays one dollar for each piece of fruit. However, with price variance, the consumer will buy the cheaper fruit, making her strictly better off. As price variance increases, the better off she becomes. This intuition, that price variance benefits the consumer by allowing her to stock up on cheaper goods, extends to goods in general (see Lemma 1).

Waugh (1944) was the first to demonstrate, using consumer surplus, that a consumer could gain from price instability.[24] As Turnovsky, Shalit, and Shmitz (1980) demonstrate, consumers *gain* from price *stability* (i.e., consumers prefer less variance) if

$$(s_j(\eta_j - \rho) - e_j) < 0, \tag{22}$$

where $\eta_j$ is the income elasticity of demand for the good in question, $e_j$ is the price elasticity, $s_j$ is the budget share of the good, and $\rho$ is the coefficient of relative risk aversion. Empirical evidence suggests that this equality will be positive for almost any good. Conventional estimates of income and price elasticities for public services also suggest that this equality is positive.[25] Thus, it appears unlikely that taxpayers would prefer less tax-price volatility if they can freely choose the quantities of public services they consume. (This only addresses variance, and does not suggest that taxpayers would not prefer a lower average tax price or lower expected changes in tax price.)

[24] Samuelson (1972) argued that the price instability discussed by Waugh (1944) and Oi (1961) was not realistic (i.e., feasible) and that feasible price instability did not benefit consumers. This discussion of feasibility is emphasized less in subsequent work and the work of Turnovsky, Shalit, and Shmitz (1980) considers mean-preserving price variance as feasible. Waugh later emphasized, however, that his work was meant to elucidate issues of volatility in market prices as opposed to the government induced price volatility that Samuelson criticized. The argument over feasibility is not relevant to the example of property taxation.

[25] The price elasticity is certainly negative making the last term a positive number.

We demonstrate this preference for price volatility for a non-Giffen good whose price varies about its mean.[26] We show, that the value of buying more when the good is cheaper makes her strictly better off. Note that she observes all prices when she makes her consumption choice.

**Lemma 1** *An agent is made strictly better off by increased mean-preserving variance between any two prices for the same good if and only if the good is not a Giffen good.*

**Proof.** Without loss of generality, suppose the first price of the same good is lower than the second: $p_1 < p_2$. Let $\bar{p} = \frac{p_1+p_2}{2}$. Then define $\delta > 0$ such that:

$$p_1 = \bar{p} - \delta \tag{23}$$
$$p_2 = \bar{p} + \delta \tag{24}$$

This analysis restricts discussion of variance to an analysis of $\delta$. It should be clear to the reader that an increase in $\delta$ is equivalent to an increase in variance.[27]

As usual, define $v(p, w)$ as the agent's indirect utility at prices $p$ and wealth $w$, and $x_1^\star$ and $x_2^\star$ as consumer demand for the good in periods 1 and 2. It is shown that the following two claims are equivalent:

1. The agent is made better off by an increase in price variance: $\frac{\partial v}{\partial \delta} > 0$, and
2. The good in question is not a Giffen good, that is the agent buys more of the good when it is cheaper: $x_1^\star > x_2^\star$

First note that:

$$\begin{aligned} v(p,w) &= v(p-\delta, p+\delta, p_3, \ldots, p_L, w) && (25) \\ \Longrightarrow \frac{\partial v}{\partial \delta} &= \frac{\partial v}{\partial p_2} - \frac{\partial v}{\partial p_1} && (26) \end{aligned}$$

then discover that $\frac{\partial v}{\partial \delta} > 0$ and $x_1^\star > x_2^\star$ are equivalent:

$$\begin{aligned} \frac{\partial v}{\partial \delta} &> 0 && (27) \\ \Longleftrightarrow \frac{\partial v}{\partial p_2} - \frac{\partial v}{\partial p_1} &> 0 && (28) \\ \Longleftrightarrow \frac{\partial v}{\partial p_2} &> \frac{\partial v}{\partial p_1} && (29) \\ \Longleftrightarrow -\frac{\frac{\partial v}{\partial p_2}}{\frac{\partial v}{\partial w}} &< -\frac{\frac{\partial v}{\partial p_1}}{\frac{\partial v}{\partial w}} && (30) \\ \Longleftrightarrow x_2^\star &< x_1^\star && (31) \end{aligned}$$

by Roy's Identity. This completes the proof. ■

[26] Consumers of a Giffen good will increase consumption of that good as its price increases.
[27] See appendix, lemma 2 section A.1 for a demonstration of this equivalence.

The most crucial assumption in the above results is that consumers are able to freely select the quantity of the price-volatile good that they wish to consume. While this result certainly applies to the consumption of, for example, apples and oranges, it seems less applicable to consumption of public services such as education, law enforcement, fire protection, and street maintenance. Even when these are not pure public goods it is difficult to argue that each individual selects their specific level of consumption over time.

Individuals do select an exact quantity of public service consumption in models inspired by Tiebout (1956). These models (e.g., Hamilton (1975)) often feature communities whose taxpayers have identical and static tax prices. Thus these models do not readily lend themselves to a consideration of individual tax prices changing over time. Indeed, the zoning restrictions assumed in these models would likely prevent changes in the distribution of tax shares over time.

Other models allow for individuals within a single community to have different tax shares (e.g., Epple and Platt (1998)). These models allow voters to have tax shares different from the decisive voter, implying that, given their location, all voters are not consuming their most preferred level of public services.[28] These are static models, however, and they do not address how voters might respond to volatility in their tax prices over time. Fernandez and Rogerson (1996) examine the same issues with an overlapping-generation model, thereby incorporating time explicitly into these types of models. They don't, however, consider how tax prices within the same community might change over time. Furthermore, neither these models or the models inspired by Tiebout (1956) explicitly consider risk and uncertainty.

The evidence cited above from Minnesota and Chicago suggests that individual tax shares do change over time and that these changes can produce substantial changes in individual tax liabilities. Furthermore, it also the case that tax shares do vary within the same community at one point in time. Figure 1 displays the distribution of tax shares within the residential property class for four Chicago suburbs. The distributions exhibit substantial variation in tax shares, with both Evanston and Oak Park having much larger tails at the high-end of the tax share distribution. To provide some context, consider that using state-wide averages for property tax collections, a 0.01 difference in tax share implies over a $100 difference in property tax liability.[29] The standard deviation of tax shares in Oak Park is almost 0.04, implying a one standard deviation move from the mean tax share implies an over $400 difference in property tax liability.

As is hopefully clear, *changes* over time in individual tax share result from differential rates of appreciation among properties within the same jurisdiction. If all properties appreciate by 10%, all tax prices will remain the same. If, however, some properties appreciate by 5% while others appreciate by 10%, the tax prices of high appreciating home will increase and the tax price of the low appreciating homes will decrease (assuming that the marginal

[28]Across locations, an individual is in her most preferred community, with her most preferred tax policy. But this does not imply that she would not prefer more or less public service given her tax price *within* her current location.

[29]Consider that in 2000, school districts in Illinois collected an average of $10,665,821 in property taxes in 2000. In this same year residential property paid approximately 58% of the total property taxes. This suggests that a difference in tax share of 0.01 in the average school district would imply a difference in residential property tax of approximately $62. Considering that school district property tax revenues represented 58% of all property tax revenues, this implies an average total difference in property tax liability of $106.

costs of service provision remain the same). The empirical observations on tax payments in Chicago and Minnesota suggest that these differential rates of appreciation occur within the residential property class and between residential and commercial property within the same jurisdiction.

## 3.2 Introduction to the dynamic model

As the Turnovsky, Shalit, and Shmitz (1980) result, equation (22), makes clear, when consumers with standard preferences have complete freedom to select consumption amounts, these consumers prefer more variance in prices. Rather than insist that preferences for public services are unusual or that taxpayers have an unusually high degree of risk-aversion, the model below explores how a lack of freedom to completely adjust the quantity of consumption in response to changing tax prices implies that taxpayers prefer lower variance in tax prices. This lack of freedom also creates a desire to limit government revenues.

As noted above, this lack of consumptive freedom at one point in time is a feature of models allowing individuals in a single community to have different tax shares at one point in time. In the simplest model, a level of public service quality is selected through majority voting, allowing the decisive voter (e.g., median voter) to select his or her most preferred level of public services. The level of public services most preferred by the decisive voter will be different from the most preferred bundle of the other voters (with different prices, tastes, or incomes) in that same jurisdiction. Although the selection of public service levels in a community is not a single-dimensional problem, attention is restricted to the single-dimension, majority voting, median voter framework to highlight the role played by changes in tax price over time.

The first step in analyzing taxpayers preferences in the presence of variance in tax prices is to allow for tax prices to change by introducing time into a model of choice regarding private consumption and public services. Time is introduced by examining a two-period model. The Turnovsky, Shalit, and Shmitz (1980) result on price variance implies that if the decisive voter in one time period is certainly decisive in each subsequent period that this always-decisive voter will prefer more price variance to less. An always-decisive voter prefers higher price variance because she can always optimally adjust her consumption in response to changes in prices.[30] In the presence of an always-decisive voter, every other (never-decisive) voter will not be able to fully respond to changes in their tax prices by selecting a new optimal bundle. Taxpayers that are never-decisive or sometimes-decisive are classified as not-always-decisive.[31]

The model below demonstrates that any risk-averse taxpayer that is a not-always-decisive voter will prefer less price variance to more variance. The analysis also demonstrates that given the potential for changing tax prices, that taxpayers may prefer to limit government revenues. The analysis highlights the effects of price uncertainty and rigid consumption

[30] Again assuming convex indirect utility with respect to price.

[31] This analysis ignores mobility across communities, but it is unlikely that any voter could always move and find the perfectly optimal tax-consumption bundle in every period. By optimal, we mean a particular voter's optimal tax-consumption bundle within a community at their given tax price as opposed to their optimal selection of a community (described by a specific bundle) that is preferred over all other locations (and their specific bundles).

choices on internal equilibria. It does not address the issue of external equilibria that presents itself in the context of multi-community sorting and voting models.

Suppose there is a decisive-in-revenue voter, voter $i$, at time 1. (This decisive-in-revenue voter might be median in some parameter–presumably house value or tax price–but the parameter is not important for this result.) Decisiveness in revenue indicates that under the majority voting system, this time 1 decisive voter selects the level of government revenue in period 1.[32]

All voters receive idiosyncratic shocks to real estate value in period 2. Hence, with virtual certainty, voter $i$ will *not* be the decisive-in-revenue (e.g., median) voter at time 2 if the parameter determining decisiveness is either home value or tax price. Regardless of the mechanism, suppose this voter assumes that she will not be the decisive-in-revenue voter in period 2. This makes her a not-always-decisive voter. Thus, voter $i$ views $R_2 \geq 0$ as out of her direct control. Voter $i$ is able to select private consumption in both periods and is able to transfer funds without cost across the two periods. All voters have the same two-period exogenous income and there is only one community.

Below it is shown that this not-always-decisive voter $i$ is interested in limiting the volatility of her tax price.

## 3.3 Voters prefer less tax-price volatility

In order to understand how shocks to tax price and risk-aversion create a desire for tax limits, we demonstrate that in general a not-always-decisive voter would approve of a plan to reduce her variance in tax prices in order to reduce her variance in tax liability.

Consider a decisive-in-revenue voter $i$ at who is decisive at time 1 and not decisive at time 2. The voter selects the level of government revenue in period 1 but not in period 2. Again, this voter is called *not-always-decisive*.

Below it is shown that the not-always-decisive voter $i$ is interested in limiting the volatility of her own tax price. Suppose there are two probability distributions of her own tax price. Call them $F$ and $G$, and suppose $G$ is a mean-preserving spread of $F$. This is the case if, for example, $F$ and $G$ have the same mean and $G$ has a larger variance. It is shown that the not-always-decisive voter prefers limiting volatility by demonstrating that she prefers $F$ to $G$.

### 3.3.1 Model Assumptions

1. Suppose there are two goods: private consumption $c$ and government services $R$.

2. Suppose voter $i$ has von Neuman-Morgenstern utility $u(c, R)$ over consumption and services so that the expected utility of a gamble over consumption which follows distribution $\Phi(c)$ can be evaluated according to:

$$Eu(\Phi, R) = \int u(c, R) d\Phi(c)$$

[32]Selecting government revenue is equivalent to voters selecting a level of service quality and assuming that the marginal cost of a unit of quality is $1.

Suppose further that voter $i$ is risk-averse (e.g. concave) in the first argument (consumption.)

3. Suppose voter $i$ **can choose**:
   - Consumption $c_1$ and services $R_1$ at time 1.
   - Consumption $c_2$ at time 2.
4. Suppose that tax prices $p_1$ is known. Suppose that the price of consumption in both periods is normalized to 1.
5. Suppose revenue $R_2$ is known.[33]
6. Suppose the each voter has a fixed budget $B$ over the two-periods and there is no interest rate/discounting between periods.

### 3.3.2 Theorem and Proof

**Theorem 1** *Suppose assumptions 1-6.*

*Suppose there are two distributions $F$ and $G$ of possible values for the price $p_2$. Suppose that $G$ is a mean-preserving spread of $F$.*

*Then the not-always-decisive voter would prefer distribution $F$ over $G$. That is, the not-always-decisive voter would prefer less variance.*

**Proof.**

Since $G$ is a mean-preserving spread of $F$, then $G$ can be related to $F$ in the following way:

For each $p$ in the support of $F$, the $p$ is further randomized to $p + z_p$, where $z_p$ has a distribution function $H_p(z_p)$ with a mean of zero. (Note that there is a separate distribution for each level of $p$. That is why any mean preserving spread can be simulated this way.)

Consider not-always-decisive voter $i$'s problem as choosing, at time 1, three variables: consumption $c_1$, services $R_1$, and savings to time 2, called assets $a_1$. Note that the government budget constraint (must balance its budget) has been substituted into the period 1 and period 2 budget constraints for the consumer. Then individual $i$'s problem is:

$$\max_{c_1,a_1,R_1} \quad u(c_1, R_1) + Eu(c_2, R_2) \tag{32}$$

$$c_1 + p_1 R_1 + a_1 \leq B, c_2 \leq a_1 - p_2 R_2 \tag{33}$$

Please note again that she does not consider that she has any influence on $R_2$.

By local non-satiation (and substituting the second period budget constraint into the problem), this problem is equivalent to:

$$\max_{c_1,a_1,R_1} \quad u(c_1, R_1) + Eu\left(a_1 - p_2 R_2, R_2\right) \tag{34}$$

$$c_1 + p_1 R_1 + a_1 = B \tag{35}$$

[33] To illustrate this assumption, suppose the voter believes she is the decisive voter now, and knows that the voter ordering will change, but that the decisive voter will have (on average) the same price (share) next period. This means that $R_1 = R_2$.

Let $(c_1^J, R_1^J, a_1^J)$ be the optimal values chosen by this voter under a distribution $J$, where $J$ can be $F$ or $G$.

Now we demonstrate that the indirect utility under $F$ exceeds that under $G$.

$$v(G) = u(c_1^G, R_1^G) + Eu(a_1^G - p_2 R_2, R_2) \tag{36}$$

Dropping the time subscript on $p_2$,

$$= u(c_1^G, R_1^G) + \int u(a_1^G - pR_2, R_2) dG(p) \tag{37}$$

by definition of expected utility. Then:

$$v(G) = u(c_1^G, R_1^G) + \int \left( \int u\left(a_1^G - (p + z_p)\, R_2, R_2\right) dH_p(z_p) \right) dF(p) \tag{38}$$

by the construction of the mean-preserving spread. Then, since $u$ is concave in its first argument:

$$\begin{aligned} & u(c_1^G, R_1^G) + \int \left( \int u(a_1^G - (p + z_p) R_2, R_2) dH_p(z_p) \right) dF(p) \\ & \leq u(c_1^G, R_1^G) + \int u\left( \int \left(a_1^G - (p + z_p) R_2\right) dH_p(z_p), R_2 \right) dF(p) \end{aligned} \tag{39}$$

$$= u(c_1^G, R_1^G) + \int u\left( a_1^G - pR_2 - R_2 \int z_p dH_p(z_p), R_2 \right) dF(p) \tag{40}$$

$$= u(c_1^G, R_1^G) + \int u(a_1^G - pR_2, R_2) dF(p) \tag{41}$$

since $H_p(z_p)$ is mean zero.

But

$$u(c_1^G, R_1^G) + \int u(a_1^G - pR_2, R_2) dF(p) \leq u(c_1^F, R_1^F) + \int u(a_1^F - pR_2, R_2) dF(p) \tag{42}$$

$$\begin{aligned} \implies v(G) = u(c_1^G, R_1^G) + Eu(a_1^G - p_2 R_2, R_2) & \leq u(c_1^G, R_1^G) + \int u(a_1^G - pR_2, R_2) dF(p) \\ & \leq u(c_1^F, R_1^F) + \int u(a_1^F - pR_2, R_2) dF(p) = v(F) \end{aligned} \tag{43}$$

since $\left\{c_1^F, R_1^F, a_1^F\right\}$ are the chosen optimally.

Hence, not-always-decisive voter $i$ is made worse off by a mean-preserving spread of her own tax prices. That is, voter $i$ is made worse off by variance in her tax price. ■

In essence, since $R_2$ is out of the voter's control, adding noise to that term, given that she is risk averse, will make her worse off.

The choice of voter $i$ is arbitrary and that this is a two-period model is also arbitrary. This result demonstrates that any not-always-decisive voter prefers *now* to reduce price variance in the *future*. This applies to any not-always-decisive voter at any time she has the opportunity to vote.

## 3.4 The effect of risk aversion and voters prefer revenue limits

Theorem 1 established that a risk-averse, not-always-decisive voter is made worse off by more tax-price variance in the second period. Two additional results are shown in a more restrictive setting. First, the utility penalty is increasing in the level of risk aversion. Second, the utility penalty is increasing in the amount of revenue raised in the second period ($R_2$).

### 3.4.1 Additional Model Assumptions

The analysis assumes CARA utility and normally distributed shocks to tax price. Here are the additional assumptions formally stated:

Assumption 7. Voter $i$ has CARA utility of the following form:

$$\begin{aligned} utility(c, R) &= u(c) + u(R) \qquad (44) \\ \text{where } u(x) &= -e^{-\alpha x} \qquad (45) \end{aligned}$$

where $\alpha$ is the coefficient of absolute risk aversion (which is constant.)

Assumption 8. The shock to price is normally distributed:

$$p_2 = p_1(1 + \epsilon) \qquad (46)$$

where $\epsilon$ is distributed $N(\mu, \sigma^2)$.

Under assumptions 1-8, the problem facing the agent is:

$$\begin{aligned} &\max_{c_1, a_1, R_1} u(c_1) + u(R_1) + Eu(c_2) + u(R_2) \\ &\text{s.t.: } c_1 + p_1 R_1 + a_1 \leq B \\ &\qquad c_2 \leq a_1 - p_1(1 + \epsilon) R_2 \end{aligned} \qquad (47)$$

By local non-satiation (and substituting the budget constraints into the problem), this problem is equivalent to:

$$\begin{aligned} &\max_{c_1, a_1, R_1} u(c_1) + u(R_1) + Eu\left(a_1 - p_1(1 + \epsilon) R_2\right) + u(R_2) \\ &\text{s.t.: } c_1 + p_1 R_1 + a_1 = B \end{aligned} \qquad (48)$$

One well-known property of CARA utility is that, if a utility function is CARA ($u(x) = -e^{-rx}$), and $x$ is distributed $N(\mu, \sigma^2)$, then:

$$E\left(u(\alpha x + \beta)\right) = u\left(\alpha\mu + \beta\frac{1}{2} r \alpha^2 \sigma^2\right) \qquad (49)$$

Applying this property,

$$\begin{aligned} Eu\left(a_1 - p_1(1 + \epsilon) R_2\right) &= u\left(a_1 - p_1 R_2 - p_1 R_2 \mu - \frac{1}{2}\alpha p_1 R_2 2\sigma^2\right) \qquad (50) \\ &= u\left(a_1 - p_1 R_2\left[1 + \mu + \alpha\sigma^2\right]\right) \qquad (51) \end{aligned}$$

Therefore, the agent's problem (equation 48) is equivalent to:

$$\begin{aligned} &\max_{c_1,a_1,R_1} u(c_1) + u(R_1) + u\left(a_1 - p_1 R_2\left[1+\mu+\alpha\sigma^2\right]\right) + u(R_2) \\ &\text{s.t.: } c_1 + p_1 R_1 + a_1 = B \end{aligned} \tag{52}$$

The first order conditions of this problem are:

$$\begin{aligned} u'(c_1^\star) &= \lambda^\star \\ u'\left(a_1^\star - p_1 R_2\left[1+\mu+\alpha\sigma^2\right]\right) &= \lambda^\star \\ \frac{u'(R_1^\star)}{p_1} &= \lambda^\star \end{aligned} \tag{53}$$

Consider, now, the voter's indirect utility as a function of $\sigma^2$:[34]

$$v(\sigma^2) = u(c_1^\star) + u(R_1^\star) + u(R_2) + u\left(a_1^\star - p_1 R_2\left[1+\mu+\alpha\sigma^2\right]\right) \tag{54}$$

$$\implies \frac{\partial v}{\partial \sigma^2} = u'\left(a_1^\star - p_1 R_2\left[1+\mu+\alpha\sigma^2\right]\right)\cdot[-\alpha p_1 R_2] < 0 \tag{55}$$

Inequality 55 states that indirect utility is decreasing in variance. This is also an implication of Theorem 1.

The structure of the voter's problem is referred to in the proofs below.

### 3.4.2 Theorems and Proofs

**Theorem 2** *Suppose assumptions 1-8. Then the penalty to utility of variance is increasing in risk aversion ($\alpha$). Namely:*

$$\frac{\partial^2 v}{\partial\sigma^2\partial\alpha} < 0 \tag{56}$$

**Proof.**

$$\frac{\partial v}{\partial \sigma^2} = u'\left(a_1^\star - p_1 R_2\left[1+\mu+\alpha\sigma^2\right]\right)[-\alpha p_1 R_2] \tag{57}$$

$$\implies \frac{\partial^2 v}{\partial\sigma^2\partial\alpha} = -u''(\cdot)\left[\frac{\partial a_1^\star}{\partial\alpha} - p_1 R_2 \sigma^2\right]\alpha p_1 R_2 - u'(\cdot) p_1 R_2 \tag{58}$$

Now $-p_1 R_2 \sigma^2$ is negative and $-u''(\cdot)$ is positive. So if $\left[\frac{\partial a_1^\star}{\partial\alpha} - p_1 R_2\sigma^2\right]$ is (weakly) negative, then $\frac{\partial^2 v}{\partial\sigma^2\partial\alpha}$ is negative.

We establish that $\frac{\partial a_1^\star}{\partial\alpha} \leq p_1 R_2 \sigma^2$ here:

Suppose towards a contradiction that $\frac{\partial a_1^\star}{\partial\alpha} > p_1 R_2\sigma^2$. By the first order conditions of the problem (equations 53),

$$u'(c_1^\star) = u'(a_1^\star - R_2 p_1\left[1+\mu+\alpha\sigma^2\right]) = \frac{u'(R_1^\star)}{p_1} \tag{59}$$

[34] The envelope theorem implies that the indirect effects through $c_1^\star$, etc. can be ignored. Furthermore, $\sigma^2$ does not appear in the budget constraint, and hence can also be ignored in the derivative of the Lagrangean.

which holds at all optima. (These are the Euler equations of this problem.)

Note that:

$$\frac{\partial}{\partial \alpha}\left(a_1^\star - R_2 p_1\left[1+\mu+\alpha\sigma^2\right]\right) = \frac{\partial a_1^\star}{\partial \alpha} - p_1 R_2 \sigma^2 \tag{60}$$

By the assumption toward contradiction:

$$\frac{\partial a_1^\star}{\partial \alpha} - p_1 R_2 \sigma^2 > 0 \tag{61}$$

$$\implies \frac{\partial}{\partial \alpha}\left[a_1^\star - \left[1+\mu+\alpha\sigma^2\right] p_1 R_2\right] > 0 \tag{62}$$

Then by the Euler equations:

$$\implies \frac{\partial c_1^\star}{\partial \alpha} > 0, \frac{\partial R_1^\star}{\partial \alpha} > 0 \tag{63}$$

But note that, at a maximum,

$$c_1^\star + p_1 R_1^\star + a_1^\star = B \tag{64}$$

$$\implies \frac{\partial c_1^\star}{\partial \alpha} + p_1 \frac{\partial R_1^\star}{\partial \alpha} + \frac{\partial a_1^\star}{\partial \alpha} = \frac{\partial B}{\partial \alpha} \tag{65}$$

$$\implies \frac{\partial c_1^\star}{\partial \alpha} + p_1 \frac{\partial R_1^\star}{\partial \alpha} + \frac{\partial a_1^\star}{\partial \alpha} = 0 \tag{66}$$

Since all terms on the left are strictly positive, this is a contradiction. Hence it must be the case that $\frac{\partial a_1^\star}{\partial \alpha} \leq p_1 R_2 \sigma^2$. In that case, equation 58 allows us to conclude that $\frac{\partial^2 v}{\partial \sigma^2 \partial \alpha}$ is negative.[35] ■

**Theorem 3** *Suppose assumptions 1-8. Then the penalty to utility of variance is increasing in period two revenues ($R_2$). Namely:*

$$\frac{\partial^2 v}{\partial \sigma^2 \partial R_2} < 0 \tag{68}$$

**Proof.** This proof follows the proof of Theorem 2, so the exposition here is relatively brief. Please refer to that proof for details.

[35] An aside. The result, that $\frac{\partial a_1^\star}{\partial \alpha} \leq p_1 R_2 \sigma^2$, implies that $\frac{\partial c_1^\star}{\partial \alpha} \leq 0$ and $\frac{\partial R_1^\star}{\partial \alpha} \leq 0$. One might be tempted to conclude that equation 66 could be violated because all terms on the left side are weakly negative. However, not all terms on the left side are weakly negative. $\frac{\partial a_1^\star}{\partial \alpha} \leq p_1 R_2 \sigma^2$ does not imply that $\frac{\partial a_1^\star}{\partial \alpha} \leq 0$. On the contrary, we can conclude that

$$0 \leq \frac{\partial a_1^\star}{\partial \alpha} \leq p_1 R_2 \sigma^2 \tag{67}$$

In this case, equation 66 is not violated.

$$\frac{\partial v}{\partial \sigma^2} = u'\left(a_1^\star - p_1 R_2\left[1+\mu+\alpha\sigma^2\right]\right)\left[-\alpha p_1 R_2\right] \tag{69}$$

$$\implies \frac{\partial^2 v}{\partial \sigma^2 \partial R_2} = -u''(\cdot)\left[\frac{\partial a_1^\star}{\partial R_2} - p_1(1+\mu+\alpha\sigma^2)\right]\alpha p_1 R_2 - u'(\cdot)p_1\alpha \tag{70}$$

Now $-u'(\cdot)p_1\alpha$ is negative and $-u''(\cdot)$ is positive. So if $\frac{\partial a_1^\star}{\partial R_2} - p_1(1+\mu+\alpha\sigma^2)$ is (weakly) negative, then $\frac{\partial^2 v}{\partial\sigma^2\partial R_2}$ is negative.

We establish that $\frac{\partial a_1^\star}{\partial R_2} \leq p_1(1+\mu+\alpha\sigma^2)$. Suppose towards a contradiction that $\frac{\partial a_1^\star}{\partial R_2} > p_1(1+\mu+\alpha\sigma^2)$. Let $K = (1+\mu+\alpha\sigma^2)$. Then, by the Euler equations of this problem:

$$u'(c_1^\star) = u'(a_1^\star - K p_1 R_2) = \frac{u'(R_1^\star)}{p_1} \tag{71}$$

which holds at all optima. Then:

$$\frac{\partial}{\partial R_2}\left[a_1^\star - K p_1 R_2\right] = \frac{\partial a_1^\star}{\partial R_2} - K p_1 \tag{72}$$

By the assumption toward contradiction:

$$\implies \frac{\partial}{\partial R_2}\left[a_1^\star - K p_1 R_2\right] > 0 \tag{73}$$

By the Euler equations:

$$\implies \frac{\partial c_1^\star}{\partial R_2} > 0, \frac{\partial R_1^\star}{\partial R_2} > 0 \tag{74}$$

At a maximum,

$$c_1^\star + p_1 R_1^\star + a_1^\star = B \tag{75}$$

$$\implies \frac{\partial c_1^\star}{\partial R_2} + p_1\frac{\partial R_1^\star}{\partial R_2} + \frac{\partial a_1^\star}{\partial R_2} = 0 \tag{76}$$

Since all terms on the left are strictly positive, this is a contradiction. Hence it must be the case that $\frac{\partial a_1^\star}{\partial R_2} \leq p_1(1+\mu+\alpha\sigma^2)$. In that case, equation 70 allows us to conclude that $\frac{\partial^2 v}{\partial\sigma^2\partial R_2}$ is negative. ■

# 4 Discussion of Results

Theorem 3 demonstrates that a higher level of revenue in period 2 exacerbates the costs of variance in prices for the non-decisive voter. Variances in prices are also seen to exacerbate the potential costs to a non-decisive voter of increases in revenue. Increases in revenue in a period when voter $i$ is not decisive, $R_2$, can have negative or positive effects on individual utility. Positive variance in prices, however, decreases any benefit (and magnifies any loss) associated with the increased revenue.[36]

These results demonstrate that, under the CARA and normality assumptions, not-always-decisive voters will support measures to limit tax revenues, $R_2$, given variance in their tax prices. Since at any moment non-decisive-in-revenue voters out number the always-decisive-in-revenue voter, limits on tax revenues can win a majority vote. Importantly, that a voter is decisive in revenue does not imply that she is decisive on all issues. In fact, an always-decisive-in-revenue voter is the *only* voter that would always reject any limits on revenue growth in the presence of variance in prices.

These results suggest that a limit on tax revenues or tax rates can win a majority vote in the context of a median voter model. There is no waste or inefficiency in this model as the government selects $R$ by majority vote in each period. The not-always-decisive voters must pass legislation limiting revenue since an always-decisive in revenue voter (or any decisive voter in revenue in any period), can always win a majority vote in the current period to receive their preferred level of public revenues and expenditures. That is, the non-decisive voters will not be able to effectively limit changes in revenues in each period.

It is clear that measures limiting variance in prices, all else equal, will win in a majority voting framework. Assessment limits might resemble such measures, although assessment limits are likely to affect expected values of changes in tax prices as well as the variance of tax price changes.

The desire for limits on revenues and price variance arises out of the preference for smoothing marginal utility across periods that is common in multiple period models. While a revealed lower price in period 2 offers a benefit to a not-always-decisive voter in the form of increased private consumption (assuming the same $R$), a revealed higher price in period 2 costs the consumer private consumption. Risk-aversion then implies that the consumer will prefer to avoid the possible loss, and importantly, forgo the possible benefit in order to better equate marginal utility across periods. This argument does not apply to a voter who can optimally adjust $R_2$ to any change in prices.

One important caveat to these results is the availability of overrides. An override gives voters an opportunity to collect revenues or increase tax rates in excess of the legislated limit, often only temporarily. Overrides are an important component in most revenue limitation legislation. As the analysis above indicates, within any single period a decisive-in-revenue voter should always win a majority vote to exceed the limit if this is what she desires. This would seem to make revenue limits completely useless in the context of this model. The actual override process, however, often also limits the amount by which revenue can exceed a limit. For example, in Illinois school districts many tax rate limitations can be exceeded for one period but only up to a new maximum rate level that cannot be exceeded. In Massachusetts voters can override revenue limitations with a simple majority but they cannot exceed tax rate limitations. These types of super-limits insure that the non-decisive voters have protections against large and sudden increases in property taxes. It is important, however, to have overrides if, for example, a common shock to income were to affect taxpayers in a jurisdiction and they all wish to spend more than previously. The three main types of

[36] In a model with constant tax prices over time and an always-decisive voter there is no reason to expect changes in $R$ over time unless something else changes to alter demand (e.g., income).

property tax limits protect voters from particularly bad outcomes caused by shocks to price (or something else) that are not equally distributed across the jurisdiction's population.

# 5 Conclusion

Revenue limits serve to dampen the effects of the changes in real estate value that can cause volatility in individual tax payments. Theorem 1 shows that voters prefer to limit tax price variance. Theorem 3 shows that, when direct policy measures limiting the mean and variance of price changes are not available, the majority of voters within a jurisdiction will support at least some form of revenue limitation. It is demonstrated formally that individuals prefer changes in tax prices to have less variance and lower expected values and that revenue increases exacerbate the potential negative effects of these price changes. However, these results do not yet formally demonstrate the existence of a political equilibrium where revenue limits survive a majority vote.

The lack of an ability to adjust consumption of public services, $R$, is the driving force behind the conclusions thus far. Of course, the lack of ability to adjust consumption of $R$ could arise from sources different from what is described above. For example, if a decisive voter was forced to choose a constant level of revenue or service quality for several periods, she may vote to restrict variance in prices.

The traditional Leviathan model provides a motivation for voters to limit variance in prices. If voters cannot force the government to collect less revenue, variance in prices would only make revenue limits more of an urgent priority. The results in this paper demonstrate that a voter belief in wasteful government is *not necessary* for the existence of limitations on revenue, tax rates, and assessed values.

A notable fact about property tax limitations is that nearly all of them are instituted politically by the state legislature or by state referendum rather than at the local level.[37] The model outlined above suggests the possibility of individual jurisdictions supporting limitations on revenues at the local level. If the model is insightful then why aren't local revenue limits observed? Perhaps this is a legal issue. Since local governments exist at the privilege of state government, perhaps state government is able to limit them but they cannot create separate laws for themselves.

Suppose that voters did vote to eliminate price variance, creating a system similar to the acquisition based system in California. Although uncertainty in property tax payments would be minimized, systems of fixed property values do create inequities.[38] The model above does not consider that voters also might value the fairness or equality of the tax system as well as the certainty of their tax payments. Hopefully the model illuminates the issue of uncertainty in a relatively rigorous theoretical framework. Since voters care about both uncertainty and equality, voters would not maximize equality at the expense of maximizing uncertainty.

An important extension to this research is a discussion of how the different sources of

[37]Some cities pass their own tax limitations. For example, following cuts to state aid in 2002, the Minneapolis city council voted to restrict growth in tax levies to 8% annually until 2010.

[38]These inequities may be eliminated though capitalization of property taxes into property values. Full capitalization may be more likely to obtain when tax payments are relatively predictable.

variation in local property tax payments make it more difficult for individuals to monitor local government. As the above analysis makes clear an increase in an individual's tax bill is not necessarily a signal of increased government expenditure. If voters are unsure how to interpret changes in their tax bills they may support constraints on government spending because they falsely interpret the causal mechanism of changes in their tax bills. This can be considered as causal confusion on the part of voters (see Pape (2006)).

An important assumption as yet undiscussed is the affects of increases in property value on the wealth of consumers. Even if wealth increases, an unequal change in tax prices still appears to cause the same problems for the non-decisive voters. They cannot optimally respond to their changes in wealth by selecting their most desired reallocation of income between private consumption and government services. It is the non-decisiveness that produces the limitation results, not the inability of taxpayers to finance consumption with gains in the asset value of their property.

The next step in this research is to demonstrate empirically that changes in tax prices are large enough to engender these types of limitations. Thus tax shares can exhibit substantial variation without extreme variations in property valuation. While evidence from Minnesota and Illinois suggests that tax shares are volatile, there is still a need for more empirical studies that measure this extent of the problem. It is still possible that while theoretically important the variance of tax prices is just not large enough to be empirically relevant. We find this unlikely and hope to demonstrate that these shocks to tax price can be surprisingly large. Of course, if voters are concerned (even mistakenly) about shocks to tax price they may support certain limitations even when the likelihood of the shocks is relatively small.

When discussing property tax limitations it is important to remember that property assessors can also mitigate the problem of uncertainty by not accurately updating assessed values. This probably still happens but it happened even more often in the past. There is some evidence that prior to assessment reforms aimed at increasing equality in the 1970s that many California assessors would smooth out assessments over time. When assessors forced to have higher quality assessments than are no longer able to smooth out changes as effectively.

Another interesting fact is that current measures to reduce the possible negative effects of uncertainty in tax payments, such as tax deferrals or tax loans, are incredibly unpopular and under utilized. If the demand for certainty is so high, then why don't voters use these programs to help smooth their consumption? While one could certainly take the lack of demand of these instruments as evidence that voters do not care about certainty one could also take the very existence of the programs as evidence that voters care about uncertainty. It seems likely that voters are averse to borrowing money to pay their property taxes, which is what most of these programs require that they do. This aversion to borrowing to pay taxes should be investigated further.

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# A Appendix

The appendix features a proof of delta and variance equivalence as well as a detailed discussion of the role of assessment limits. We do not include these discussions in the main text because they are much more speculative and, at the moment, seem to detract from our main point above concerning revenue limitations.

## A.1 Delta, Variance Equivalence

**Lemma 2** *Evaluating a change in δ is equivalent in sign to evaluating a change in variance.*

**Proof.** The variance between the prices is:

$$\begin{aligned} \text{var}(p_1, p_2) &= (p_1 - \bar{p})2 + (p_2 - \bar{p})2 && (77) \\ &= \delta 2 + \delta 2 = 2\delta 2 && (78) \\ \implies \delta &= \frac{1}{2}\sqrt{\text{var}} && (79) \\ \implies \frac{\partial \delta}{\partial \text{var}} &= \frac{1}{4\sqrt{\text{var}}} && (80) \\ \implies \frac{\partial \delta}{\partial \text{var}} > 0 &\iff \text{var} > 0 && (81) \end{aligned}$$

This establishes that an increase in var is equivalent to an increase in $\delta$; hence, to evaluate the effect of an increase in variance, it is equivalent to evaluate the effect of an increase in $\delta$. ∎

## A.2 Assessment Limits and Price Variance

As of 2006, 20 states have some form of assessment limit for property taxation. Although idiosyncratic to each state, most assessment limitations designed to limit increases in the taxable value of individual pieces of real estate. Since the policies are not explicitly designed to limit variance of tax prices it is important to understand how assessment limitations might affect the variance of individual tax prices.

Assessment limitations affect the expected value of an individual's future tax price as well as its variance. If a policy lowers variance in tax price but increases its expected value voters will be more likely to oppose the assessment limitation. Of course, if a policy lowers variance in tax price and also decreases its expected value, voters will be more likely to support the assessment limitation.

### A.2.1 Simulating the Effects of Assessment Limits

In order to understand how assessment limits limit variance and affect the mean of individual tax shares it is helpful to simulate the effects of assessment limits under a different assumptions about the nature of shocks to value. We ran simulations based on the assessed values of residential properties in Oak Park, IL from the year 2000. The values of individual homes in Oak Park in 2000 imply a set of initial property tax shares. These initial values were then subjected to shocks to value over 10 periods (e.g., years). In each period new tax shares were calculated for each home. Each simulation was run 10 times and the results discussed here are the average results from the 10 simulations.

Figures $A1 - A3$ display results assuming that shock in each period is randomly drawn from a normal distribution with mean zero and variance 0.04. Each year's shock is independent of the previously year's shock and shocks do not depend on previous value of the home. There is no reason to believe that this is an accurate portrayal of housing price changes within a community. In fact, we suspect there are reasons to believe that this is not an accurate portrayal of price changes. We hope, however, that these simulations provide an example of how assessment limits might affect tax shares over time. These simulations were also run assuming independent uniformly distributed shocks across homes with similar results.[39]

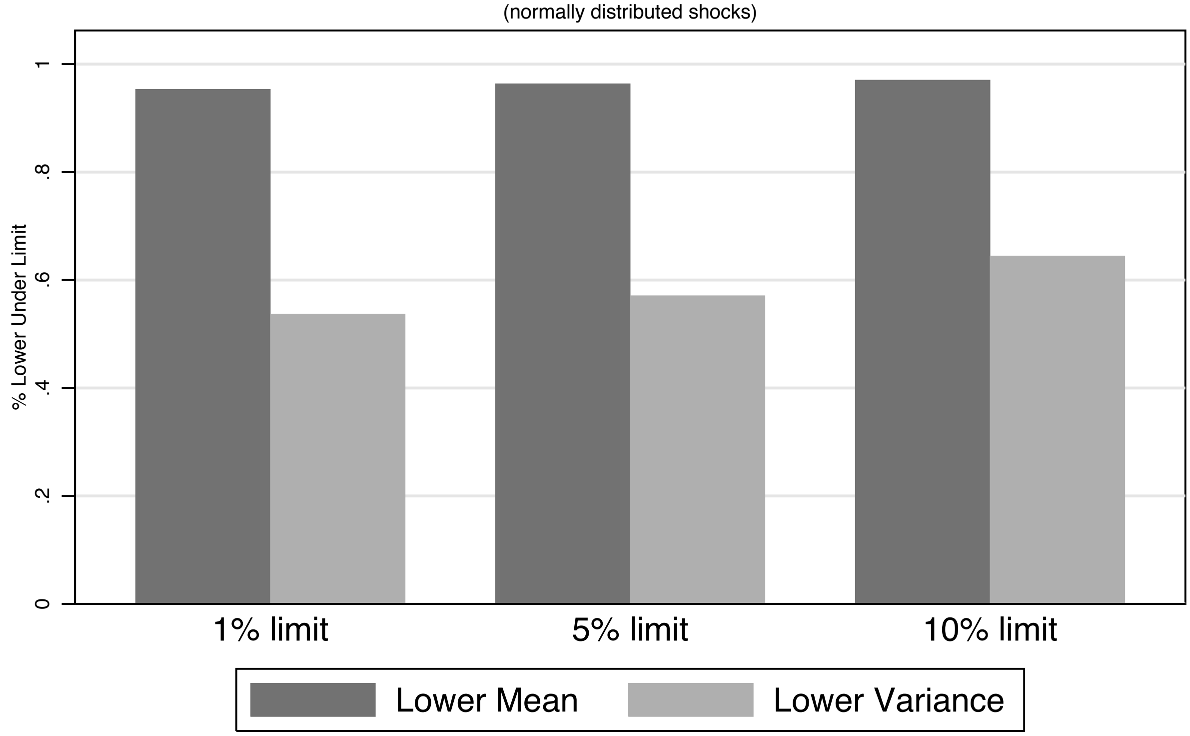
Under these very strong assumptions, Figure $A1$ demonstrates that over 90% of the homes in the simulation experience a lower average percentage change in tax price across the 10 periods with limits than without limits. Thus, for most taxpayers the limits would reduce the average magnitude of tax price changes. This figure also demonstrates that the variance of percentage changes in prices across all the periods was reduced for over half of the homes. Figure $A2$ demonstrates that within each period, the standard deviation of price changes across individuals is lower under limits than without limits. Figure $A3$ demonstrates that within each period the average percentage change in individual tax price is lower under limits than without limits.

There is limited empirical evidence on the affects of assessment limitations. Furthermore, most of the existing research focuses on so-called "tax shifts" rather than any changes in the variance of either tax shares or tax payments over time. The clearest implication of the evidence is that substantial tax-shifting does occur between classes of property. There is not much research and therefore less evidence regarding the extent of within-class shifting of statutory tax burdens. The next step is clearly to examine changes in individual tax shares over time in jurisdictions where limits exist. These changes under limits could in principal be compared to some estimation of the tax shares that would have existed if not for the limits.

[39]We've also examined simulations using an AR(1) process to generate the percentages changes to each home value. This has produces slightly different results than those discussed above. The assessment limits only effective in reducing the mean and variance for roughly half the population and the average variance in each period is not significantly affected.

Figure A1: Effects of Assessment Limits on Mean and Variance of Price Changes

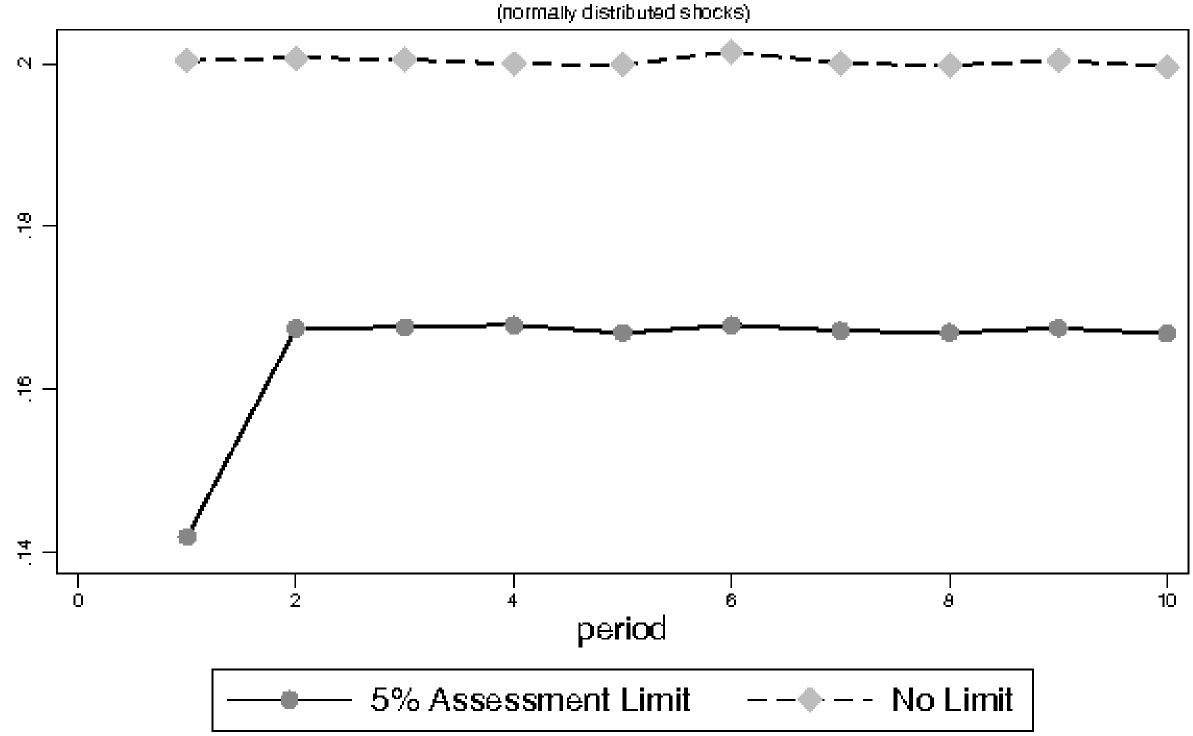
(normally distributed shocks)




Source: Author's simulations.
Annual shocks to individual real estate value are assumed to be normally distributed
distributed with mean 0 and variance 0.04.
In period 0, all real estate values are from Oak Park, IL in 2000.
The shocks begin in period 1 and are independent across time and over pieces of real estate.

Figure A2: Avg Standard Deviation In % Price Changes With and Without Limits

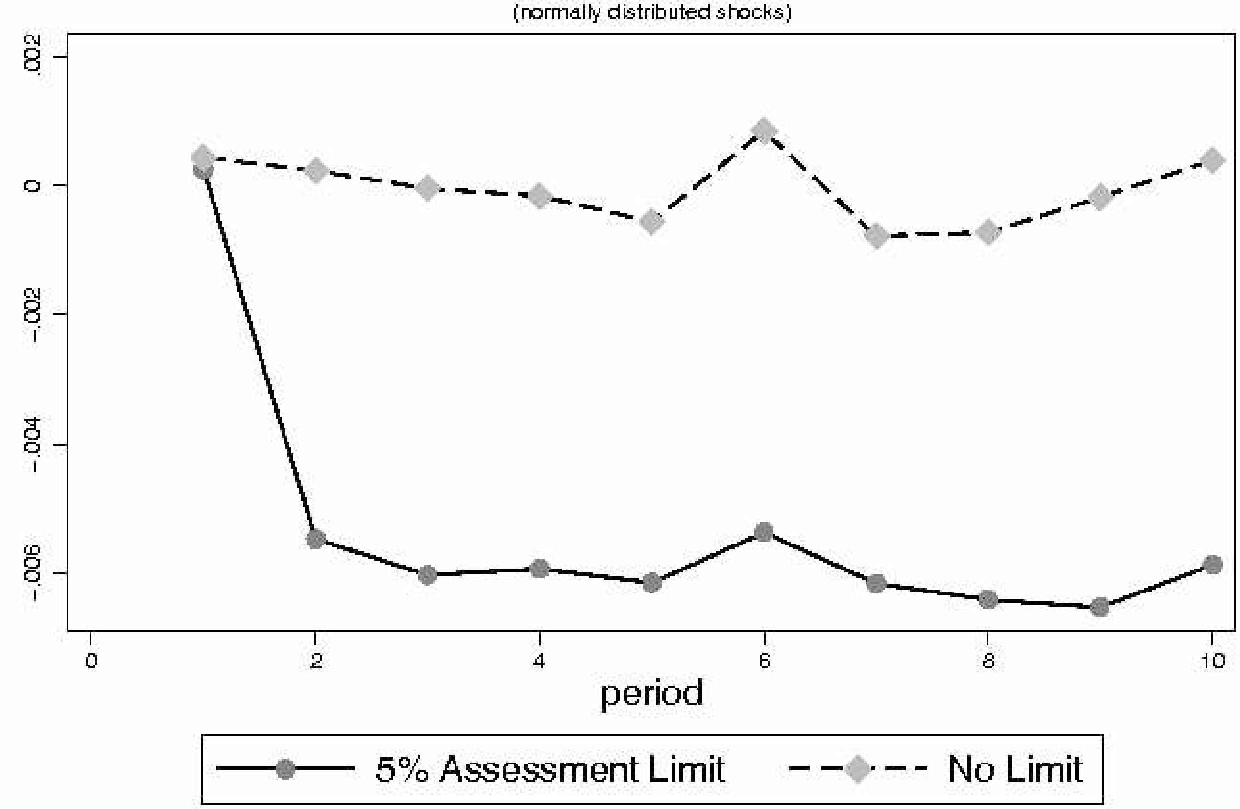
(normally distributed shocks)




Source: Author's simulations.
Annual percentage shocks to individual real estate value are assumed to be normally distributed
distributed with mean 0 and variance 0.04.
In period 0, property values are the values in Oak Park, IL in 2000.
The shocks begin in period 1 and are independent across time and over individual pieces of real estate.

Figure A3: Avg % Change in Tax Share With and Without Limits

(normally distributed shocks)




Source: Author's simulations.
Annual percentage shocks to individual real estate value are assumed to be normally distributed with mean 0 and variance 0.04.
In period 0, the property values are the property values in Oak Park, IL in 2000.
The shocks begin in period 1 and are independent across time and over individual pieces of real estate.