

# Improving Turbocharged Diesel Engine Operation with Turbo Power Assist System

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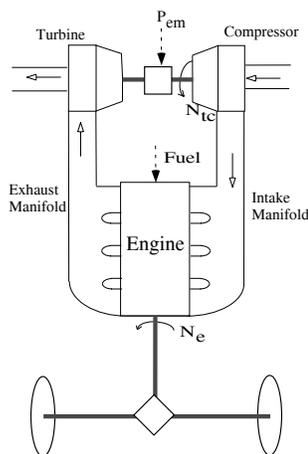
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## Abstract

The paper investigates improvements in the turbocharged diesel engine transient response that are possible when a turbocharger power assist system, consisting, for example, of an electric motor and a battery, is coupled to the turbocharger shaft. The method of investigation relies on formulating and solving numerically an appropriate minimum time optimal control problem. Comparison with a conventional turbocharged diesel engine reveals the mechanism by which acceleration improvements are attained while maintaining high fuel efficiency and equivalent smoke emission levels. A feedback controller that generates responses similar to optimal is also presented.

## 1 Introduction

The idea of coordinating a heat engine with a power assist system, such as an electric motor and a battery, is commonly encountered in hybrid vehicles. In parallel hybrid applications, the power assist system may supply power directly to the crankshaft as required to meet driver and accessory torque demand. In this paper we investigate the coupling of a power assist system at the turbocharger shaft of a diesel engine. Figure 1 shows the assisted turbocharger configuration, where  $P_{em}$  denotes the supplemental power.



**Figure 1:** Schematic representation of a diesel engine with a turbocharger power assist system.

A turbocharger power assist system (TPAS) is any device capable of bi-directional energy transfer to the tur-

bocharger shaft and energy storage. During engine acceleration from low engine speed and load conditions the TPAS can be used to add a positive torque on the turbocharger shaft thereby forcing a rapid increase in the turbocharger shaft speed. Faster response of the turbocharger results in faster increase of the fresh air charge delivery to the engine. Since the amount of fuel that can be burnt in a diesel engine without generating visible smoke is limited by the amount of the fresh air charge, faster rise in the air charge delivery results in the improved diesel engine acceleration performance. In particular, an important transient performance characteristic of turbocharged engines, the so called *turbo-lag*, can be reduced. At higher engine speed and load conditions the TPAS can be used to absorb and store in a battery some of the excess energy provided by the exhaust gas to the turbocharger, essentially, emulating a wastegate and preventing engine overboost.

Examples of a turbocharger power assist system include an electrically actuated *Dynacharger* manufactured by TurboDyne, Inc., [7] and a *TurboGenerator* manufactured by the Allied Signals [1]. The Allied Signals have also reported the use of a similar device for a turbocharger in a fuel cell application [4]. As these references suggest the electric motor technology is mature enough to enable the operation at very high motor speeds as would be required for turbocharging applications. In [2] the use of an electric generator integrally coupled with the turbocharger shaft is proposed as a way of generating additional power for the vehicle electric applications.

The focus of this paper is on the optimization of the acceleration performance of a turbocharged diesel engine with an electric TPAS and the comparison in terms of achievable acceleration performance with the conventional vehicle. Specifically, we seek to gain understanding of supplemental power/energy addition issues in transient operation without reference to specific properties of the devices that are used for this energy addition. Consequently, an ideal situation is considered whereby the motor and battery energy addition/regeneration efficiencies are neglected.

The organization of the paper is as follows. In Section 2 we provide a brief description of the system model and introduce the basic notations. In Section 3 we formulate an optimal control problem for a vehicle with the TPAS that aims at minimizing the time that it takes for this vehicle in a fixed gear to reach a specified final ve-

locity. Fixed gear acceleration tests are often used to characterize car “highway passing” performance or a “launch” performance from idle, see e.g. [6]. The fixed gear acceleration scenario eliminates the complexity associated with the optimization of gear selection and allows us to focus on the engine/TPAS optimal coordination. The minimum time problem is solved under two constraints. The first constraint is on the maximum power that can be delivered or absorbed by the TPAS. The second constraint is that the total energy spent by the TPAS over the acceleration interval must be zero. The solution approach relies on the reduction of the problem to a finite-dimensional mathematical programming problem that is solved numerically. The TPAS is shown to improve car acceleration performance with better or equivalent fuel consumption. This conclusion is reached even though all the energy spent by the TPAS is regenerated during the same acceleration event. In Section 4 we describe some of the suboptimal strategies for operating the TPAS that are based on the use of a feedback controller. Concluding remarks are made in Section 5.

## 2 Preliminaries

### 2.1 Powertrain model

The study is based on a mean-value model of a turbocharged diesel engine described in [5]. To focus on the main issues involved with a relatively simple model we assume zero exhaust gas recirculation (EGR) and a fixed position of the vanes of the variable geometry turbocharger. The dynamics of the intake and exhaust manifold, assuming zero EGR, can be described by four states: the gas pressures  $p_1$ ,  $p_2$  (kPa) and the gas densities  $\rho_1$ ,  $\rho_2$  (kg/m<sup>3</sup>). The fifth state is the turbocharger rotor speed,  $N_{tc}$  (rpm), and the sixth state is the engine crankshaft speed,  $N_e$  (rpm). The engine speed  $N_e$  is determined with an augmented vehicle dynamics model which generates the load torque on the engine crankshaft from aerodynamic and rolling resistance forces and known gear ratio; then the engine acceleration is proportional to the difference between the engine brake torque and the load torque. If the gear ratio is fixed the vehicle speed,  $N_v$ , is proportional to the engine speed. The state vector is defined as

$$x = [ \rho_1 \quad p_1 \quad \rho_2 \quad p_2 \quad N_{tc} \quad N_e ]^T. \quad (1)$$

The cycle averaged fueling rate,  $W_f$ , (kg/hr) is generated by the control system and we let

$$r \triangleq W_f. \quad (2)$$

The power supplied by the TPAS to the turbocharger shaft,  $P_{em}$  (kW), is a control input to the system and we let

$$u = P_{em}.$$

The power is absorbed from the turbocharger shaft if  $u$  is negative. Hence,

$$\dot{N}_{tc} = \frac{P_t - P_c + u}{I_{tc} N_{tc}}, \quad (3)$$

where  $P_t$  (kW) is the power generated by the turbine,  $P_c$  (kW) is the power consumed by the compressor and  $I_{tc}$  is the turbocharger inertia (in appropriate units). The supplemental power directly affects  $N_{tc}$  (Eq. (3)) which, in turn, affects the mass air flow rate and the rate of change of pressure in the intake manifold. The maximum power that can be delivered or absorbed by the TPAS is constrained by a constant value. From the fundamental laws of mass and energy conservation for the intake and the exhaust manifolds and from the torque balance on the turbocharger shaft and on the crankshaft we obtain the equations for the engine and the vehicle in the following general form

$$\dot{x} = f(x, u, r), \quad (4)$$

where  $f$  is a vector function.

### 2.2 Smoke-limited acceleration

The generation of visible smoke emissions must be avoided during the vehicle acceleration. This is achieved by restricting the fueling rate when not enough air is available for combustion without visible smoke generation. If  $W_{f,req}$  is the fueling rate requested by the driver through the pedal depression then the actual fueling rate delivered to the engine,  $W_f$ , is

$$W_f = \min\left\{\frac{W_{1e}}{\phi_s}, W_{f,req}\right\}. \quad (5)$$

In (5):  $W_{1e}$  is the engine intake airflow (proportional to  $\eta_{vol} N_e p_1$ , where  $\eta_{vol}$  is the engine volumetric efficiency), and  $\phi_s$  is a value of the in-cylinder air-to-fuel ratio that guarantees smoke-free combustion. In this study a conservative constant value of  $\phi_s = 25$  has been assumed. For the maximum vehicle acceleration demand the operation on the fueling rate limiter is assumed:

$$r(t) = W_f(t) = \frac{W_{1e}(t)}{\phi_s}. \quad (6)$$

The TPAS can affect the car acceleration performance through its effect on (i)  $N_{tc}$  and, hence,  $p_1$  thereby increasing  $W_{1e}$  and allowing more fuel to be burned in the engine without generating visible smoke; (ii) the pressure difference between the exhaust and intake manifolds  $p_2 - p_1$  that determines the pumping losses and affects the engine brake torque  $\tau_e$ . As a secondary effect, the TPAS also affects the engine volumetric efficiency  $\eta_{vol}$  that depends on the intake and exhaust manifold pressures; the changes in  $\eta_{vol}$  translate into changes in  $W_{1e}$  and affect the car acceleration performance.

## 3 Optimal Acceleration with TPAS

### 3.1 Minimum-time problem formulation

The optimization objective is to minimize the time that it takes for the vehicle, in a fixed gear, to reach a specified final velocity subject to the constraint that the total energy consumption by the TPAS does not exceed zero. That is, all the energy spent by the TPAS has to be regenerated and returned back to the battery by

the end of the acceleration period so that the battery is never depleted. Since the engine speed is proportional to the vehicle speed when the gear ratio is fixed, this is equivalent to maximizing the engine speed. The acceleration is assumed to take place with the fuel limiter active so that (6) applies. Mathematically, the problem is formulated as

Minimize  $J(u, T) \triangleq T$  subject to

$$u \in C_{[0, T]}^0, \quad |u(t)| \leq u_{max}, \quad 0 \leq t \leq T, \quad (7)$$

$$r(t) = \frac{W_{1e}(t)}{\phi_s}, \quad (8)$$

$$\int_0^T u(t) dt \leq \bar{E}_{max} = 0, \quad (9)$$

$$g(u, T) \triangleq N_e(T) - N_e^d = 0, \quad (10)$$

$$\dot{x}(t) = f(x(t), u(t), r(t)), \quad x(0) = x^0. \quad (11)$$

Here  $x^0$  is the initial equilibrium, e.g. corresponding to the engine in neutral idle when  $N_e(0) = N_e^0$ ,  $r = r^0$ ,  $u = 0$ ;  $u_{max}$  is an upper limit on the TPAS power and  $N_e^d$  is the desired final engine speed that corresponds to the desired final vehicle velocity.

### 3.2 Reduction to a finite-dimensional optimization problem

We first recast the problem as a fixed-time problem by re-scaling time,

$$\tau = \frac{t}{T}, \quad 0 \leq \tau \leq 1.$$

Then, (11) becomes

$$\frac{dx}{d\tau} = Tf(x, u, r), \quad x(0) = x^0. \quad (12)$$

The basic idea is to parametrize  $u$  within a finite dimensional class of functions and optimize the finite number of parameters in this parametrization. We use linear B-splines in this parametrization that are defined as

$$\phi_0(\Delta; \tau) = \begin{cases} 1 - \frac{|\tau|}{\Delta}, & |\tau| \leq \Delta, \\ 0, & \text{otherwise,} \end{cases}$$

$$\phi_k(\Delta; \tau) = \phi_0(\tau - k\Delta), \quad k \in Z^+.$$

Let  $\Delta > 0$  and an integer number  $n$  such that  $1 = n\Delta$ . We parametrize  $u$  as

$$u(\tau) = \sum_{i=0}^n \alpha_i \phi_i(\Delta; \tau), \quad 0 \leq \tau \leq 1. \quad (13)$$

Note that the coefficients in this parametrization are precisely the values of  $u(\tau)$  at the time instants  $\Delta i$ ,  $i = 0, \dots, n$ . This property facilitates the development of iterative algorithms where  $n, \Delta$  can be changed (e.g. to obtain better accuracy) and the solution obtained in the previous iteration is used as an initial

guess for the next iteration. In this situation, linear B-splines allow to rapidly move from coefficients of the old parametrization to the coefficients of the new parametrization. Furthermore, to simulate the model in Matlab/Simulink it is only necessary to supply the values of  $\alpha_i$ ,  $i = 0, \dots, n$  as a vector in Matlab/Simulink while the interpolation (13) is done automatically by the Matlab/Simulink.

Consequently, we have reduced the problem to a finite dimensional optimization problem where we optimize the  $(n + 1)$  parameters  $\alpha_i$ . The constraint  $u \in C_{[0, T]}^0$  is satisfied automatically due to our choice of the parametrization. The constraint  $|u| \leq u_{max}$  translates into  $|\alpha_i| \leq u_{max}$ ,  $i = 0, \dots, n$ . The constraint (9) becomes  $T\Delta \sum_{i=1}^{n-1} \alpha_i \leq \bar{E}_{max}$ . The constraint  $g(u, T)$  is evaluated by simulating the model (12) over the time-interval  $0 \leq \tau \leq 1$ .

### 3.3 Numerical optimization procedure

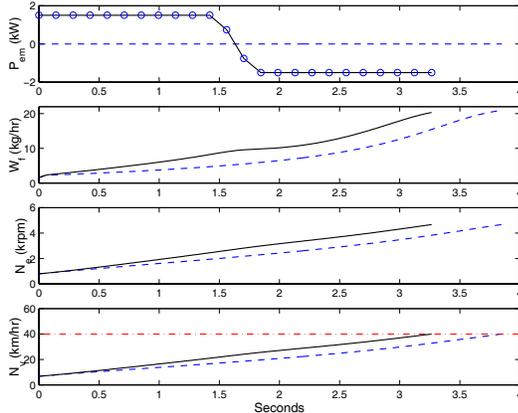
The numerical optimization was performed using `constr.m` function of the Matlab optimization toolbox which is based on an SQP algorithm for constrained optimization. Special routines have been written to provide the gradients of the objective functions and constraints to the optimization function. The difficult part is to calculate the derivatives of  $g$  (defined in (10)) with respect to  $\alpha_i$ 's and  $T$ . The procedure is fairly standard but cumbersome (see [3]) and involves the following steps: (i) evaluating the forward trajectory of the system (12) for the specified  $\alpha_i$ 's,  $T$  and  $x_0$ ; (ii) integrating the adjoint equations backward in time along the forward trajectory of the system with the initial condition equal to the sensitivity of  $g$  to changes in the final state  $x(1)$  evaluated for the forward trajectory; (iii) evaluating weighted convolution integrals that involve the adjoint variables and the basis functions  $\phi_i$  with matrix weights that depend on the forward trajectory of the system. The linearized equations required to formulate the adjoint equations have been obtained symbolically from the nonlinear system equations.

### 3.4 Numerical optimization results

In this section we apply the optimization procedure to two scenarios of acceleration requests. The first scenario reflects the ‘‘launch performance’’ requirements associated with acceleration from idle in the first gear. The second scenario reflects ‘‘highway passing’’ performance associated with acceleration in the third gear.

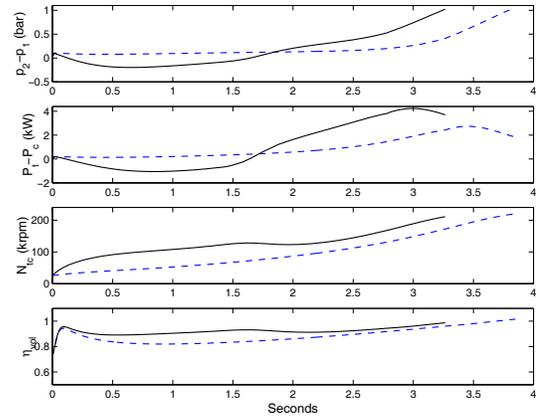
In the first scenario we consider the vehicle acceleration from neutral idle conditions and we initialize accordingly the model states and inputs. The idle conditions are determined by fixing the engine speed value at  $N_e^0 = 800$  rpm, and setting the fueling rate value,  $W_f$ , to fully balance the frictional losses in the engine. We selected  $n = 23$  in the parametrization of  $u$  with linear B-splines. Based on preliminary information about an experimental TPAS device we selected the maximal power limit as  $u_{max} = 1.5$  kW. The operation of the TPAS was optimized as to minimize the time for the vehicle in the first gear to reach the final velocity of

40 km/hr under the constraint that the net energy expenditure by the TPAS is zero. The optimization (initialized with different sets of initial values for  $\alpha_i$ 's) converged to trajectories shown in Figures 2-3. The circles superimposed on the  $P_{em}$  trajectory indicate the values of  $\alpha_i$ . The optimized trajectory for  $P_{em}$  achieves the acceleration time of 3.27 seconds for the vehicle with the TPAS as compared to 3.85 seconds for the conventional vehicle (i.e. with the TPAS turned off). At the beginning of the acceleration the TPAS supplies the energy to the turbocharger shaft, thereby helping the engine to accelerate since (i) more fuel can be burnt without generating visible smoke due to increased values of the intake manifold pressure  $p_1$  and the volumetric efficiency  $\eta_{vol}$  and (ii) the pumping losses (that are proportional to the difference  $p_2 - p_1$ ) are reduced. At higher engine speeds and loads the TPAS absorbs some of the energy from the turbocharger, essentially, acting as a wastegate. All energy spent by the TPAS is regenerated during the same acceleration event so that the battery depletion is avoided. Since the TPAS reduces the pumping losses at the beginning of the acceleration (i.e. at low engine speeds) where the diesel engine efficiency is low, the total fuel consumption is improved. It is 8.55 gram for the vehicle with the TPAS as compared to 8.76 gram for the conventional vehicle. This is an improvement of 2.4 percent.



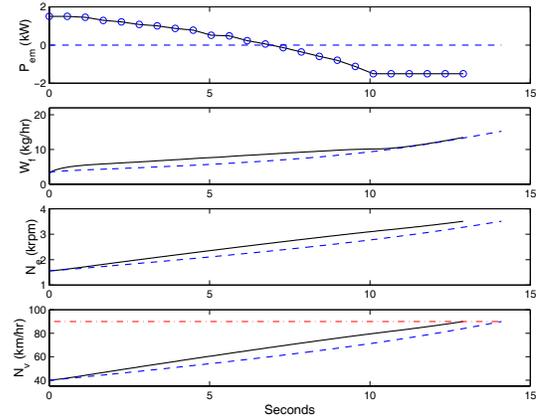
**Figure 2:** Time histories of TPAS power, fueling rate, engine speed and vehicle speed for the acceleration in the first gear: operation with TPAS optimized for minimum time acceleration (solid) and operation with TPAS turned off (dashed).

For the second scenario the optimization was repeated to treat the acceleration in the third gear. The vehicle starts from a steady-state cruise condition at 40 km/hr and accelerates to 90 km/hr. The acceleration time with the TPAS was 12.91 seconds as compared to 14.10 seconds for the conventional vehicle, see Figures 4-5. The qualitative features of the optimized  $P_{em}$  trajectory are the same as for the case of acceleration in the first gear. The fuel consumption of the vehicle with the TPAS was 30.27 gram as compared to 30.26 gram for the conventional vehicle. The fuel economy advantage of the TPAS is lost because the engine acceleration is confined to a medium engine speed range



**Figure 3:** Time histories of manifold pressure difference, turbine and compressor power difference, turbocharger speed and volumetric efficiency for the acceleration in the first gear: operation with TPAS optimized for minimum time acceleration (solid) and operation with TPAS turned off (dashed).

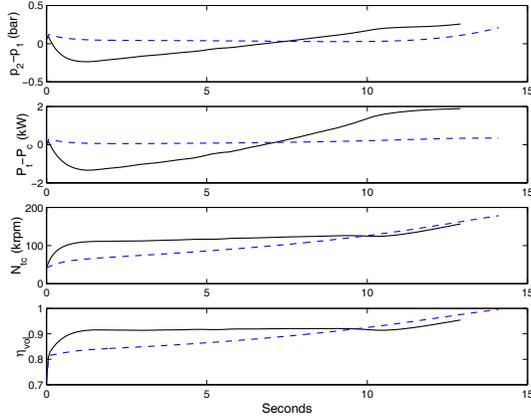
and the pumping losses reduction no longer improves the engine efficiency as much as for the first gear case where the acceleration was from a low engine speed value.



**Figure 4:** Time histories of TPAS power, fueling rate, engine speed and vehicle speed for the acceleration in the third gear: operation with TPAS optimized for minimum time acceleration (solid) and operation with TPAS turned off (dashed).

#### 4 Feedback Controller

Previous developments indicate that the optimal TPAS strategy requires electric energy addition during the first part of the acceleration period and electric energy regeneration during its later part. A feedback implementation that immitates and approximates the suggested optimal electrical and heat energy coordination is now presented. The following feedback law realizes both the transient objective of fast engine torque response and the steady-state objective of net zero energy consumption by the TPAS.



**Figure 5:** Time histories of manifold pressure difference, turbine and compressor power difference, turbocharger speed and volumetric efficiency for the acceleration in the third gear: operation with TPAS optimized for minimum time acceleration (solid) and operation with TPAS turned off (dashed).

First, consider a situation when the fueling rate to the engine is being severely limited at a tip-in due to an initial lack of air, i.e.  $W_f < W_{f,req} - \Delta_1$ , where  $W_f$  is the actual fueling rate,  $W_{f,req}$  is the fueling rate requested by the driver through the pedal depression and  $\Delta_1$  is a threshold parameter. Then, a simple Proportional-plus-Integral feedback controller driven by the intake manifold pressure error can be used to prescribe the power,  $P_{em}$ , to be delivered by the TPAS:

$$P_{em}(t) = u_1(t) \triangleq \text{sat} \left( k_p(p_{1,c} - p_1(t)) + k_i \int_0^t (p_{1,c} - p_1(\tau)) d\tau \right). \quad (14)$$

In (14):  $p_1$  is the intake manifold pressure,  $p_{1,c}$  is the set-point for the intake manifold pressure,  $k_p > 0$  is the proportional gain,  $k_i > 0$  is the integral gain and the function  $\text{sat}(\cdot)$  enforces the maximum TPAS power limits. The integral action is implemented with an anti-windup that does not update (i.e. clips) the integrator state when the saturation limits are reached.

Once the strategy (14) for the TPAS combined with the actual turbocharger and manifold filling dynamics results in meeting the driver's fueling rate demand, i.e.  $W_{f,req} - W_f < \Delta_2$ ,  $0 < \Delta_2 < \Delta_1$ , then a separate strategy can be enacted to regenerate the energy. This strategy ensures that the total electric energy consumption in response to an increase in  $W_{f,req}$  is zero:

$$P_{em}(t) = u_2(t) \triangleq \text{sat} \left( -k_{em} \int_0^t P_{em}(\tau) d\tau \right), \quad (15)$$

where  $k_{em} > 0$  is a gain and where the integrator state is not updated (i.e. clipped) if the saturation limits are reached.

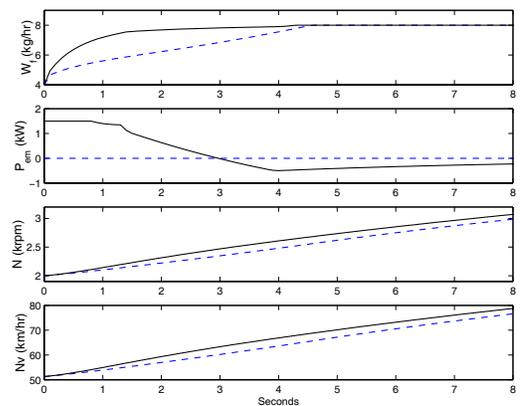
In the transition interval,  $\Delta_1 \geq W_{f,req} - W_f \geq \Delta_2$ , an interpolation between the values of  $u_1$  and  $u_2$  can be

performed so that

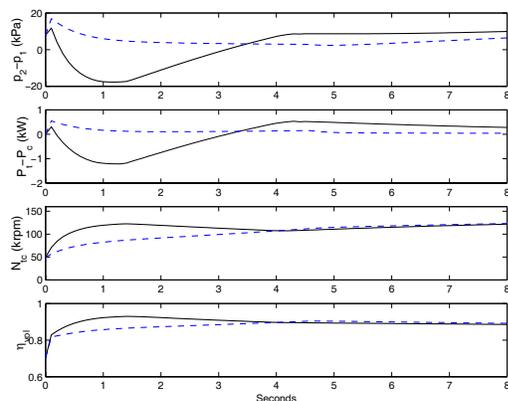
$$P_{em}(t) = u_1(t)\phi(\chi(t)) + u_2(t)(1 - \phi(\chi(t))), \quad (16)$$

where  $\chi = (W_{f,req} - W_f - \Delta_2)/(\Delta_1 - \Delta_2)$ ,  $0 \leq \phi(\chi) \leq 1$ ,  $\phi(\chi) = 0$  if  $\chi \leq 0$ ,  $\phi(\chi) = 1$  if  $\chi \geq 1$ . When  $\chi \leq 0$  the integrator state of the controller (14) is clipped.

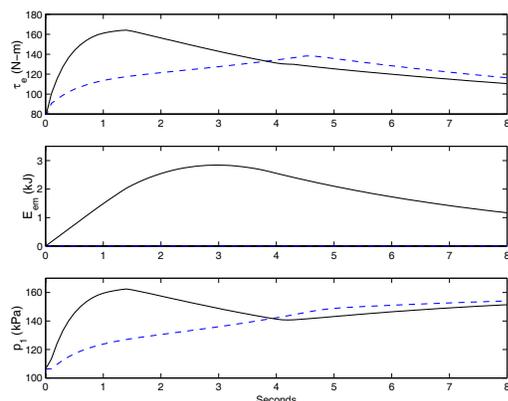
We evaluate the response of this controller to a step in the requested fueling rate quantity,  $W_{f,req}$ , from 1 kg/hr to 8 kg/hr applied to the vehicle in the third gear (see Figures 6-8). The values  $\Delta_1 = 0.16$ ,  $\Delta_2 = 0.1$ ,  $\phi(\chi) = 1 - \chi^{1.1}$ ,  $0 \leq \chi \leq 1$  were used,  $p_{1,c}$  was set to a sufficiently large value, and the saturation limits for  $|P_{em}|$  were set at 1.5 kW. The initial condition was the engine equilibrium corresponding to  $W_f = 1$  kg/hr,  $N = 2000$  rpm,  $P_{em} = 0$ . The fueling rate and the engine torque responses are, clearly, much faster with the TPAS and the controller enabled than without the TPAS. Note that, initially, the fueling rate jumps from 1 kg/hr to about 4 kg/hr due to a sufficient amount of air already available in the engine intake manifold to support this increase in the engine fueling rate. The ‘‘jump’’ in  $W_f$  leads to an initially similar looking responses of the engine torque and of the manifold pressure difference  $p_2 - p_1$  for both cases with and without the TPAS. The action of the TPAS becomes transparent in the engine responses somewhat later. It results in the increased airflow to the engine and, hence, enables the increased delivery of fuel to the engine. It also results in a smaller manifold pressure difference  $p_2 - p_1$  and reduced pumping losses. These two factors contribute to a much faster engine torque response with the TPAS and a smaller turbo-lag. When the difference  $W_{f,req} - W_f$  is sufficiently reduced the controller (15) is gradually phased in to regenerate the energy. The action of this controller cannot be very aggressive (i.e.  $k_{em}$  cannot be very large) to avoid a significant drop in the engine torque. Hence, the action of (15) extends over a sufficiently large time period as the net energy consumed by the TPAS,  $E_{em}$ , asymptotically approaches zero, see Figure 8.



**Figure 6:** Time histories of fueling rate, TPAS power, engine speed and vehicle speed due to a step in requested fueling rate in the third gear: operation with TPAS and feedback controller (solid) and operation with TPAS turned off (dashed).



**Figure 7:** Time histories of manifold pressure difference, turbine and compressor power difference, turbocharger speed and volumetric efficiency due to a step in the requested fueling rate in the third gear: operation with TPAS and feedback controller (solid) and operation with TPAS turned off (dashed).



**Figure 8:** Time histories of engine torque, total energy spent by the TPAS and intake manifold pressure due to a step in the requested fueling rate in the third gear: operation with TPAS and feedback controller (solid) and operation with TPAS turned off (dashed).

## 5 Concluding Remarks

The development of operating strategies for hybrid powertrains often proceeds on a steady-state basis. That is, the appropriate regions in terms of engine speed and demanded powertrain torque are identified where the ancillary power is applied or the energy is regenerated. In this paper a different approach has been taken whereby both the application of the ancillary power and energy regeneration take place over a single *transient* event. For the specific application of a Turbocharger Power Assist System (TPAS) we have shown that the turbo-lag reduction can be attained even though the total energy spent by the TPAS during an acceleration event is zero.

It is also possible to interpret the benefits of the TPAS in terms of smoke and particulate emission reduction. Specifically, by adding a TPAS and optimally control-

ling its operation in an existing vehicle, lower particulate and smoke levels can be achieved with no deleterious effects on vehicle acceleration. This is possible due to operation at higher values of the air-to-fuel ratio for the vehicle equipped with the optimized TPAS. That is, by adding a TPAS to an already existing powertrain this powertrain can now accelerate at the same rate with less smoke and particulate emissions due to operation at higher values of the air-to-fuel ratio.

Finally, we make a remark about the value of optimal control methods for system development. The proper selection of system configuration and components is an important task facing the automotive engineers. This selection has to be made to meet multiple and sometimes conflicting requirements such as emissions, fuel economy, driveability and cost. When the transient performance of the system is of importance the decision about which one of several configurations is better can be based on solving an appropriate dynamic optimal control problem. The specific application treated in this paper is thus an example of a wider spectrum of problems that can be treated using similar techniques.

## 6 Acknowledgements

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