# Ph-507. Homework 8 (due: Wednesday, April 16).

**PROBLEM 8-1 (15 pts.)**

Perform numerical study of the pendulum subjected to time-dependent torque:

$$H = \frac{p^2}{2} + \varpi_0^2 \cos\theta + \varepsilon\theta\cos\Omega t$$

a) Investigate the transition from periodic to chaotic behavior by constructing a Poincare map of the system for several values of $\varepsilon$. (one way of doing this is to run the simulation several times with random initial conditions).

b) Study the sensitivity of the dynamics to the initial conditions. In particular, consider simultaneous motion of two or more identical systems which are originally very close. Follow the evolution of their relative "distance" (in phase space). Compare results for chaotic and periodic orbits. In the chaotic case, estimate the Lyapunov exponents, as functions of $\varepsilon$.

c) After running the simulation for certain time $T$, run the system "backward in time" over the same time interval $T$. Similarly to part (b) follow the evolution of the "distance" between two points in phase space. Perform this study both in periodic and chaotic regimes. In the latter case, investigate the dependence of the behavior on the choice of $T$. Interpret your results.