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## Issues in Modelling and Control of Intake Flow In Variable Geometry Turbocharged Engines

### 1 Introduction

Advanced hardware components are increasingly being considered for production of passenger car internal combustion engines to meet stricter emission regulations and customer demands of improved fuel economy and drivability. When integrated in a single engine configuration, these advanced hardware components may result in significant nonlinearities and interactions thereby requiring advanced control methods. In this paper we consider one such situation when demands for increased engine power, improved fuel economy and drivability provide a rationale for utilizing a variable geometry turbocharger while emission regulations necessitate an external exhaust gas recirculation system. However, careful modelling and analysis of the integrated system (Figure 1) uncovers difficult issues that a control designer must face in order to realize the benefits of advanced hardware components.

An automotive turbocharger consists of a compressor and a turbine coupled by a common shaft. The engine exhaust gas drives the turbine which drives the compressor which, in turn, compresses ambient air and directs it into intake manifold [5]. Since the increased quantity of air can be delivered to engine cylinders, a larger quantity of fuel can be burnt thereby providing larger torque output as compared to non-turbocharged engines. The turbocharging also improves fuel economy due to improved efficiency of engine operation at lean air-to-fuel ratios.

Turbocharging also affects regulated engine emissions such as particulates (PM), oxides of nitrogen (NO<sub>x</sub>), hydrocarbons (HC) and carbon monoxide (CO). Specifically, increased air-to-fuel ratio, air charge density and temperature resulting from turbocharging tend to reduce particulate emissions. However, increased charge temperature and oxygen availability tend to increase NO<sub>x</sub> formation. An intercooler is used to reduce air charge temperature and partially offset NO<sub>x</sub> increase caused by turbocharging. The intercooler also increases air charge density and thus reduces particulate emissions.

However, the use of intercooling alone is not sufficient to reduce NO<sub>x</sub> emissions to regulated levels. The exhaust gas recirculation (EGR) system is used to divert a portion of the exhaust gas back to the engine intake manifold to dilute the air supplied by the compressor. In the cylinders the recirculated exhaust gas acts as an inert gas and it increases the specific heat capacity of the charge. This reduces the burn rate, lowering peak flame temperatures, and, hence, decreases formation of NO<sub>x</sub>. The exhaust gas recirculation is, typically, accomplished with an exhaust gas recirculation valve that connects the exhaust manifold with the intake manifold. Some diesel engine

configurations also include an EGR throttle between the compressor and the intake manifold to lower the manifold pressure and thus create a sufficient pressure drop across the EGR valve needed for high EGR rates. A judicious selection of EGR rates is important because excessive EGR rates lead to excessive particulate emissions. The EGR cooler is used to decrease EGR temperature and thus it contributes to further NOx reduction.

A turbocharger is typically sized to a particular engine so that it provides fast airflow response when the driver demands acceleration at low engine speeds. Fast increase in fresh air charge delivered to engine cylinders allows to rapidly increase fueling rate and, hence, engine torque. To achieve this effect the turbine flow area has to be relatively small to produce fast exhaust manifold pressure rise and, hence, fast increase in power generated by the turbine in response to a fueling rate increase. At high engine speeds and loads such a turbine may result in an excessive difference between exhaust and intake manifold pressures and thus impair fuel economy, and, in extreme cases, the resulting very high value of intake manifold pressure may damage the engine. One approach that avoids negative consequences of a small turbine area at high engine speeds and loads is to use a wastegate. The wastegate is opened at high engine speeds and loads to allow some of the exhaust gas to bypass the turbine. An alternative approach which avoids using the wastegate and grants more flexibility in shaping engine torque response, improving fuel economy and reducing emissions, is to use a variable geometry turbocharger (VGT). The turbine stator of a variable geometry turbocharger is equipped with a system of pivoted guide vanes. By operating the guide vanes the turbine flow area and the angle at which the exhaust gas is directed at the turbine rotor blades can be changed. By moving the guide vanes the turbine can be optimally sized for each engine operating condition to meet the requirements of fast torque response, fuel economy, low emissions and engine safety. An excellent description of various aspects of VGT operation can be found in [3].

Hereafter, we focus on the diesel engine configuration shown in Figure 1 without the EGR throttle and heat exchangers (intercooler and EGR cooler) and with nominal injection timing. The omissions do not change the fundamental aspects of the engine behavior.

## 2 Model

The investigation relies on a mean-value engine model developed by techniques described in [1, 2, 4]. The model has seven states. The six states,  $\rho_1$ ,  $F_1$ ,  $p_1$ ,  $\rho_2$ ,  $F_2$ ,  $p_2$  represent the gas dynamics in intake manifold and exhaust manifold. Specifically,  $\rho$  stands for gas mass ( $\text{kg}/\text{m}^3$ ),  $F$  for burnt gas fraction, and  $p$  for pressure (kPa). The subscript 1 identifies the intake manifold and the subscript 2 identifies the exhaust manifold. The burnt gas fractions,  $F_1$  and  $F_2$ , are defined as the density fraction of the inert combustion products in their mixture with air for the intake manifold and for the exhaust manifold, respectively. They are used to account for the amount of

fresh air and burnt gas recirculated back to the engine. Because of lean combustion, as much as half of the flow through the EGR valve may be fresh air that can participate in combustion and, if properly accounted for, can be used to burn additional fuel. The seventh state is the turbocharger speed,  $N_{tc}$  (rpm). Very high turbocharger speeds (up to 200 krpm) are not unusual for medium size diesel engines. The control inputs are the EGR valve position  $\chi_{egr}$  (between 0 and 1, where 1 is fully open) and the VGT actuator position  $\chi_{vgt}$  (between 0 and 1, where 1 is fully open). Other external inputs include the fueling rate,  $W_f$  (kg/hr), and the engine speed,  $N$  (rpm).

The plant model is represented by the following equations which follow from the fundamental laws of mass and energy conservation for intake and exhaust manifolds and from the torque balance on the turbocharger shaft:

$$\begin{aligned}
\dot{\rho}_1 &= \frac{1}{V_1} (W_{c1} + W_{21} - W_{1e}), \\
\dot{F}_1 &= \frac{W_{21}(F_2 - F_1) - W_{c1}F_1}{\rho_1 V_1}, \\
\dot{p}_1 &= \frac{\gamma R}{V_1} \left( W_{c1}T_{c1} + W_{21}T_2 - W_{1e}T_1 - W_{12}T_1 - \frac{\dot{Q}_1}{c_p} \right), \\
\dot{\rho}_2 &= \frac{1}{V_2} (W_{e2} - W_{2t} - W_{21} + W_{12}), \\
\dot{F}_2 &= \frac{W_{e2}(F_{e2} - F_2)}{\rho_2 V_2}, \\
\dot{p}_2 &= \frac{\gamma R}{V_2} \left( W_{e2}T_{e2} - W_{2t}T_2 - W_{21}T_2 + W_{12}T_1 - \frac{\dot{Q}_2}{c_p} \right), \\
\dot{N}_{tc} &= \frac{30^2 c_p}{\pi^2 I_{tc}} \left( \frac{\eta_{tm} W_{2t} (T_2 - T_{tout}) - W_{c1} (T_{c1} - T_{amb})}{N_{tc}} \right).
\end{aligned} \tag{1}$$

The variables in the right-hand side of these equations are either constant parameters or can be expressed as nonlinear functions of the seven states and inputs.

Specifically,  $W$  stands for a mass flow rate (kg/sec) where the first subscript identifies the flow upstream location while the second subscript identifies the flow downstream location. The subscript  $c$  stands for compressor,  $t$  for turbine,  $e$  for engine, 1 for intake manifold and 2 for exhaust manifold. The backflow through the EGR valve is represented by  $W_{12}$  and either  $W_{21} = 0$  or  $W_{12} = 0$ . We do not model the backflow through either the compressor or the turbine since during normal operation these backflows do not occur.

The temperatures (K) of the flows are denoted by  $T$  with two subscripts and the same convention as for the flows. The temperature,  $T_{tout}$ , is the turbine outlet temperature. The temperatures inside intake and exhaust manifolds are denoted by  $T_1$  and  $T_2$ , respectively. Due to low gas velocities the differences between static and dynamic pressures and temperatures are neglected.

The heat transfer rate to the surroundings for the intake manifold is denoted by  $\dot{Q}_1$  and for the exhaust manifold it is denoted by  $\dot{Q}_2$ . The heat transfer effects in the intake manifold can be neglected,  $\dot{Q}_1 = 0$ . Because of high temperatures the heat

transfer effects in the exhaust manifold can be quite significant. However, we still may use  $\dot{Q}_2 = 0$  in Equations (1) because the temperature of the flow out of the engine,  $T_{e2}$ , is specified by a static map that already accounts for heat transfer effects in steady-state. Indeed, this map is developed from the experimental data for the gas already in the exhaust manifold. A better approximation is to use a transient heat correction term that is zero in steady-state, introduced e.g. by modelling exhaust manifold wall temperature dynamics.

The constant parameters include the volumes ( $\text{m}^3$ ) of intake manifold,  $V_1$ , and of exhaust manifold,  $V_2$ , the specific heats at constant pressure and constant volume (kJ/kg/K),  $c_p$  and  $c_v$ , and their ratio and difference,  $\gamma = \frac{c_p}{c_v}$ ,  $R = c_p - c_v$ , the turbocharger inertia ( $\text{kg m}^2$ ),  $I_{tc}$ , and the turbocharger mechanical efficiency (between 0 and 1),  $\eta_{tm}$ , which accounts for turbine power losses due to friction. Strictly speaking, both temperature and composition affect  $c_p$ ,  $c_v$ ,  $R$  and  $\gamma$ . However, this dependence is relatively weak and may be treated as an uncertainty. The ambient pressure and temperature are denoted by  $p_{amb}$  and  $T_{amb}$ .

We summarize the dependencies of the intermediate variables in the right hand side of Equations (1) on state variables and inputs variables in Table 1. Some of these dependencies are obtained by fitting steady-state experimental engine mapping data with appropriate nonlinearities while others follow from physics. Similar static maps are employed to represent engine brake torque and emissions (smoke, PM, HC, CO and NOx). Engine speed is treated as a time-varying parameter.

We now make several observations pertinent to variable geometry turbine modelling. The lack of space prevents us from going further into details. The experimental data indicated that the variable geometry turbine mass flow rate,  $W_{2t}$ , is not a strong function of the turbocharger speed,  $N_{tc}$ . Therefore, this dependence was omitted (see Table 1) and a modification of the orifice flow equation with effective flow area, modelled a polynomial function of  $\chi_{vgt}$ , was used to calculate  $W_{2t}$ . The turbine isentropic efficiency map, used to calculate  $T_{tout}$  however, does depend on  $N_{tc}$ .

The engine model also needs to account for engine cycle delays. For example, the change in fueling rate does not instantaneously affect the temperature of the exhaust gas leaving the engine since it takes one engine event until the exhaust valve opens. The engine cycle delays are engine speed (and, hence, time) dependent and they can be important for low engine speeds (e.g. at idle) and are less important for medium and high engine speeds. The delays can be included by using finite-dimensional approximations or their effect can be approximately captured by slightly increased exhaust manifold volume.

### 3 Control Objectives

A control system for a diesel engine must meet driver torque demand while satisfying constraints on emissions. There are two types of emission constraints. The first type are pointwise-in-time constraints on smoke emissions arising from customer re-

quirements that no visible smoke emissions can be tolerated. The second type are cumulative over time constraints on NOx, HC and particulate emissions that need to be met during the government emission testing cycles (e.g. FTP cycle in the US and the Euro cycle in Europe). Visible smoke can be avoided by keeping the air-to-fuel ratio sufficiently lean. The treatment of constraints on NOx, HC and particulates is more complex because they are integral constraints only on a certain class of trajectories of the closed loop system. Neither emissions nor torque can be measured in production vehicles due to the cost and reliability issues associated with sensors.

It is important to understand the main steps in the diesel engine controller development. First, a static map is developed that provides the demanded steady-state fueling rate,  $W_f$ , as a function of  $N$  and the driver’s pedal position. The analysis of the emission cycles yields optimal steady-state set-points for  $\chi_{egr}$  and  $\chi_{vgt}$  for each  $N$  and  $W_f$ . These set-points may also be uniquely defined by specifying the values of two internal process variables instead of  $\chi_{egr}$ ,  $\chi_{vgt}$ . If sensors or estimators for these internal variables are available, a feedback controller can be designed that generates commands for  $\chi_{egr}$  and  $\chi_{vgt}$  to force these two internal variables to follow the set-points. The use of internal variables for feedback often assures better robustness properties of the engine operation. Typically, diesel engines are equipped with sensors for intake manifold pressure,  $p_1$ , and compressor mass flow rate,  $W_{c1}$ , and the feedback on these two variables is used. During transients caused, for example, by the driver’s pedal tip-in the main objective of the control system is to provide sufficient fresh air charge to the engine as soon as possible so that the fuel demanded by the driver can be burnt without causing visible smoke. Lack of fresh air causes the control system to limit the fueling rate and is responsible for undesirable “turbo-lag” or sluggish diesel engine torque response. The secondary objective during transients is to provide the engine, whenever possible, with a sufficient amount of recirculated burnt gas to reduce NOx emissions. Large tracking errors may be admissible and, in fact, may be required during transients to meet these objectives.

## 4 Plant Properties

We use the diesel engine model to exhibit important properties of VGT/EGR diesel engine dynamics. These observations may be useful early on in the control design process when selecting an appropriate control system configuration.

First, note that for each quadruple  $(N, W_f, \chi_{egr}, \chi_{vgt})$  there exists a unique equilibrium of the diesel engine model which is asymptotically stable. Figure 2 shows the equilibrium values of  $p_1$ ,  $W_{c1}$ ,  $F_1$ , in-cylinder air-to-fuel ratio  $AFR = W_{1ea}/(W_f)$ ,  $W_{1ea} = (1 - F_1)W_{1e}$ , and  $p_2$  as functions of  $\chi_{egr}$  and  $\chi_{vgt}$  for  $N = 2000$  rpm,  $W_f = 5$  kg/hr. Both  $F_1$  and  $AFR$  are important because they affect engine emissions. Increasing the value of  $F_1$  tends to reduce NOx emissions while small values of  $AFR$  lead to excessive smoke and high levels of particulate emissions. Opening the EGR valve increases  $F_1$  but decreases  $AFR$  in a monotonic way. On the other hand, the depen-

dence of  $AFR$  and  $F_1$  on  $\chi_{vgt}$  is nonmonotonic: For a fixed  $\chi_{egr}$ , smallest values of  $F_1$  result when  $\chi_{vgt}$  is approximately 0.5. Typically,  $AFR$  decreases when  $F_1$  increases. Since a feedback controller relies on sensor measurements of  $p_1$  and  $W_{c1}$  it is of interest to examine equilibrium values of these variables. The effect of  $\chi_{egr}$  on steady-state values of  $p_1$  and  $W_{c1}$  is monotonic throughout the operating region: increase in  $\chi_{egr}$  causes  $p_1$  and  $W_{c1}$  to decrease. An increase in  $\chi_{vgt}$  causes reduction in  $p_1$ . Hence, VGT can act as a wastegate and prevent overboosting the engine at high fueling rates. Note that  $p_2$  is well-correlated with  $p_1$  and the difference  $p_2 - p_1$  increases when  $p_1$  increases. Hence, opening the VGT for high fueling rates may improve fuel economy by reducing pumping losses associated with large values of the difference  $p_2 - p_1$ . Closing the VGT for low fueling rates helps maintain a larger pressure ratio across the EGR valve and thus increase the exhaust gas recirculation and reduce NOx emissions. The effect of  $\chi_{vgt}$  on steady-state values of  $W_{c1}$  is nonmonotonic when the EGR valve is large open: when VGT is almost closed, an increase in  $\chi_{vgt}$  causes  $W_{c1}$  to increase (point “b”) while when VGT is almost completely open an increase in  $\chi_{vgt}$  causes  $W_{c1}$  to decrease (point “c”). If the EGR valve is almost closed but  $\chi_{vgt}$  is the same as for point “b”, an increase in  $\chi_{vgt}$  causes  $W_{c1}$  to decrease (point “a”). Since the regions where this dc-gain reversal takes place are uncertain, the analysis suggests that to avoid possible loss of stability it is best not to use the VGT to track setpoints in  $W_{c1}$  but use the EGR valve for this purpose.

To understand the dynamic properties of the plant, consider the Bode magnitude plots in Figure 3 for a plant linearization at four operating points specified by the quadruples  $(N, W_f, \chi_{egr}, \chi_{vgt})$ : (a) [1000, 2, 0.8, 0.2]; (b) [3500, 7, 0.8, 0.2]; (c) [2500, 5, 0.8, 0.2]; (d) [2500, 5, 0, 0.2]. The outputs have been scaled as  $\bar{W}_{1ea} = 60W_{1ea}$ ,  $\bar{W}_{c1} = 60W_{c1}$  and  $\bar{p}_1 = p_1/100$ . In all cases the same amount of fuel is injected per single engine stroke. Comparing cases (a), (b) and (c) we verify that the dynamics become faster as the engine speed increases. In case (a), the slowest eigenvalue is at  $-0.26$ , in case (b) it is at  $-2.34$ , and in the case (c) it is at  $-1.44$ . The dynamics slow down when the EGR valve opens up. In case (d) the slowest eigenvalue is at  $-2.28$  as compared to  $-1.44$  for case (c). From Figure 3 we see that at low engine speeds (point (a)) the EGR valve can increase fresh air flow to the engine,  $W_{1ea}$ , at a much slower rate than at higher engine speeds (point (b)), and that at low engine speeds the VGT can increase  $W_{1ea}$  more rapidly than the EGR valve. Consequently, a coordinated action of EGR valve and VGT is beneficial in the low speed region to rapidly increase fresh airflow in response to a driver’s tip-in.

From Figure 3 it is obvious that we are dealing with a highly nonlinear plant that is also strongly coupled. For example, the dc-gains of the plant with inputs  $(\chi_{egr}, \chi_{vgt})$  and outputs  $(\bar{W}_{c1}, \bar{p}_1)$  at the four operating points are,

$$H_a = \begin{bmatrix} -0.56 & 0.68 \\ -0.11 & -0.05 \end{bmatrix}, H_b = \begin{bmatrix} -1.41 & 2.53 \\ -0.30 & -1.07 \end{bmatrix}, H_c = \begin{bmatrix} -0.80 & 1.27 \\ -0.11 & -0.68 \end{bmatrix},$$

$$H_d = \begin{bmatrix} -2.27 & -1.26 \\ -0.50 & -1.25 \end{bmatrix}.$$

Note that the dc-gain from VGT to  $\bar{W}_{c1}$  changes its sign from point (c) to point (d), consistent with the previous discussion. The relative gain array analysis at operating points (b), (c), (d) suggests that if a decentralized control architecture is desired, EGR valve should be used to control  $W_{c1}$  and VGT should be used to control  $p_1$ . However, at the operating point (a) the relative gain array analysis suggests that the preferred pairing is to use EGR valve to control  $p_1$  and VGT to control  $W_{c1}$ .

Because the plant is strongly coupled the existence of right-half plane zeros should not be surprising. Specifically, the transfer function from  $\chi_{egr}$  to  $p_1$  has a right-half plane zero while the transfer function from  $\chi_{vgt}$  to  $W_{c1}$  has a right-half plane zero when EGR valve is closed or nearly closed. These nonminimum phase properties can be explained from physics, by demonstrating that the step response exhibits an undershoot. When the EGR valve opens, first the flow through the EGR valve increases thereby increasing the intake manifold pressure,  $p_1$ . However, since a smaller portion of the exhaust gas is supplied to the turbocharger and the exhaust manifold is emptied at a higher rate when the EGR valve opens, the intake manifold pressure, eventually, decreases. The latter effect is, however, delayed because of the turbocharger dynamics. Similarly, when the VGT opens instantaneously, the flow through the VGT increases instantaneously thereby increasing the power transferred to the compressor and the compressor mass flow rate,  $W_{c1}$ . Since the exhaust manifold is emptied at a higher rate, eventually, the exhaust manifold pressure will decrease causing the power transferred to the compressor and the compressor mass flow rate to drop. If the EGR valve opening is sufficiently small, the compressor mass flow rate drops to a steady-state value that is lower than the initial one. However, when the EGR valve is large open and we open the VGT, the final value of the compressor mass flow rate might be larger than the initial one and the response will no longer exhibit an undershoot but it will exhibit an overshoot. The MIMO system with inputs  $(\chi_{egr}, \chi_{vgt})$  and outputs  $(\bar{W}_{c1}, \bar{p}_1)$  can also be shown to be nonminimum phase. For the four operating points the system has a single right-half plane zero at (a) 68.97, (b) 62.0, (c) 73.19, (d) 64.7. A physics-based explanation for this nonminimum phase property is also available, thanks to M. Jankovic. Suppose a controller has been designed that holds  $p_1$  and  $W_{c1}$  fixed, the system is initially at an equilibrium and there is an instantaneous increase in the exhaust manifold pressure  $p_2$ . To instantaneously compensate for possible increase in  $W_{c1}$  and  $p_1$  the controller is forced to close EGR valve and VGT. But this leads to a further increase in  $p_2$ . Therefore the dynamics consistent with fixed  $p_1$  and  $W_{c1}$  are unstable and the system is nonminimum phase.

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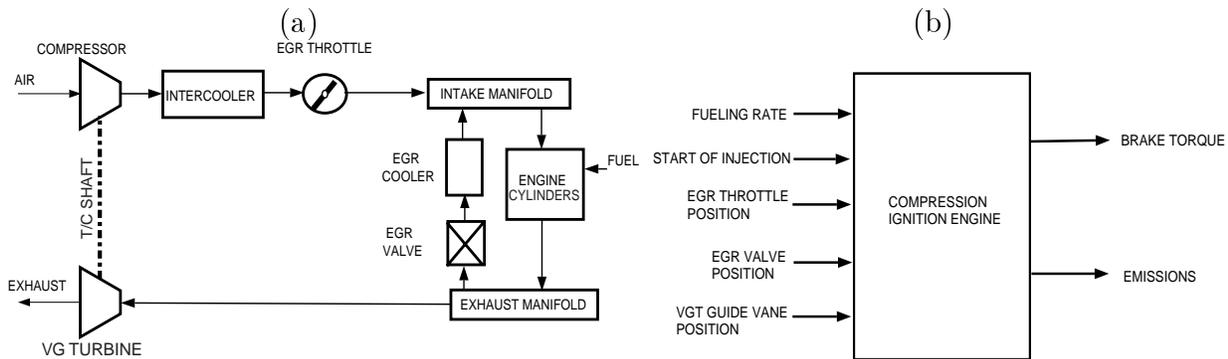


Figure 1: VGT diesel engine: (a) Main subsystems. (b) Inputs and outputs.

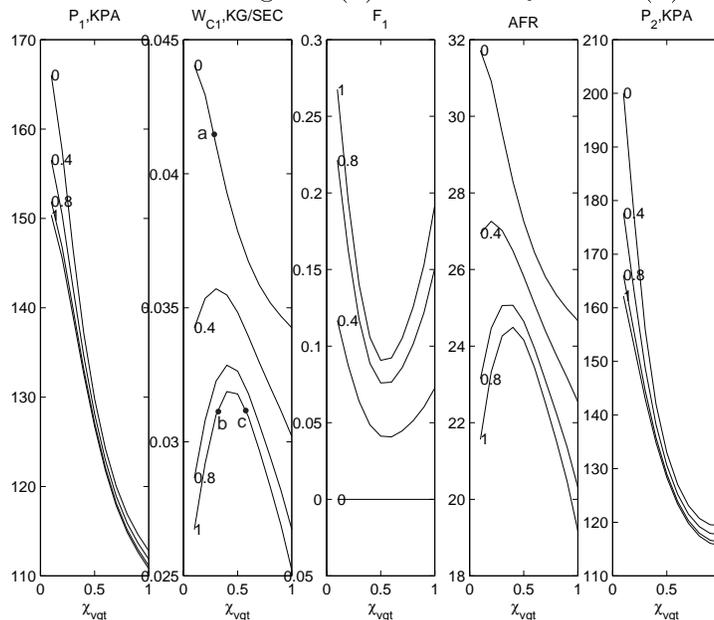


Figure 2: Equilibria for  $N = 2000$  rpm,  $W_f = 5$  kg/hr. Each line corresponds to constant  $\chi_{egr}$  (shown on the plot) and varying  $\chi_{vgt}$ .

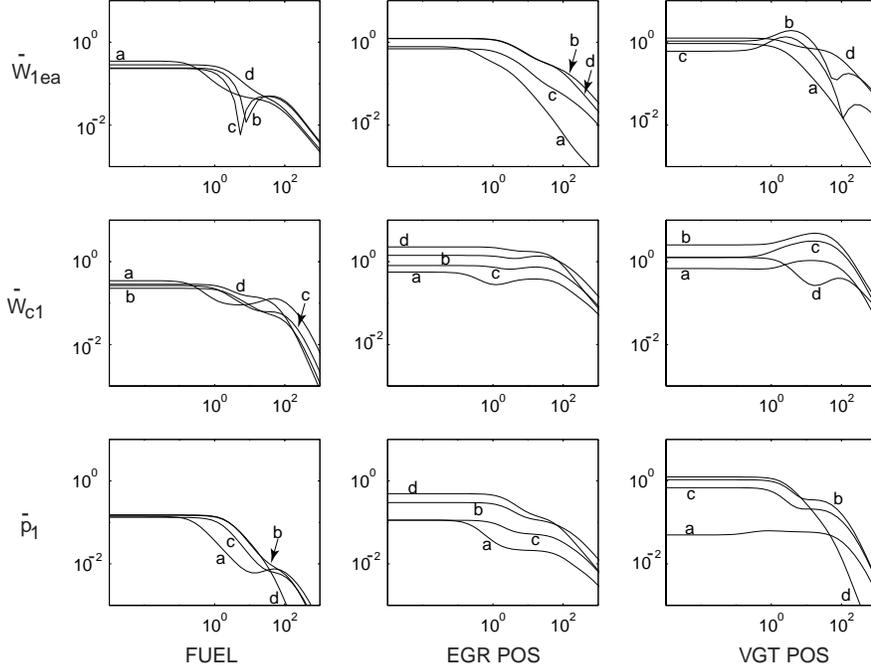


Figure 3: Bode magnitude plots at four operating points.

Variable	Depends as/on	Source
$T_1$	$T_1 = p_1/(\rho_1 R)$	ideal gas law
$T_2$	$T_2 = p_2/(\rho_2 R)$	ideal gas law
$T_{c1}$	$p_1, N_{tc}$	compressor isentropic efficiency map
$T_{tout}$	$p_2, T_2, N_{tc}, \chi_{vgt}$	turbine isentropic efficiency map
$W_{c1}$	$p_1, N_{tc}$	compressor flow map
$W_{21}$	$p_1, p_2, \rho_2, \chi_{egr}$	orifice equation
$W_{12}$	$p_1, p_2, \rho_1, \chi_{egr}$	orifice equation
$W_{2t}$	$p_2, \rho_2, \chi_{vgt}$	turbine flow map
$W_{1e}$	$\rho_1, N, T_1, p_2$	engine volumetric efficiency map
$W_{e2}$	$W_{e2} = W_f + W_{1e}$	engine mass conservation
$T_{e2}$	$T_1, F_1, W_f, W_{1e}$	engine temperature rise map
$F_{e2}$	$F_1, W_f, W_{1e}$	stoichiometric combustion balance
$\dot{Q}_1$	$\dot{Q}_1 = 0$	neglected
$\dot{Q}_2$	$\dot{Q}_2 = 0$	heat transfer is partially accounted for in the map that specifies $T_{e2}$

Table 1: Static maps used in diesel engine model.