Abstract—In this paper, a natural gas fuel processor system (FPS), a proton exchange membrane fuel cell (PEM-FC) and a catalytic burner (CB) are integrated in a combined heat and power (CHP) generation plant. The FC provides power based on the electrochemical reaction of hydrogen (H2). The FPS generates the hydrogen from natural gas and the CB provides the energy for preheating the FPS inlet flows by burning any excess H2 from the FC exhaust. The coupling of these three systems poses a challenging optimization and control problem. The goal is to analyze the open loop dynamics and design a controller that achieves optimal steady state operation and acceptable transient performance, i.e., mitigates the H2 starvation and regulates reactor temperatures. We show in simulations that an observer based feedback controller, which relies on temperature measurements of two reactors, speeds up the transient response fourfold, as compared to the baseline when no feedback control is employed.

Index Terms—Fuel processor, fuel cells, dynamics, modeling, feedback control.

I. INTRODUCTION

FUEL cells can utilize fuels like methane, ethane or diesel instead of pure hydrogen, but require a fuel processor system (FPS) to convert those fuels into a hydrogen-rich flow. Fuel conversion to hydrogen is commonly achieved either through oxidation or steam reforming. Fuel processor systems have great potential both for mobile and stationary usage including vehicles, marine and generator applications.

In addition to the benefits offered by implementing fuel cell and fuel processor technology, such as cleaner power generation, flexible design, silent operation and low thermal acoustic signature, higher efficiency can be achieved by incorporating a catalytic burner. For steady state operations, as much as 16% efficiency improvement can be achieved with the CHP systems over the stand alone fuel cell power generation systems [1].

The CHP system, depicted in Fig. 1a, consists of a proton exchange membrane fuel cell (PEM-FC) stack that generates electric power, a fuel processing system (FPS) that converts a hydrocarbon fuel to a H2 rich mixture to be fed to the fuel cells, and a CB that recuperates any excessive H2 leaving the fuel cells to provide the preheating energy for the reformer. Other auxiliaries, such as heat exchangers (HEX), are also integral parts of the CHP system. The complicated configuration and closely coupled dynamics, together with the large thermal inertia and therefore slow transients associated with the fuel processor, impose a very challenging control problem. In Fig. 1b, the externally preheated FPS (EFPSS) is also shown, where there is no H2 recirculation loop and the heat is provided by burning additional fuel directly in the CB.

In order to achieve the overall maximum efficiency, the integrated CHP system has to work very close to its operating boundary as shown later in this paper. This optimal setpoint selection, however, puts the system in a very vulnerable position during load1 transitions. When a load is suddenly applied, the CHP system may not be able to provide sufficient H2 and heat quickly enough to sustain the operation under the new load condition, thereby leading to temporarily H2 starvation2 or system shutdown. These problems can be avoided by slowing down the current drawn from the fuel cell through a rate limiter or a load governor [2]. The load deficit in this case can be provided through hybridization [3], with an additional electrical power source which will increase cost and complexity. Another typical approach that does not rely on hybridization is to increase the hydrogen production, in order to provide excess hydrogen than that required, and thus lower the FC utilization but also the CHP efficiency.

Operating at optimal steady state and thus capitalizing the benefits of the CHP system will be made possible only if the controlled system responds very fast to FC load changes. Otherwise, one has to resort to sub-optimal setpoints, i.e., to trade efficiency for improved safety margin. In this paper we design a controller that speeds-up the natural CHP dynamics and supplies hydrogen to the fuel cell while maintaining optimum reactor temperatures. The feedback controller is based on measuring reactor temperatures and estimating the spatially averaged composition of reactant flow through the series of CHP components.

II. BACKGROUND AND OVERVIEW OF THE CHP SYSTEM

A low pressure CHP system with a rated power (i.e. max. FC load) of 200kW is used as the platform for our investigation in this paper. A nonlinear, control oriented, dynamic model of the FPS is developed in order to analyze its behavior. In [4], the initial model of the fuel processor was developed. The model in [4] assumed constant inlet temperature and did not include the heat exchangers and the catalytic burner. A control study for that model was performed in [5], which yields satisfying H2 production during load transitions by utilizing the CPOX

1In this paper, the term “load” is synonymous to the current drawn from the fuel cell.

2Starvation period is defined as the time period in which the produced amount of H2 from the FPS is lower than the required from the FC.
temperature \(T^{\text{CPOX}}\) and \(H_2\) partial pressure \(p_{H_2}\) as feedback variables for the designed control scheme. The CPOX reactor model is based on \([6]\), where kinetic model simulations are employed along with experimental data to define the products of the CPOX reactor as a function of its temperature and the inlet gas composition.

A similar FPS configuration is studied in \([7]\), where a dynamic model of the system suitable for observer design and start up analysis is presented. Sommer in \([8]\) develops a dynamic model of an autothermal reformer. The effects of volume sizes on the system’s transient performance, as well as the benefit of the heat capacity of each volume that acts as a buffer during transients are analyzed. The need for precise inlet flow control can also be inferred. An explicit dynamic model for direct reforming carbonate fuel cell stack is developed in \([9]\). Using mainly thermodynamic principles and mass/energy conservation, the authors of \([9]\) develop a 10-state model suitable for transient analysis and validate it with a higher order model and experimental results. During transient operation temperature overshoot problems are observed, alike to our analysis. The authors of \([10]\) develop a model to study the performance on a steam reformer and a PEM FC based system. Using a simple rate limiter control of the input flows they also note the possibility of overheating the catalyst bed.

In \([11]\), a molten carbonate dynamic model is developed and the authors note the coupling between the load following capabilities and the input flow scheduling during a transient as well as the temperature and \(H_2\) production fluctuations. In \([12]\), after using a dynamic model to examine the transient system behavior of a 1 MW FC power plant, the authors reach the conclusion that feedback control is required to enhance the load following capabilities of the system. Finally, similar studies using dynamic FPS-FC models can also be found in \([13]\), \([14]\), \([15]\), \([16]\)

All dynamic studies of FPS-FC CHP systems mentioned in this literature review concur, explicitly or implicitly, to the need of feedback control of the inlet air and fuel flows to the system. Several authors also mention that feedforward scheduling of those flows is inadequate in order for the system to meet the load following requirements.

The CHP system utilized in this work, shown in Fig. 1a, is composed of four main reactors, namely, the hydrodesulfurizer (HDS), the catalytic partial oxidation (CPOX), the water gas shift (WGS), and the preferential oxidation (PROX). Natural gas, rich in methane \(\text{CH}_4\), is supplied to the FPS from a tank. All FPS and FC components operate at low pressures of up to 110 kPa. The HDS is used to remove the sulfur from the natural gas \([17]\), \([18]\). The main air flow is supplied to the system by a blower (BL) which draws humidified air. The air and the fuel are pre-heated in separate heat exchangers (HEX) and the fuel is desulphurized. Then, the two flows are mixed in the mixer (MIX). The mixture is then passed through the catalytic partial oxidizer (CPOX) where \(\text{CH}_4\) reacts with oxygen to produce \(\text{H}_2\). There are two main exothermal chemical reactions taking place in the CPOX: partial oxidation (POX) and total oxidation (TOX) given in \([19]\), \([6]\) with their corresponding energy released per mole of reactant \(\Delta H^0\).

\[
\begin{align*}
\text{(POX)} & \quad \text{CH}_4 + \frac{1}{2}\text{O}_2 \rightarrow \text{CO} + 2\text{H}_2 \quad \Delta H^0_{\text{POX}} = -0.036 \times 10^6 \text{ J/mol} \\
\text{(TOX)} & \quad \text{CH}_4 + 2\text{O}_2 \rightarrow \text{CO}_2 + 2\text{H}_2\text{O} \quad \Delta H^0_{\text{TOX}} = -0.8026 \times 10^6 \text{ J/mol.}
\end{align*}
\]

Hydrogen is produced only by the POX reaction while heat is mostly generated by the TOX reaction. As shown in Fig. 2, the distribution between the two is dictated by the reactor temperature \(T^{\text{CPOX}}\) and the molar ratio of \(\text{O}_2\) to \(\text{CH}_4\):

\[
\lambda_{\text{O}_2\text{C}} = n_{\text{O}_2}/n_{\text{CH}_4},
\]

where \(n_i\) is the number of moles of the species \(i\). Moreover, since the CPOX products are also highly dependent on the CPOX reactor temperature \(T^{\text{CPOX}}\), the optimum balance between the two reactions has to be determined.

Carbon monoxide (CO) is also created along with \(\text{H}_2\) in the POX reaction, as can be seen in \((1)\). Since CO poisons the PEM fuel cell catalyst, it has to be eliminated using water in the water gas shift reactor (WGS) and air in the preferential oxidizer (PROX). The latter are assumed to operate perfectly thus eliminating all the CO in the stream. The \(\text{H}_2\)-rich mixture leaving the PROX enters the anode of the fuel cell stack where the electro-chemical reaction takes place to convert \(\text{H}_2\) to electrical power. The flow from the anode is then supplied.
to the catalytic burner (CB) where the excess H₂ is burnt using the air supplied through a blower. Finally, the flow from the CB is fed to two separate heat exchangers (HEX), one to preheat the air and one to preheat the fuel before they enter the FPS and thereby increasing the overall fuel utilization of the system.

A 19 state, nonlinear, dynamic and control oriented model for the CHP system has been developed. Details on the model can be found in [1], [20], [21]. This model is employed in this work for optimization analysis, performance evaluation and control design. The dynamic states of the model are indicated inside the volumes in Fig. 3 while other important variables are also shown such as $W_f$, $W_a$, $W_{wrox}$ and $W_{H2}$.

Some important assumptions about the developed model include that all gases obey the ideal gas law and that the reactors were modeled as homogenous, lumped parameter and spatially invariant volumes. The model is not suitable for start up or shut down simulations since such dynamics were not included. Finally, the model and its operating setpoints are valid for the range of FC loads between 20 to 80% (i.e. 50-160kW or 70-250A). Within this range of loads the FC stack voltage ranges between 0.71V and 0.64V per cell with a total of 1000 0.04m² cells [21].

III. Steady State Efficiency Optimization

For the system to work efficiently in an integrated fashion, each component has to be conditioned properly in terms of its operating temperature, humidity, and pressure. This is achieved by controlling the air and fuel intakes of the FPS. The strong physical coupling of the CHP components will dictate the optimal set-points for the system.

To determine the optimal steady state operating points with respect to the overall system efficiency, the following optimization problem is formulated

$$\max_{(u_f,u_a)} \left( \eta_{CHP} = \frac{V \cdot I_{st}}{W_f \cdot Q_{LHV}} \right).$$

The objective is to maximize the overall efficiency, $\eta_{CHP}$, which is defined as the ratio of the FC electric power output $V \cdot I_{st}$ over the energy used $W_f \cdot Q_{LHV}$, where $W_f$ is the amount of fuel used and $Q_{LHV}$ its lower heating value. The optimization variables are the fuel valve command, $u_f$, and the air blower command, $u_a$, both ranging from 0 to 100%, corresponding to fully closed or fully open actuators respectively.

The gradient descent method was employed to solve the optimization problem [22]. The corresponding iterative algorithm is given as

$$u_{k+1} = u_k - a_k \cdot \nabla \eta_{CHP}(x_k) \quad (5)$$

where $a_k$ is the iteration step, $\nabla \eta_{CHP}(x_k)$ is the gradient vector which corresponds to the $\eta_{CHP}$ increasing direction and $u = [u_f \ u_a]^T$. The iteration step size, $a_k$, is kept constant until no further increasing steps can be found and is then reduced by the bisection method up to the desired accuracy.

Convergence of the gradient algorithm to a global maximum can be verified given the convex form of the efficiency map for the whole range of FC loads. An example of the efficiency map is given in Fig. 4 for the FC load of 100A. The maximum efficiency for this load is 33.6% while the optimal setpoint is $[u_f^* \ u_a^*] = [20.75 \ 29.00]$. Using the same procedure for each operating load the optimal steady state map can be determined for the actuator inputs $u^* = [u_f^* \ u_a^*]^T = f_u(I_{st})$ and other critical operating variables (ex. $T_{c/\text{POX}}^* = f_{T_{c/\text{POX}}}(I_{st})$) that can serve later as controller setpoints. The optimization
results can be approximated by the regression expressions:

\[
\begin{align*}
    u_f &= 7 \times 10^{-7} I_{st}^4 - 0.0003 I_{st}^3 + 0.0637 I_{st}^2 - 4.9581 I_{st} + 149.12 \\
    u_a &= 0.3153 I_{st} - 2.3897 \\
    W_f &= 5 \times 10^{-5} I_{st} - 0.0005 \\
    W_a &= 0.0003 I_{st} - 0.0023
\end{align*}
\]

where \( W_f \) and \( W_a \) are the corresponding optimal flows in \((\text{kg/s})\) of air and fuel when using \( u_f \) and \( u_a \). Those four curve fits on the optimal setpoints are valid for the range of FC loads between 20 and 80%.

\[\text{Fig. 4. Efficiency map of the CHP system at load } L_{st}=100A\]

As illustrated in Fig. 4, the optimal operating setpoint lies close to the operating boundary of the system. This trend is observed for the whole range of operating loads of the system\(^3\). As a result, the system is susceptible to steady state \( H_2 \) starvation when there are uncompensated loads during steady state and transient operation. To avoid modeling errors and to react fast to load variations, a combination of feedforward and feedback control is required.

It is important to point out that according to the optimization results, all the optimal operating points, independent of the load drawn from the FC, occur at \( \lambda_{O2C} = 0.69 \) and \( T^{\text{CPOX}} = 980\text{K} \). Note here that the optimization of the overall CHP efficiency (4) leads to a CPOX oxygen to carbon ratio (\( \lambda_{O2C} \)) that is greater than the value for maximum \( H_2 \) production at the CPOX reactor (\( \lambda_{O2C} = 0.5 \) as indicated in Fig. 2).

In order to explain this result, one has to notice that while both POX and TOX reactions in (1)-(2) are exothermal, the TOX releases 20 times more heat than the POX reaction. For the integrated CHP system the CPOX temperature, which is highly coupled to the \( H_2 \) production, is a function of both the heat released by the reactions inside the CPOX and the temperature of the incoming air and fuel flows which are heated by the energy supplied through the CB. Moving \( \lambda_{O2C} \) towards 0.5 will promote \( H_2 \) production but suppress the TOX reaction which only occurs for \( \lambda_{O2C} > 0.5 \). Thus the contribution of the TOX reaction to the CPOX temperature will be reduced and, as a result, the CPOX reactor has to rely on preheating the inlet flows by the CB. Since both variables (the \( H_2 \) and heat) are essential for the system to function properly, the optimal point (\( \lambda_{O2C} = 0.69 \)), reflects a balance between the \( H_2 \) production in the FPS and heat generation in the CB and the CPOX reactors for steady state operation. Thus, the overall CHP optimum cannot be defined through optimization of individual components.

Another interesting optimization result is the \( H_2 \) utilization \((U_{H_2})\) in the FC seen in Fig. 5 and defined as

\[
U_{H_2} = \frac{H_2 \text{ reacted}}{H_2 \text{ supplied}} = \frac{W_{H_2}^{\text{react}}}{W_{H_2}^{\text{supplied}}}. \tag{10}
\]

where \( W_{H_2}^{\text{react}} \) is the amount of hydrogen supplied to the FC (i.e. exiting the WROX) and \( W_{H_2}^{\text{supplied}} \) is the amount of \( H_2 \) consumed in the FC at a given load. The need for decreasing the \( H_2 \) utilization as load increases is dictated not only by the need for excess \( H_2 \) for preheating as load increases, but also by the fuel cell efficiency. At higher loads, excess \( H_2 \) promotes the fuel cell efficiency and in turn the overall system efficiency of the CHP. Indeed, the \( H_2 \) utilization in both the CHP (Fig. 1a) and the EPFPS (Fig. 1b) systems is found to be equal after dedicated EPFPS optimization was performed. Thus, the need for decreasing the utilization at high loads is mostly due to the requirement for high FC efficiency (37% at max load). This is the case for both FPS and EPFPS, independent of the \( H_2 \) recirculation loop.

\[\text{Fig. 5. } H_2 \text{ Utilization vs. FC Load}\]

The CHP system will exhibit higher efficiency and lower fuel consumption when compared against the EPFPS as shown in Fig. 6. Note that the efficiency shown in Fig. 6 is the FPS efficiency defined as

\[
\eta_{FPS} = \frac{W_{H_2}^{\text{react}} \cdot Q_{LHV}^{H_2}}{W_f \cdot Q_{LHV}^{CH}}. \tag{11}
\]

At high loads efficiency increase of up to 12% and fuel consumption decrease of up to 16% can be achieved. Thus the addition of a hydrogen recirculation CB is quite beneficial for such a fuel processing unit.

\section*{IV. Open Loop Dynamic Analysis}

Examining now the transient performance of the system using static feedforward control enables us to gain insight

\(^3\)The operating boundary (\( \eta_{CHP} = 0\% \)) is defined as the locus of points where the produced \( H_2 \) is less than that required by the FC, given a specific load at steady state operation.
on the system dynamics. The open loop system dynamics are examined by utilizing the optimal steady state setpoint maps derived from the optimization results where for a given load, the fuel and air operating setpoint are defined by the feedforward maps (eq. (6),(7)).

The open loop response of the system for two consecutive load steps is shown in Fig. 7. For the initial small step of 90-100A, the fuel processor provides the fuel cell with the required amount of H\(_2\) in order to meet the load demand. For the second larger step of 100-150A though, the H\(_2\) generation is below the demanded H\(_2\) level for a considerable period of 7 seconds. Starving a fuel cell for 7 seconds can cause power loss and membrane damage while it jeopardizes the life span of the stack [23]. Moreover, a 65 K overshoot in the CPOX temperature within 15 sec is observed, which can have damaging consequences for the CPOX reactor. Both issues are highlighted on Fig. 7. Since the feedforward maps correspond to steady state optimal operation, it is not surprising that the open loop control is inadequate in preventing H\(_2\) starvation when a large load step is applied.

In order to identify the root cause of the H\(_2\) starvation and temperature overshoot of the CHP system, we consider three critical processes that affect the generation of H\(_2\) during load changes. Analyzing those processes will provide insight in the control problem and the system design. The first critical process was found to be the CB temperature variation during a step change in load. When a step increase in load is applied, the H\(_2\) flow is depleted at a rate faster than it is produced, due to the slow time constant of the FPS. This results in reduction or even elimination of the H\(_2\) flow to the catalytic burner, which in turn results in a temperature reduction in the CB and eventually a temperature reduction of the inlet air and fuel flows. However, the thermal inertia and large time constant of the CB prevents the temperature from dropping quickly, therefore helps maintain the temperature at a level that does not affect the H\(_2\) production of the FPS severely. A comparison of the CHP system, where the CB temperature is a function of the H\(_2\) present in the anode exhaust, with an imaginary system where the CB temperature is maintained constant at a nominal value, is given in Fig. 8a and b. The two responses are almost identical, with the constant CB temperature response (Fig. 8b) exhibiting slightly less H\(_2\) starvation. Consequently, the CB temperature variation during load increase is not the main cause of the H\(_2\) starvation problem.

Furthermore, the feedforward controller, which is based on steady state optimization, places the system close to its operating boundaries and therefore makes it susceptible to H\(_2\) starvation. A suboptimal map implies increased excess fuel usage which implies increased H\(_2\) production in steady state. As a result the difference between the required H\(_2\) and the produced is expanded leading to increased safety margins at the price of reduced efficiency. In the case of a suboptimal setpoint, shown in Fig. 8c, there is an efficiency reduction from 33.5\% to 28.7\% but the severity of the H\(_2\) starvation is reduced substantially.

The third and most important cause of the H\(_2\) starvation can be attributed to reactor sizing issues. The large residence time of the HDS, due to the slow kinetics of the fuel desulphurization [24], imposes the requirement of a relatively large volume compared to the adjacent volumes of the MIX and the HEX. The combination of the large volume in the fuel path, the small volume of the air path and the large flow of air compared to the fuel flow causes the MIX pressure to build up at a rate.
faster than the HDS pressure during transients. In turn the pressure difference between these volumes initially exhibits an undershoot until the HDS pressure manages to build up again as illustrated in Fig. 9. Since flow is a function of the pressure difference ($P_{\text{hds}} - P_{\text{mix}}$), the same undershoot is observed for the fuel flow ($W_f$) which causes the oxygen-to-carbon ratio, $\lambda_{\text{O2C}}$, to overshoot.

Based on the CPOX reaction map, given in Fig. 2, an overshoot in $\lambda_{\text{O2C}}$ from its nominal steady state operating point of 0.69 to 1.15, implies a steep decrease in $H_2$ production. One way to avoid this is by decreasing the HDS volume in the model. Fig. 8d compares the response of the original system, with $V_{\text{hds}} = 0.3 \text{ m}^3$ [25], with an imaginary system that has a considerably smaller HDS volume of $0.1 \text{ m}^3$. For the latter case, the $H_2$ starvation problem disappeared. If future advances in the desulphurization process produce more compact HDS reactors, then the transient performance of the system would improve \(^4\).

A straight forward solution to the transient problems is to apply an air rate limiter since it was found that the air flow chokes the fuel flow during transients. Such a configuration is seen in Fig. 10. A rate limiter to restrict the current drawn from the fuel cell is also required since the air rate limiter alone does not eliminate the $H_2$ starvation problem due to the system dynamics. For a load step of 100 to 150A it was found that a $2\%$/sec rate limiter and a 10A/sec rate limiter for the air command and the current demand respectively, are required.

Obviously a constant rate limiter is not suitable for all load transitions. If this solution was purused, a more elaborate load governor would be required as presented in [2] or a scheduled filter rate as in [28]. However, limiting the rate of load change would slow down the system response and lead to deteriorated performance. Rate limiters or load governors are add-on mechanisms that are applied to systems whose control capabilities have otherwise been full explored.

\section*{V. Feedback Control Design & Analysis}

In this section we investigate the effectiveness of using observer based feedback control in improving the transient performance of the CHP system. In particular, our objective is to reduce the $H_2$ starvation problem and to control the CPOX temperature overshoot while maintaining efficient steady state operation by utilizing the optimized feedforward maps.

The control architecture is based on setpoint error regulation, using the setpoint maps defined in Sec. III through the plant optimization. The controller is implemented by augmenting integrators to the estimator based feedback controller. In deciding which signals need to be regulated and are best suited as feedback variables in the controller, one has to consider the control requirements (namely, $P_{\text{H}_2}$ and $T_{\text{cPOX}}$ regulation), the sensitivity of measured signals to the fuel and air actuators, as well as the ease of measuring those signals. Ideally we would choose the CPOX temperature $T_{\text{cPOX}}$ and the partial pressure of $H_2$ leaving the anode $P_{\text{H}_2}$ as the feedback variables, as they are linked directly to the control objectives [21]. Since hydrogen partial pressure $P_{\text{H}_2}$ is difficult to measure, while estimating it requires elaborate modeling of the fuel cell polarization characteristics [21], [29], we choose the CB temperature $T_{\text{cb}}$ instead. The CB temperature $T_{\text{cb}}$ is closely coupled to the $H_2$ starvation problem. During transient operation, reduction of $T_{\text{cb}}$ from its optimal steady state value $T_{\text{cb}}^*$ implies reduction of the $H_2$ leaving the anode exhaust and in turn, $H_2$ starvation. The only drawback of using $T_{\text{cb}}$ instead of $P_{\text{H}_2}$ is the slow dynamics due to the associated thermal inertia. As we show later, this drawback can be eliminated by a model based closed loop estimator that compensates for the slow $T_{\text{cb}}$ dynamics.

The CHP model developed can be expressed as a function of the states $x$, the inputs $u$ and the disturbance $w$ as

$$\dot{x} = f(x, u, w).$$

(12)

with the linear expression of the CHP plant being

$$\delta \dot{x} = A_p \cdot \delta x + B_p \cdot \delta u + B_w \cdot \delta w$$

(13)

$$\delta z = C_2 \cdot \delta x$$

(14)

where $\delta(\cdot) = (\cdot) - (\cdot)_0$. The model has 19 states (Fig. 3) and two inputs, namely the fuel and air command. The current is treated as a measured disturbance to the system.

\(^4\)Recent advances in desulphurization technology allow the use of multiple smaller HDS reactors [26], [27] which would require precise switching control
The performance variables considered are the CPOX and CB temperatures (15).

\[ u = [u_f u_a]^T, \quad w = I_{st}, \quad z = [T_{cpx}^O T_{cby}^O]^T \]  

The medium load of 100A was chosen as the linearization point. At that load, the optimization yielded steady state fuel and air input commands of 20.75% and 29% respectively.

By looking at the eigenvalues of the linearized CHP plant, a large difference was found between the smallest (−0.024786) and the largest one (−7062.4), indicating a very stiff system (\(\text{cond}(A_p) = 8.4313 \times 10^{15}\)) with potential difficulties in tuning and robustness issues. In addition the normalized condition numbers of controllability and observability matrices are very large:

\[ c_N = \frac{\text{cond}[\text{cont}(A_p, B_p)]}{\text{cntr}[\text{cont}(A_p, I_{19})]} = 1.5 \cdot 10^{10} \]  

\[ o_N = \frac{\text{cond}[\text{obs}(A_p, C_p)]}{\text{cond}[\text{obs}(A_p, C_p, I_{19})]} = 2.0 \cdot 10^{14}. \]

where \(\text{cond}, \text{cont}\) and \(\text{obs}\) indicate the condition number, the controllability matrix and observability matrices respectively.

A balanced realization [30][pp.372-376] shows that only 5 states are needed to describe the dynamics of the chosen performance variables. This plant will be referred to as the bt-plant and can be expressed as:

\[ \delta \dot{x}_{bt} = A_{bt} \delta x_{bt} + B_{bt} \delta u + B_{wbt} \delta w \]  

\[ \delta z = C_{bt} \delta x_{bt} \]

where \(\delta x_{bt} = T \delta x\). Analyzing the balanced transformation matrix \(T\) we conclude that the important original states are \(P_{hds}, T_{cby}, T_{cpx}^O, P_{hpx}^O\) and \(m_{hox}\). This was inferred by examining the rows that correspond to relatively large Hankel singular values, namely the first five rows of the transformation matrix \(T\) in this case. As expected, since we want to monitor the static and dynamic behavior of \(T_{cpx}^O\) and \(T_{cby}\), the balanced states in turn depend on \(T_{cpx}^O\) and \(T_{cby}\) as well. Moreover, the transient behavior of \(H_2\) production and thus \(T_{cpx}^O\) and \(T_{cby}\) is highly coupled with \(P_{hds}\) due to the transient effects analyzed earlier in Sec IV. Finally, the fact that \(P_{hpx}^O\) is important can be attributed to its correlation to the \(H_2\) production from the FPS and in turn to the \(T_{cpx}^O\).

Checking the condition number of the controllability and observability matrices of the bt-plant we have

\[ \text{cond}([B_{bt} A_{bt} B_{bt}; \ldots; A_{bt} B_{bt}]) = 25.265 \]  

\[ \text{cond}([C_{bt} C_{bt} A_{bt}; \ldots; C_{bt} A_{bt}]) = 43.396 \]

The step responses of the linear bt-plant and the non-linear full order plant are shown in Fig. 11. It is interesting to note that even though the DC-gain of the \(u_o\)-to-\(T_{cby}\) transfer function is zero, the transient dynamics are captured while for the rest, the responses of the bt and nonlinear plants are similar. Thus, as far as \(T_{cpx}^O\) and \(T_{cby}\) are concerned we can conclude that no dynamic (transient) or static (DC-gain) information was lost by the truncation of the original plant.

\[ A_{bt} S + S A_{bt}^T - SC_{bt}^T R_{L}^{-1} C_{bt} S + B_{bt} Q L B_{bt}^T = 0. \]

A. Controller Design

A first approach to the controller design is the derivation of full state feedback control which is designed using the LQR technique. The estimator design follows next. The estimator is based on the bt-plant (eq.(18),(19)) and is expressed as

\[ \dot{\delta \hat{x}}_{bt} = (A_{bt} - L \cdot C_{bt}) \delta \hat{x}_{bt} + [L \cdot B_{bt} B_{wbt}] \begin{bmatrix} \delta z \\ \delta u \\ \delta w \end{bmatrix}, \]

where the estimator gain is defined as

\[ L = SC_{bt}^T R_{L}^{-1} \]

and \(S\) is the solution to

\[ A_{bt} S + S A_{bt}^T - SC_{bt}^T R_{L}^{-1} C_{bt} S + B_{bt} Q L B_{bt}^T = 0. \]

with process noise covariance \(Q_L\) and the measurement noise covariance \(R_L:\)

\[ Q_L = I_{(m \times m)} + 100 \cdot B_{bt} B_{bt}^T, \quad R_L = 100 \cdot I_{(2 \times 2)} \]

The estimator based linear control law is

\[ \delta u = -K_p \cdot \delta \hat{x}_{bt} - K_I \cdot q \]

with \(q\) being the integrator states of \(T_{cpx}^O\) and \(T_{cby}\) defined as

\[ \delta \dot{q} = z - z^* = \begin{bmatrix} T_{cpx}^O - T_{cpx}^O^* \quad T_{cby} - T_{cby}^* \end{bmatrix}^T \]

where \(T_{cpx}^O^*\) and \(T_{cby}^*\) are the desired steady state points as obtained in Sec. III. The control gain for the augmented bt-plant is

\[ [K_p, K_I] = PB_{aug}^T R_K^{-1} \]

and \(P\) is the solution to the Riccati equation

\[ A_{aug}^T P + PA_{aug}^T - PB_{aug}^T R_K^{-1} B_{aug} P + Q_K = 0. \]

The weighting matrices \(Q_K\) and \(R_K\) used for the controller inputs and the outputs respectively are

\[ Q_K = \text{diag}([10 0.5 1 0.1]), \quad R_K = \text{diag}([10 10]) \]

The implementation of the control law for the nonlinear plant is modifed to

\[ u = u_w - K_p \cdot (\delta \hat{x}_{bt} + T_{(m \times n)}(x_o - x_w)) - K_I \cdot q \]
where \( T_{n \times n} \) denotes the first \( m = 5 \) rows of the \( n \times n \) transformation matrix \( T \) (where \( n = 19 \)) and \( x_w, u_w \) satisfy the steady state (12) as

\[
f(x_w, u_w, w) = 0
\]

with \( u_w \) being the optimal steady state operating setpoint as defined by the optimization. Note that when implementing the linear control law to the nonlinear CHP plant, the \( (\cdot)_w \) terms are given by a feedforward map as a function of the load. At the linearization load of 100A, \( x_w=100 = x_o \) and \( u_w=100 = u_o \), and \( x_{blo} = T_{x_o} \). Using (31) has several advantages over using constant setpoints. Since at steady state both \( u - u_o \) and \( x - x_o \) go to zero, \( q \) has to go to zero as well. Thus, even if the feedforward \( u_w \) and \( x_w \) maps are not perfect, it will take a longer time for the integrators to become saturated. Compared to \( u - u_o \) and \( x - x_o \), another advantage is the initial step at the time of the load step that comes through the feedforward maps, which helps speed up the transient performance of the system. A schematic of the application of the estimator based controller to the nonlinear model is given in Fig. 12.

![Fig. 12. Estimator based controller applied to nonlinear plant](image)

**B. Performance Evaluation of the closed loop system**

The feedback controller manages to improve the transient performance of the CHP system as shown in Fig. 13. It overshoots the fuel and slows down the air command initially, in order to regulate \( \lambda_{O_{2C}} \), around its optimal value and reduce the undershoot of fuel flow that was observed during open loop operation. As a result, the \( H_2 \) production increases smoothly and the \( H_2 \) starvation problem is reduced.

Given the need for overshooting the fuel to overcome the fuel flow chocking, it is critical for the fuel actuator (i.e. the fuel valve) to react very fast. Therefore, it is important to choose the controller tuning matrices based on the individual fuel actuator dynamic characteristics in order to fully exploit the actuator capabilities.

Furthermore, the CPOX reactor temperature overshoot is substantially reduced. Application of the observer based feedback controller yields a small overshoot which is negligible compared to the open loop performance where \( T^{\text{cpox}} \) overshoots to 1060 K within 15 sec.

![Fig. 13. Comparison of Estimator FB, Open Loop (OL) and State FB Performance](image)

It is important to note that the performance achieved with the proposed controller, that utilizes only temperature measurements, is comparable to the performance of the controller developed in [31] for the FPS-FC system, that utilizes \( T^{\text{cpox}} \) and \( y_{H_2} = P_{H_2}^{\text{an}} / P_{H_2}^{\text{an}} \) (i.e. \( H_2 \) partial pressure sensor). The latter measurement is significantly faster than \( T^{\text{cpox}} \), since it involves pressure dynamics instead of temperature but is only available for experimental investigations. Thus, implementing a CB into an FPS, besides increasing significantly the steady state efficiency, also provides an indirect measurement of the \( H_2 \) starvation that is easy to measure and can be utilized to control the transient response.

Finally, as shown in Fig. 13, even thought the \( H_2 \) starvation problem is substantially alleviated compared to the open loop case, there is still a short period of hydrogen starvation of about 0.9 sec. Obviously, the \( H_2 \) production, due to the system dynamics (mainly due to the volumes involved), cannot follow the desired step response. Thus, a load rate limiter is still required to eliminate \( H_2 \) starvation.

In combination with the designed estimator feedback controller, a very fast rate limiter of 40 A/sec is adequate to mitigate the \( H_2 \) starvation problem completely, while a 10 A/sec was required for the open loop case. Thus, with feedback control, the maximum load transition speed was increased by a factor of 4.

**C. Robustness Evaluation of the closed loop system**

A well recognized problem for CPOX-FPS systems is CPOX reactor clogging due to carbon build up and deformation caused by excess temperature. Risk of CPOX clogging due to carbon formation is increased when reforming diesel or gasoline fuels, given their increased carbon concentration, but is still an issue when reforming natural gas. Deformation of the CPOX catalyst can easily occur if the CPOX temperature
an efficiency boost of up to 16% and fuel economy of up to 12%. In addition, the CB serves as a H₂ sensor to the system. During transients the CB temperature variations exhibit the same trends with the H₂ flow entering the CB and can be used as a feedback control variable. A feedback controller is designed to mitigate the transient problems identified. Using only reactor temperature measurements, namely the CPOX and CB temperatures, as inputs to the feedback controller, it is shown that satisfactory transient performance can be achieved with the closed loop control schemes. Fast \( \lambda_{O2C} \) regulation and a fourfold increase in H₂ production rate is observed during closed loop FC load transitions compared to the open loop operation.

VII. Future Work

The control design and dynamics analysis reported in this paper can be further leveraged to explore other issues associated with CHP systems, such as controller tuning and online adaptation. The insights developed in this paper will also facilitate several other studies including the development of adaptive control schemes to compensate for CPOX clogging and sulphur build up. Control of more generic CHP systems which will include both electric and heat loads will also be explored as part of our future research.

REFERENCES


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TABLE I

H₂ STARVATION PERIOD (SP) AND Tₘₐₓ CPoX VS. CPOX OUTLET ORIFICE SIZE REDUCTION DURING A 100-150A LOAD STEP

exceeds the meltdown temperature of the catalyst or backbone material (1000-1100 K), which is caused by \( \lambda_{O2C} \) increase [32].

Clogging of the CPOX reactor leads to increased CPOX pressure drop. Given the low operating pressure of the system examined in this work, even small increase in the CPOX reactor pressure drop can increase the H₂ starvation problem during a transient. In order to simulate the CPOX clogging scenario using the model developed in this work, the CPOX outlet orifice is decreased up to 30% and the corresponding starvation period is recorded.

In Table I those results are shown for the CHP system both for the closed and the open loop CHP configuration during a 100-150A FC load step. As far as H₂ starvation, it can be seen the closed loop control scheme exhibits satisfactory performance for up to 30% valve reduction while the open loop scheme is not able to produce enough H₂ when the valve reduction is over 8%. At 30% reduction the feedback (FB) control scheme exhibits almost the same period of hydrogen starvation as the open loop scheme exhibits at 0% orifice reduction. As far as \( T_{max}^{CPoX} \) overshoot is concerned, the FB control scheme exhibits an acceptable value (less than 2%) for CPOX orifice reduction up to 30%. We can conclude that the robustness of the designed FB scheme is satisfactory against CPOX clogging.

VI. CONCLUSIONS

The integration of a fuel processor system with a proton exchange membrane fuel cell and a catalytic burner was examined in this work. The system reforms methane to a hydrogen rich gas to be utilized in the fuel cell. The steady state operating setpoints that yield the maximum overall efficiency of the system are determined for a wide range of FC loads. Utilizing those setpoints the open loop transient characteristics of the CHP system are analyzed. It is shown that the open loop transient issues are strongly coupled to the size of the desulphurizer (HDS). The significant pressure drop of the fuel flow across the HDS reactor degrades the transient performance. Due to the relatively slow dynamics along the fuel path associated with the HDS, \( \lambda_{O2C} \) deviates from its steady state value during a transient. In turn, this causes a large overshoot in the CPOX reactor temperature and an undershoot in the H₂ production flow rate during open loop operation.

Our analysis shows that implementing a CB in the FPS system does not degrade the transient performance but provides...


